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Phenomenology of the proton spin

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Abstract. We analyze the various inputs that go into computing the recently measured first moment of the proton spin structure function $g_1^p(x)$. The basic inputs are the various valence and sea quark polarisations and the gluonic contribution coming through axial anomaly. We show that the quark model predictions for valence quark polarisations, suitably modified to accommodate Bjorken sum rule, are consistent with measured value of moment of the spin structure function.

Keywords. Structure functions; polarization; sum rules; quantum chromodynamics.

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1. Introduction

The European Muon Collaboration (Ashman *et al* 1988) has recently measured the asymmetry in polarized muon deep inelastic scattering on polarized protons. The measured asymmetry together with the data on unpolarized proton structure function directly leads to the determination of the spin structure function of the proton $g_1^p(x)$. We are interested in the first moment of this structure function,

$$G_p = \int \mathrm{d}x g_1^p(x) = 0{\cdot}114 \pm 0{\cdot}012\,(\text{stat.}) \pm 0{\cdot}026\ (\text{syst.}), \tag{1}$$

where x is the Bjorken scaling variable. This result is in disagreement with the Ellis-Jaffe sum rule (Ellis and Jaffe 1974) which predicts,

$$G_p = \frac{1}{12} G_A \left[1 + \frac{5}{3}\frac{3F - D}{F + D} \right] = 0{\cdot}2 \pm 0{\cdot}005, \tag{2}$$

where $G_A = 1{\cdot}254 \pm 0{\cdot}006$ is the neutron β decay coupling constant and F and D are SU(3) couplings, $F/D = 0{\cdot}632 \pm 0{\cdot}024$ (Gaillard and Sauvage 1984). Much of the controversy generated regarding the spin structure of the proton has its origin in this discrepancy (see for example, Soffer 1989; Jaffe and Manohar 1989).

Bjorken's (1966) analysis relates the first moment G_p to a weighted sum of axial vector matrix elements of various quark flavours $(u, d, s, \ldots)$ between proton states,

$$G_p = \tfrac{1}{18}[4\Delta u + \Delta d + \Delta s], \tag{3}$$

where

$$\Delta q = -S^\mu \langle P, S | \bar{q}\gamma_\mu\gamma_5 q | P, S \rangle. \tag{4}$$

Here q is any quark flavour and S^{μ} is the spin vector of the proton. In general G_p gets contribution from both flavour singlet as well as non-singlet current matrix elements. The flavour non-singlet part of G_p can be related to,

$$G_A = \Delta u - \Delta d = F + D \tag{5}$$

$$G_8 = \Delta u + \Delta d - 2\Delta s = 3F - D. \tag{6}$$

The Ellis-Jaffe sum rule is obtained by setting $\Delta s = 0$ in which case G_A, F/D values are sufficient to determine G_p. On the other hand using the measured values of G_A, F/D and G_p one obtains,

$$\Delta u = 0{\cdot}74 \pm 0{\cdot}08, \quad \Delta d = -0{\cdot}51 \pm 0{\cdot}08, \quad \Delta s = -0{\cdot}23 \pm 0{\cdot}08. \tag{7}$$

As a result $\Sigma = \sum \Delta q_i \cong 0$, $i = u, d, s$, which has aroused much interest in the high energy physics community. A naive interpretation of this would be that quarks together carry very little of the proton spin. An even more surprising result is that the strange quark contribution is substantial which could imply large violation of the OZI rule (Ellis *et al* 1988). At the outset these conclusions rebel against well known results from the quark models based on the $SU(3)_f$ flavour symmetry where quarks are expected to carry nearly 75 per cent of the proton polarization.

In a recent paper by Gupta *et al* (1989), it has been argued that the non-zero value of Δs need not be in contradiction with known hadronic physics. The matrix elements in (3) and (4) are essentially non-perturbative and can be computed using the method of QCD sum rules. The method yields,

$$\Delta u = 0{\cdot}8, \quad \Delta d = -0{\cdot}45, \quad \Delta s = -0{\cdot}165, \tag{8}$$

within errors consistent with (7). With QCD radiative corrections these values yield G_p in good agreement with EMC measurements. It was also noted in the above analysis that the large non-zero value of Δs could be a result of the role of axial anomaly in singlet axial vector matrix elements. This, for example, contributes through the use of phenomenological values of chiral symmetry breaking parameter $\langle q\bar{q} \rangle$ which arises from anomaly in instanton based models of QCD vacuum. It is also worth pointing out that the importance of such an anomaly in preventing large isospin violations in Bjorken sum rule (eq. (5)) was first pointed out by Gross *et al* (1979)). In a series of papers recently, Altarelli and Ross (1988), Efremov and Teryaev (1988) and Carlitz *et al* (1988) have also emphasized the role of axial anomaly in understanding the spin structure of the proton. These authors explicitly show how gluons contribute to the first moment via the axial anomaly. A similar analysis through the QCD evolution equations had been carried out earlier by Bajpai and Ramachandran (1980) who overlooked the contribution of the anomaly but predicted large sea and gluon polarization.

The objective of the present paper is to carry out a consistent phenomenological analysis in the light of above investigations. This we try to achieve by separately analyzing the various singlet, non-singlet and gluonic contributions to the axial matrix element. The main result is that predictions from the quark model wave functions for the valence quark polarizations can be consistent with the EMC measurement.

2. The model

Following Altarelli and Ross (1988) (see also Efremov and Teryaev 1988 and Carlitz *et al* 1988), we write,

$$\Delta u = \Delta u_v + \Delta u_\sigma + \Delta G, \tag{9}$$

$$\Delta d = \Delta d_v + \Delta d_\sigma + \Delta G, \tag{10}$$

$$\Delta s = \Delta s_\sigma + \Delta G, \tag{11}$$

where we have separately identified the contributions from valence (v) and sea quarks (σ). It has been argued in the above references that the gluonic contribution

$$\Delta G = -\frac{\alpha_s}{2\pi}\Delta g, \quad \Delta g = \int dx[g_+(x) - g_-(x)], \tag{12}$$

where Δg the moment of the spin dependent gluon density, arises through the Adler-Bell-Jackiw axial vector anomaly. However there is some disagreement (Jaffe and Manohar 1989) as to whether this gluonic contribution is calculable using perturbative QCD. For our purposes we treat ΔG as a phenomenological parameter which measures the gluonic contribution to the axial matrix elements.

As advertised our objective is to use the theoretically determined values of Δq_i (eq. (8)) or equivalently the experimentally determined values (eq. (7)) and check the consistency of various inputs Δq_v, Δq_σ and ΔG with known physics. To do this we first note that we have no information on ΔG, it is a phenomenological input to the flavour singlet part of Δq. Similarly we have no information of the sea quark spin densities. However proceeding in analogy with the unpolarized case we set,

$$\Delta u_\sigma = \Delta d_\sigma = \frac{1}{k}\Delta s_\sigma = \Delta\sigma. \tag{13}$$

The equivalence of u, d quark contributions avoids any isospin violation in the Bjorken sum rule for G_A. For a SU(3) symmetric sea $k=1$. However it is well known that in the unpolarized case the strange sea distribution is about 1/2 of the u, d, distributions (Abramowicz 1983). The factor k takes care of the break down of the SU(3) symmetry or equivalently the quark mass effects.

Next we consider the valence quark contribution. Using the $SU(3)_{\text{flavour}} \times SU(2)_{\text{spin}}$ wave function of the proton we obtain (Mani and Noman 1981),

$$\tilde{u}_v(x) = \tfrac{2}{3} u_v(x) \tag{14}$$

$$\tilde{d}_v(x) = -\tfrac{1}{3} d_v(x) \tag{15}$$

where $\tilde{q}_v(x)$ ($q_v(x)$) is the spin dependent (spin independent) quark density and the weights on the right hand side are obtained by constructing the probabilities for u or d quarks to have spin parallel or anti parallel to the proton spin (Close 1979). With these definitions,

$$\Delta u_v = \int dx \tilde{u}_v(x) = 4/3$$

$$\Delta d_v = \int dx \tilde{d}_v(x) = -1/3$$

and through Bjorken sum rule we have

$$G_A = \Delta u - \Delta d = \Delta u_v - \Delta d_v = 5/3$$

which is the standard SU(3) value in disagreement with the experimental value ($\cong 5/4$).

To avoid this problem we use the Carlitz-Kaur model (Carlitz and Kaur 1976; Kaur 1977) where the valence quark spin densities are diluted due to their interaction with the gluons and the sea. As a result,

$$\begin{aligned} \tilde{u}_v(x) &= (u_v(x) - \tfrac{2}{3}d_v(x))\cos 2\theta \\ \tilde{d}_v(x) &= -\tfrac{1}{3}d_v(x)\cos 2\theta \end{aligned} \tag{16}$$

where $\cos 2\theta$, a function of Bjorken x, is the spin dilution factor. A parametrization of $\cos 2\theta$ is then obtained using Regge arguments at small x. The large x ($x \to 1$) behaviour of $\tilde{u}_v(x)$ and $\tilde{d}_v(x)$ should be similar to $u_v(x)$ and $d_v(x)$ due to the constraint

$$|\tilde{u}_v(x)/u_v(x)|, \quad |\tilde{d}_v(x)/d_v(x)| \leqslant 1.$$

The effect of the spin dilution factor will therefore be seen only at small x and $\cos 2\theta \to 1$ as $x \to 1$. Later we demonstrate that even a simple model in which

$$\begin{aligned} \tilde{u}_v(x) &= \tfrac{2}{3}u_v(x)\cos 2\theta \\ \tilde{d}_v(x) &= -\tfrac{1}{3}d_v(x)\cos 2\theta \end{aligned} \tag{17}$$

yields results identical to the Carlitz–Kaur model.

In the next section we consider the phenomenological implications of the various inputs.

3. Phenomenology

First we compute the valence quark contribution to Δq. Recently Gupta *et al* (1989) have given parametrizations of $u_v(x)$ and $d_v(x)$ and $\cos 2\theta$ and various other distributions which fit the $g_1^p(x)$ given by EMC (Ashman *et al* 1988). We concentrate on the valence and $\cos 2\theta$ distributions given by them. The two sets of distributions given by them are,

Set I: ($Q_0^2 = 5\,\mathrm{GeV}^2$, $\Lambda_{\mathrm{QCD}} = 200\,\mathrm{MeV}$)

$$\begin{aligned} xu_v(x) &= 1{\cdot}78x^{0{\cdot}5}(1 - x^{1{\cdot}51})^{3{\cdot}5} \\ xd_v(x) &= 0{\cdot}67x^{0{\cdot}4}(1 - x^{1{\cdot}51})^{4{\cdot}5} \\ \cos 2\theta &= [1 + 0{\cdot}258\ (1 - x)^{1/2}/x^{0{\cdot}1}]^{-1} \end{aligned} \tag{18}$$

Set II: ($Q_0^2 = 15\,\mathrm{GeV}^2$, $\Lambda_{\mathrm{QCD}} = 90\,\mathrm{MeV}$)

$$\begin{aligned} xu_v(x) &= 2{\cdot}75x^{0{\cdot}588}(1 - x)^{2{\cdot}69} \\ xd_v(x) &= 8{\cdot}53x^{1{\cdot}03}(1 - x)^{6{\cdot}87} \\ \cos 2\theta &= [1 + 0{\cdot}2656(1 - x^{1/2}/x^{0{\cdot}1}]^{-1} \end{aligned} \tag{19}$$

In addition we also consider a third set, with $u_v(x)$ and $d_v(x)$ as given by Diemoz *et al* (1988) (see also Gluck *et al* 1988).

$$\begin{aligned} &\text{Set III: } (Q_0^2 = 10\,\mathrm{GeV}^2,\ \Lambda_{\mathrm{QCD}} = 200\,\mathrm{MeV}) \\ &xu_v(x) = 2{\cdot}26x^{0{\cdot}54}(1-x)^{2{\cdot}52}[1 - 1{\cdot}617(1-x) \\ &\quad + 3{\cdot}647(1-x)^2 - 1{\cdot}998\,(1-x)^3] \\ &xd_v(x) = 0{\cdot}57(1-x)xu_v(x). \end{aligned} \tag{20}$$

The corresponding $\cos 2\theta$ for set three is taken to be

$$\cos 2\theta = [1 + \eta(1-x)^2/\sqrt{x}]^{-1}, \quad \eta = 0{\cdot}078.$$

a form suggested by Carlitz and Kaur (1976) (see also Kaur (1977)) based on Regge arguments. The moments Δu_v and Δd_v calculated based on the above parametrizations are given in table 1. It is easily seen that the moments are not only independent of the various parametrizations but are insensitive to the details of the model, Carlitz-Kaur or the modified SU(3) model of eqs (17) on the other. Notice that if $\cos 2\theta = 1$, Δu_v and Δd_v given by either of these models are identical and collapse to the naive SU(3) values. The effect of spin dilution factor is therefore to reduce G_A from 5/3 of SU(3) to approximately 5/4 closer to the experimental value.

An important point that emergies from table 1 is that $\Delta u_v/\Delta d_v = -4$ which is exactly the ratio predicted by the naive SU(3). This should not be surprising since we know that the SU(3) predictions for ratios elsewhere (like magnetic moment of baryons, the F/D ratio, etc.,) have been found to be accurate. In fact if the ratios of u and d matrix elements were to be renormalized differently by the effect of sea quarks and gluons such that the ratios differed from SU(3) predictions, it would imply catastrophic isospin violations not found elsewhere. We can therefore safely choose, based on the analysis above,

$$\Delta u_v \cong 1, \quad \Delta d_v \cong -0{\cdot}25. \tag{21}$$

Similar numbers have been obtained by Fritzsch (1988) where the reduction of G_A from 5/3 to 5/4 has been attributed to the effect of sea quarks and gluons. These

Table 1. First moment of valence quark spin distribution functions

Input		Carlitz-Kaur	SU(3) modified
Set I	Δu_v	1·0	1·0
	Δd_v	−0·25	−0·25
Set II	Δu_v	1·013	1·003
	Δd_v	−0·245	−0·245
Set III	Δu_v	1·004	0·99
	Δd_v	−0·24	−0·24

Modified $SU(3)$ distributions of (17) with spin dilution factor.

however contribute differently to u and d quarks implying isospin violation, which we have avoided above.

Next we consider the sea quark contribution constrained according to (13). In the simplest case we can take the OZI rule to be exact for axial vector current matrix elements and set $\Delta s_\sigma = 0$. This would imply either $k=0$ or $\Delta\sigma = 0$. The former is unlikely since for the unpolarized distributions k is approximately 1/2 from CDHS parametrizations (Abramowicz *et al* 1983). If the latter is taken to be correct, then the non-zero value of Δs arises entirely from gluons. Using the theoretical values given in (8), and using (9–11), we find

$$\Delta G \cong -0{\cdot}165, \quad \Delta u_v \cong 0{\cdot}965, \quad \Delta d_v \cong -0{\cdot}285. \tag{22}$$

The valence quark contribution is not far off from (21). On the other hand if we take $\Delta s_\sigma \neq 0$ and using the valence quark contribution from (21) as input in (9–11), we obtain,

$$\begin{aligned} \Delta u &= 1 + \Delta\sigma + \Delta G, \\ \Delta d &= -0{\cdot}25 + \Delta\sigma + \Delta G, \\ \Delta s &= k\Delta\sigma + \Delta G, \end{aligned} \tag{23}$$

where $0 \leqslant k \leqslant 1$. The case $k=1$ corresponds to SU(3) symmetric sea which yields $G_A = 1{\cdot}25$ and $G_8 = \Delta u + \Delta d - 2\Delta s = 0{\cdot}75$. Experimentally the value of G_B is close to 0·685 using the measured value of F/D (Gaillard and Sauvage 1984). For $k<1$, we obtain

$$G_8 = 0{\cdot}75 + 2\Delta\sigma(1-k) = 0{\cdot}685. \tag{24}$$

For $k \cong 1/2$ as in the unpolarized case $\Delta\sigma \cong -0{\cdot}07$ and $\Delta s_\sigma \cong -0{\cdot}035$ which is consistent with the Regge bound (Anselmino *et al* 1988) on the strange quark polarization,

$$\|\Delta s_\sigma\| \leqslant 0{\cdot}072 \pm 0{\cdot}030. \tag{25}$$

A slightly different bound $\|\Delta s_\sigma\| \leqslant 0{\cdot}052^{+0{\cdot}023}_{-0{\cdot}052}$ has been quoted by Soffer (1989). The value of Δs_σ is consistent with either of these two bounds. The corresponding value of $\Delta G \cong -0{\cdot}13$ is slightly different from the $SU(3)$ symmetric case (eq. (22)).

In both cases discussed above the sign of the gluon contribution turns out to be negative. Following Altarelli and Ross (1988), if $\Delta G = -\alpha_s \Delta g/2\pi$, then the gluon polarization is in turn positive. Recently Cheng and Li (1989) have challenged the sign of Δg as obtained. Using the anomalous divergence equations to constrain the size of ΔG, they claim that the resulting sign of ΔG is positive; in fact,

$$\Delta G = 0{\cdot}16 \pm 0{\cdot}08. \tag{26}$$

They also conclude from this that it spells trouble for naive quark model. We show below that if any, the positive value of ΔG results in a large violation of the bound on the strange quark polarization, eq (25). Substituting the mean value of ΔG in (23) we find, without affecting the values of Δu_v and Δd_v,

$$\Delta\sigma = -0{\cdot}36, \quad \Delta s_\sigma = -0{\cdot}325, \tag{27}$$

or analogously $k \cong 0{\cdot}9$ which is very close to the SU(3) symmetric case with $k=1$. However such large strange sea polarization violates the Regge bound but is still consistent with quark model estimates.

4. Conclusions

We have recalculated the valence quark contribution Δu_v and Δd_v to the axial vector matrix elements of u and d quarks Δu and Δd consistent with the Bjorken sum rule. The ratio $\Delta u_v/\Delta d_v = -4$ obeys the SU(3) prediction. Keeping this as the input to the first moment of the proton spin structure function, we have reanalyzed the phenomenology of proton spin. We find that the gluonic contribution ΔG to be negative, as claimed by Altarelli and Ross (1988), in order to satisfy the Regge bound on the strange quark polarization. On the other hand Cheng and Li (1989) find ΔG to be positive which result they claim is just the wrong sign to accommodate naive quark model predictions. We find, however the quark model predictions for valence Δu_v and Δd_v can be accommodated with positive ΔG while violating the bound on strange quark polarisation.

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References

Ashman J *et al* [EMC] 1988 *Phys. Lett.* **B206** 364
Abramowicz H *et al* 1983 *Z. Phys. C* **17** 283
Altarelli G and Ross G G 1988 *Phys. Lett.* **B212** 391
Anselmino M, Ioffe B L and Leader E 1988 *Santa Barbara Preprint* NSF-ITP-88-94
Bajpai R P and Ramachandran R 1980 *Phys. Lett.* **B97** 125
Bjorken J D 1966 *Phys. Rev.* **148** 1467
Carlitz R D, Collins J and Mueller A H 1988 *Phys. Lett.* **B214** 229
Carlitz R D and Kaur J 1977 *Phys. Rev. Lett.* **38** 673
Cheng T P and Li L F 1989 *Phys. Rev. Lett.* **62** 1441
Close F E 1979 *An introduction to quarks and partons.* (London: Academic Press) p. 294
Diemoz M, Ferroni F, Longo E and Martinelli G 1988 *Z. Phys.* **C39** 21
Efremov A V and Teryaev D V 1988 *Dubna Preprint*-JINR EZ-88-287
Ellis J and Jaffe R L 1974 *Phys. Rev.* **D9** 1444, Erratum **D10** 1669
Ellis J, Gabathuler E and Karliner M 1988 *Preprint CERN*-TH-5188
Fritzsch H 1988 *Preprint MPI*-PAE/pTh-45/88
Gaillard J M and Sauvage G 1984 *Ann. Rev. Nucl. Part. Sci.* **34** 351
Gluck M, Godbole R and Reya E 1988 *Preprint* DO-Th-88/18
Gross D J, Treiman S B and Wilczek F 1979 *Phys. Rev.* **D19** 2188
Gupta S, Murthy M V N and Pasupathy J 1989 *Phys. Rev.* **D39** 2547
Gupta S, Pasupathy J and Szwed 1989 *Preprint* Un. Bielefeld
Jaffe R L and Manohar A 1989 *Preprint* CTP-1706
Kaur J 1977 *Nucl. Phys.* **B128** 219
Mani H S and Noman M 1981 *Phys. Rev.* **D24** 1223
Soffer J 1989 *Preprint* CPT-89/P.2220