The 1998 Physics Nobel Prize

Electrons Behave as if Split into Three!

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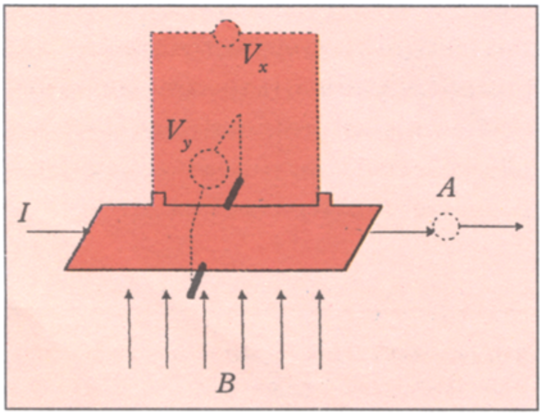
The 1998 Nobel Prize for Physics was awarded to D C Tsui, H L Störmer, and R B Laughlin, all from the USA for the experimental discovery and theoretical understanding of radically new behaviour in a layer of electrons confined by a strong magnetic field at low temperatures. We first go over some background, starting with the work of E Hall in 1879. A magnetic field B was applied normal to a rectangular gold plate, carrying current I along its length *Figure* 1. A 'Hall voltage' V_H was detected across the width of the plate. The simple explanation is that the charge carriers (say electrons) feel a sideways force due to B, but are unable to flow in a circuit in the transverse direction. They therefore accumulate at one edge and build up a potential V_H which cancels the magnetic force $qv_{||} B$. We thus are able to learn the sign of the charge carriers and the velocity with which they move. Notice that for a given current, the velocity of each carrier is *inversely* proportional to the number available per unit area in our layer (*Box* 1). Of course, in Hall's experiments, this number was determined by the properties of the material which were scarcely affected by the weak magnetic fields applied.

Figure 1. Schematic illustration of the Hall effect.



In the late seventies, Aoki and Ando in Japan realised that the situation could become very different at high magnetic fields and low temperatures in the kind of semiconductor layers used in integrated circuits. The classical orbits are circles. Quantum mechanics replaces the orbit by a wave function. For the lowest state, this wave function gets squeezed into a smaller and smaller area as the field is increased. Since each electron needs its own state, the total number n which can be accomodated in the lowest state goes up proportionally to the magnetic field B, as explained in more detail in *Box* 1.

We have just seen that the ratio $R_H = V_H/I$ (called 'Hall resistance') is proportional to B/n_e. Thus, the expectation was that B would cancel and the Hall resistance would have a universal value $h/e^2 = 25{,}813\ \Omega$. At that time, this result was regarded as a rough approximation. Crystal structure, impurities, and the Coulomb repulsion between the electrons were all neglected in the simple model. But very careful measurements by von Klitzing and colleagues in Germany in 1980 showed

Box 1.

Let the sample have a unit width, and n particles of charge q per unit area. The current I along the length (longitudinal current) is nqv, where v is the velocity. The Lorentz force qvB has to be balanced by the Hall electric force qV_H (for unit width) in the transverse direction. Thus $V_H = vB = IB/nq$, $R_H = VH/I = B/nq$. Thus Hall resistance is proportional to B/n and sensitive to the sign of q as stated in the text.

To understand the B/n ratio for a two dimensional electron gas, we recall that a classical charge q in a field B moving at a speed v describes a circle of radius r, with $mv^2/r = qBv$. The angular frequency $\omega_c = v/r = qB/m$.

The Russian physicist Landau showed that when we apply quantum mechanics to this problem, we get energy levels equally spaced by $\hbar\,\omega_c$, with the lowest at $1/2\,\hbar\,\omega_c$. The lowest Landau level corresponds to a classical orbit of energy $1/2\,\hbar\,\omega_c = mv^2/2 = m\omega_c^2\,r^2/2$, hence $r^2 = \hbar/m\omega_c = h/2\pi qB$.

We thus see that the area occupied by each orbit is inversely proportional to B. The precise result from Landau's treatment is that the maximum number of electrons which can be accomodated in the lowest level in a unit area = hB/q. One should remember that in the lowest energy state, the electron spin magnetic moment points parallel to the field and the other spin state is higher in energy.

precise integer sub-multiples of this value. The integer was clearly the number of levels filled, but the precision was a great surprise, now exploited all over the world to maintain standards and establish units. Klitzing received the Nobel Prize for Physics in 1985, for discovering this 'Integer Quantum Hall Effect'. The modern theoretical understanding of the IQH is due to Laughlin. His reasoning was based on a beautiful symmetry argument which is however too advanced to describe here. It uses a principle called 'gauge invariance'.

Meanwhile, A C Gossard at Bell Laboratories prepared some of the best samples of another semiconductor, Ga As–GaAl As, in which electrons could be confined to two space dimensions and cooled to very low temperatures in a high magnetic field. D C Tsui and H L Störmer carried out experiments on the Hall effect. They were actually looking for a state of matter conjectured to exist by E P Wigner in 1937, viz electrons avoiding each other and forming a crystalline arrangement. They did not find this. In the course of their experiments, they did find the integer Hall effect but also (for a lower magnetic field) a Hall resistance R_H three times larger. It was as if there was a new, stable configuration when the lowest level was just 1/3 filled. Subsequently, other fractions mainly with odd denominators were found (see *Figure* 2 for a modern data set). Again, Laughlin was first off the mark with a theoretical explanation. An essential point in his work is that the Coulomb repulsion between the electrons plays a vital role. Normally, when we describe electrons in solids, we can think of each as a wave in three

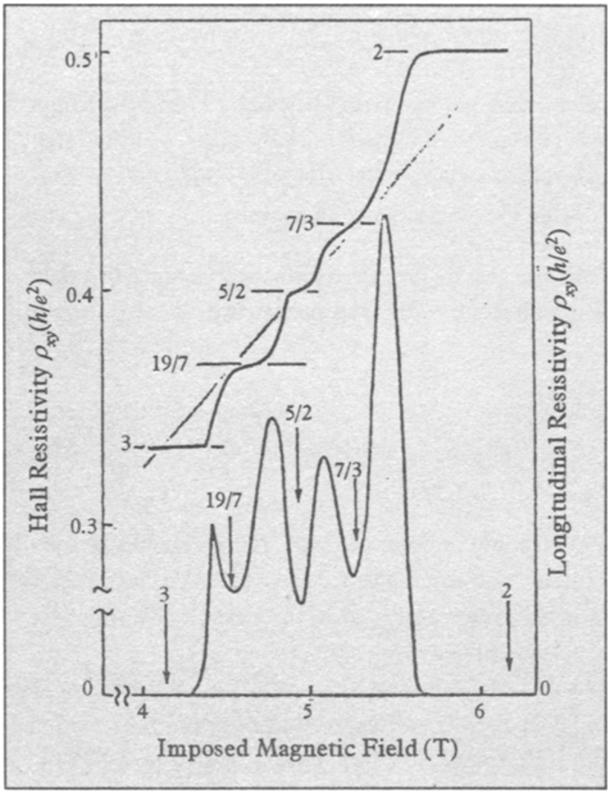


Figure 2. Emergence of the FQH state n= 5/2 at 25 mK. The Hall conductivity plateau is seen at 5/2 e^2/h and a pronounced dip in the longitudinal resistivity is also seen at the same magnetic field.
(Courtesy R Srinivasan).

dimensional space, seeing only the average effect of all the others. But in this special situation (FQH), one cannot speak of the wavelike behaviour of one electron without reference to that of the others. One speaks of a 'correlation' between the electrons. Laughlin's inspired guess about the nature of this correlation (which is too mathematical to describe at the level of this article) was confirmed by later work. In particular, he showed that the correlations would create excited states in which the total charge in a localised region was a fraction (like 1/3) of the electronic charge. (The total charge is of course an integer, the difference residing at the boundaries.) This was tested in Israel and France in 1997. Because the electron is the basic unit of charge, even a dc current has fluctuations superposed on it known as 'shot noise'. This is similar to the noise emitted by individual raindrops falling on a roof. The new experiments showed under very special conditions that the FQH state produces shot noise which can be attributed to fractional charge of $e/3$! This strange result was expected from the theory.

Interestingly, the chemistry Nobel Prize for 1998 recognized work on the *quantitative* consequences of electrons being correlated in atoms, molecules, and solids. In the same year, the Physics prize honours work revealing *qualitatively new* behaviour emerging from correlated electron motion. A good earlier example is the phenomenon of superconductivity also earning Nobel Prizes both for the experimental discovery (Kammerlingh Onnes, 1913) and the theory half a century later (Bardeen, Cooper, Schrieffer, 1972).

Acknowledgements

I thank G Baskaran and R Srinivasan for sharing with me drafts of longer articles they have written on the same subject and letting me use material from them.

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