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Theory of Stability of Tensor Operators under Perturbations and its Application to Particle Physics*

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We investigate the perturbations of tensor operators due to symmetry breaking and consequent representation mixing. A group-theoretical stability principle, valid for an arbitrary (simple, compact) group is formulated, which in many cases assures the vanishing of the first-order perturbation when it is constrained to leave a certain component unaltered. The physically interesting case of unitary symmetry is discussed in detail. All previously known results are recovered and several new results are deduced. As an application we discuss the conditions under which the universality of the Cabibbo angles for leptonic decays is valid.

INTRODUCTION

HIGHER symmetries of strong interactions that have been proposed in recent years have been remarkably successful in the organization of the data on particles and resonances. But they all share the property of being broken appreciably, either by virute of interactions of lesser strength which violate these symmetries or by virtue of some other mechanism. In the case of unitary symmetry and the spin-dependent symmetries these symmetry violations as manifested by the observed mass differences are appreciable. Nevertheless, it is remarkable that a considerable remnant of the symmetry survives in the observable features like supermultiplets and mass and coupling constant sum rules. Thus, for example, the identification of the sources of electromagnetic and weak interactions to be octet currents seems to be in quantitative agreement with experiment in spite of the large representation mixing expected in view of the departures from unitary symmetry. We should therefore search for a

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spectroscopic principle leading to the stability of the tensor type of dynamical variables.

To lowest order in the perturbation such a stability principle can be identified for those dynamical variables which are the generators of the group for an arbitrary type of perturbation. For this purpose consider a (simple compact) symmetry group G and a set of states $|\psi_\alpha\rangle$ which constitute, in the symmetric limit, an irreducible representation A of the group G. Let X_n be the generators of the group G. As soon as symmetry violations are considered the states $|\psi_\alpha\rangle$ will no longer belong to a pure representation of G but will contain admixtures of several irreducible representations of G, not all necessarily distinct. We may write an expansion, valid to first order in the perturbation parameter ϵ in the form

$$|\psi_\alpha\rangle = |\psi_\alpha{}^0\rangle + \epsilon \sum_{B,\beta,A',\alpha',a} C_\alpha{}^A{}_\beta{}^B{}_{\alpha'}{}^{A',a} |\chi_{\alpha'}\rangle + O(\epsilon^2).$$

The normalization of the unperturbed state is unchanged to lowest order in ϵ. We have considered the perturbation expanded in terms of tensors of type B and component β. The states $|\chi\rangle$ belong to the representation A' and the Clebsch-Gordan coefficient allows for multiple occurrence of the representation A'. We now observe that the generators (conserved charges) X_n have their matrix elements unaltered to first order in ϵ, since X_n do not connect $|\psi_\alpha{}^0\rangle$ with $|\chi_{\alpha'}\rangle$. This result is true for any kind of perturbation of the symmetry but it holds only for the conserved currents. If the vector current of weak interaction is identified as the conserved octet current of unitary symmetry it follows that the low-momentum transfer limit of the matrix element of these vector currents are independent of the perturbation to lowest order in the perturbation.[1,2]

While this result is interesting it does not furnish any explanation whatever of the corresponding behavior of nonconserved currents, say the axial vector current of weak interactions, or the electromagnetic decimet-octet matrix element. Similar questions arise in connection with the predictions of the $SU(6)$ theory for baryon magnetic moments, etc. We are therefore led to an investigation of stability of tensor operators perturbed by suitable perturbations even in the presence of representation mixing. As long as the tensor operator is not a generator, a straightforward calculation yields terms of first degree in ϵ coming from the transition matrix elements between the representations A and A'. Hence further constraints must be valid if these terms are to vanish. This constraint may be imposed in terms of the behavior of some components of the tensor operators. In the following sections we carry out such an investigation with particular attention to perturbations of unitary symmetry.

[1] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964).
[2] C. Bouchiat and Ph. Mayer, Nuovo Cimento 34, 1122 (1964).

I. PERTURBATIONS OF $SU(3)$

Octet Perturbations of Octet Operators

Let $J_\beta{}^\alpha$ be an octet of operators representing a suitable set of dynamical variables, say the density of the axial-vector weak interactions. In the absence of violations of unitary symmetry these eight components transform as the eight components of the adjoint representation of the group $SU(3)$. The second-rank mixed tensor $J_\beta{}^\alpha$ is traceless

$$J_\nu{}^\nu = 0. \tag{1.1}$$

In the presence of symmetry violation the operator $J_\beta{}^\alpha$ will still satisfy the tracelessness restriction, but will not any longer transform as an octet. To lowest order in the perturbation we may write

$$J_\beta{}^\alpha = T_\beta{}^\alpha + \epsilon T_\beta{}^\alpha{}_3{}^3 + O(\epsilon^2), \tag{1.2}$$

where we have assumed the perturbation to transform like the $I=Y=0$ component of an octet. We may include a unitary singlet perturbation also; such a perturbation will not alter the tensor character of $T_\beta{}^\alpha$ but can change its intrinsic character. For example, the D/F ratio of the octet could be changed. If we now consider the matrix element of the operators $J_\beta{}^\alpha$ between two octets of baryons represented as the traceless tensors $B_\beta{}^\alpha$ the general expression for the matrix element of $T_\beta{}^\alpha$ is

$$\xi(B^\dagger B)_\beta{}^\alpha + \eta B^\dagger{}_\beta{}^\nu B_\nu{}^\alpha,$$

where ξ, η are parameters. We denote by $(B^\dagger B)_\beta{}^\alpha$ the expression

$$\tfrac{1}{2}(i\gamma^0\gamma^5\gamma^\mu)_{bc}\{(B^\dagger{}_\nu{}^\alpha)_b, (B_\beta{}^\nu)_c\}$$

for the axial-vector current density. For the baryon matrix elements $\Delta_\beta{}^\alpha$ of the term $\epsilon T_\beta{}^\alpha{}_y{}^x$ we may write

$$\begin{aligned}\Delta_\beta{}^\alpha = {} & a_1(B^\dagger B)_y{}^\alpha \delta_\beta{}^x + a_2(B^\dagger B)_\beta{}^x \delta_y{}^\alpha + b_1 B^\dagger{}_\beta{}^\nu B_\nu{}^x \delta_y{}^\alpha \\ & + b_2 B^\dagger{}_y{}^\nu B_\nu{}^\alpha \delta_\beta{}^x + c B^\dagger{}_\beta{}^x B_y{}^\alpha + d B^\dagger{}_y{}^\alpha B_\beta{}^x \\ & + e(B^\dagger B)_\beta{}^\alpha \delta_y{}^x + f B^\dagger{}_\beta{}^\nu B_\nu{}^\alpha \delta_y{}^x + j(B^\dagger B)_\nu{}^\nu \delta_y{}^\alpha \delta_\beta{}^x \\ & + g(B^\dagger B)_\nu{}^\nu \delta_y{}^x \delta_\beta{}^\alpha + h_1 B^\dagger{}_\beta{}^\alpha B_y{}^x + h_2 B^\dagger{}_y{}^x B_\beta{}^\alpha \\ & + k(B^\dagger B)_y{}^x \delta_\beta{}^\alpha.\end{aligned} \tag{1.3}$$

The tracelessness of $\Delta_\beta{}^\alpha$ gives the constraints

$$\begin{aligned} a_1+a_2+c+3k &= 0, \\ b_1+b_2+d &= 0, \\ j+3g+e+f &= 0. \end{aligned} \tag{1.4}$$

We now require that the $\alpha=\beta=1$ component of $J_\beta{}^\alpha$ is unchanged to first order in the perturbation parameter ϵ. This component is the electric-charge component of the operator; for a conserved vector current it is the density of electric charge and its matrix element in the zero-momentum limit is strictly unchanged. For the general case of a tensor operator, say the axial-vector current density, this requirement of $\Delta_1{}^1$ vanishing to lowest order in ϵ is *a dynamical postulate* which has to be justified. If we impose this requirement, we get

$$cB^{\dagger}{}_1{}^3B_3{}^1+dB^{\dagger}{}_3{}^1B_1{}^3+g(B^{\dagger}B)_\nu{}^\nu+h_1B^{\dagger}{}_1{}^1B_3{}^3+h_2B^{\dagger}{}_3{}^3B_1{}^1 +e(B^{\dagger}B)_1{}^1+fB^{\dagger}{}_1{}^\nu B_\nu{}^1+k(B^{\dagger}B)_3{}^3=0.$$

This entails the constraints

$$c=d=e=f=g=h_1=h_2=k=0. \quad (1.5)$$

[It is worth pointing out that the perturbation considered is a mixture of singlet and octet perturbations. If the perturbation were pure octet type, it would be traceless in x, y and this would imply the constraints

$$a_1+a_2+d+3e=0,$$
$$b_1+b_2+c+3f=0,$$
$$j+3g+k=0.]$$

There exists a two-parameter family of perturbations which satisfy the constraints

$$a_1+a_2=b_1+b_2=c=d=e=f=g=j=h_1=h_2=0$$

of the form

$$a'\{(B^{\dagger}B)_y{}^\alpha\delta_\beta{}^x-(B^{\dagger}B)_\beta{}^x\delta_y{}^\alpha\} +b'\{B^{\dagger}{}_\beta{}^\nu B_\nu{}^x\delta_y{}^\alpha-B^{\dagger}{}_y{}^\nu B_\nu{}^\alpha\delta_\beta{}^x\}. \quad (1.6)$$

This term, however, has the opposite charge-conjugation invariance property (second class) from the unperturbed terms. [The terms with the opposite charge-conjugation property are got by putting

$$a_1+a_2=b_1+b_2=h_1+h_2=c=d=e=f=g=j=k=0.$$

If the electric interaction has no such contribution, $h_1=h_2=0$ and we get the two-parameter family given in (1.6).] Hence, if the perturbation is assumed to be charge-conjugation invariant, we may put $a'=b'=0$ so that

$$\Delta_\beta{}^\alpha=0. \quad (1.7)$$

Hence if the $\alpha=\beta=1$ term of the current has matrix elements unchanged in *tensor* character from the corresponding unperturbed matrix elements, the entire set of operators have their tensor character unchanged to first order in the perturbation.

Octet Perturbations to Second Order

We can carry out this calculation to second order in the perturbation. The correction term would now include a quantity of the type $\epsilon^2T_\beta{}^\alpha{}_3{}^3{}_3{}^3$. The new kinds of terms that have to be added to $\Delta_\beta{}^\alpha$ to obtain the expression correct to second order are of the form

$$p_1B^{\dagger}{}_3{}^3B_3{}^\alpha\delta_\beta{}^3+p_2B^{\dagger}{}_\beta{}^3B_3{}^3\delta_3{}^\alpha+q_1B^{\dagger}{}_3{}^\alpha B_3{}^3\delta_\beta{}^3 +q_2B^{\dagger}{}_3{}^3B_\beta{}^3\delta_3{}^\alpha+r(B^{\dagger}B)_3{}^3\delta_3{}^\alpha{}_\beta{}^3 +sB^{\dagger}{}_3{}^\nu B_\nu{}^3\delta_3{}^\alpha\delta_\beta{}^3+tB^{\dagger}{}_3{}^3B_3{}^3\delta_\beta{}^\alpha, \quad (1.8)$$

with the constraints

$$a_1+a_2+c+r+3k=0,$$
$$b_1+b_2+d+s=0,$$
$$p_1+p_2+q_1+q_2+3t=0, \quad (1.9)$$
$$c+d+e+f+j+3g=0,$$

following from tracelessness of $J_\beta{}^\alpha$. With a charge-conjugation invariant perturbation there would be the additional constraints

$$a_1-a_2=b_1-b_2=h_1-h_2=p_1-p_2=q_1-q_2=0. \quad (1.10)$$

If we required that the $\alpha=\beta=1$ component of $J_\beta{}^\alpha$ is unaltered we can deduce

$$c=d=e=f=g=h_1=h_2=k=t=0. \quad (1.11)$$

Hence the general perturbation of the octet $J_\beta{}^\alpha$ to second order in a charge-independent, charge-conjugation invariant perturbation is given by

$$\Delta_\beta{}^\alpha=a\{(B^{\dagger}B)_3{}^\alpha\delta_\beta{}^3+(B^{\dagger}B)_\beta{}^3\delta_3{}^\alpha-2(B^{\dagger}B)_3{}^3\delta_3{}^\alpha\delta_\beta{}^3\} +b\{B^{\dagger}{}_3{}^\nu B_\nu{}^\alpha\delta_\beta{}^3+B^{\dagger}{}_\beta{}^\nu B_\nu{}^3\delta_3{}^\alpha-2B^{\dagger}{}_3{}^\nu B_\nu{}^3\delta_3{}^\alpha\delta_\beta{}^3\} +p\{(B^{\dagger}{}_3{}^3B_3{}^\alpha-B^{\dagger}{}_3{}^\alpha B_3{}^3)\delta_\beta{}^3 +(B^{\dagger}{}_\beta{}^3B_3{}^3-B^{\dagger}{}_3{}^3B_\beta{}^3)\delta_3{}^\alpha\}. \quad (1.12)$$

In addition to the sum rules generated by charge independence, in this order there is one sum rule.

For second-class amplitudes there is a five-parameter family given by

$$a'\{(B^{\dagger}B)_3{}^\alpha\delta_\beta{}^3-(B^{\dagger}B)_\beta{}^3\delta_3{}^\alpha\}+b'\{B^{\dagger}{}_\beta{}^\nu B_\nu{}^3\delta_3{}^\alpha-B^{\dagger}{}_3{}^\nu B_\nu{}^\alpha\delta_\beta{}^3\} +h'\{B^{\dagger}{}_\beta{}^\alpha B_3{}^3-B^{\dagger}{}_3{}^3B_\beta{}^\alpha\}+p'\{B'{}_3{}^3B_3{}^\alpha\delta_\beta{}^3-B^{\dagger}{}_\beta{}^3B_3{}^3\delta_3{}^\alpha\} +q'\{B^{\dagger}{}_3{}^\alpha B_3{}^3\delta_\beta{}^3-B^{\dagger}{}_3{}^3B_\beta{}^3\delta_3{}^\alpha\}. \quad (1.13)$$

If, however, the electric interaction has no second-class contribution h' is restricted to be zero.

In third order of symmetry violation, in general only charge-independence restrictions are valid. The first-order perturbation results were obtained earlier by Ademollo and Gatto.[1] The sum rules involving only Σ and Λ hyperons that they obtain are consequences of charge independence only.

The second-order perturbation results have been derived by Zakharov and Kobzarev.[3] Both these groups of authors have restricted their consideration to vector currents which are conserved in the absence of perturbations. We have remarked in the previous section that in such a case the first-order results could be immediately obtained for an arbitrary kind of perturbation.[2] Our analysis above shows that the result is valid much more generally when the tensor operators are not generators of the group G. The essential point is the constraint imposed on the charge-like component. In a subsequent section we discuss the case of the axial-vector interaction.

[3] V. I. Zakharov and I. Yu. Kobzarev, Soviet J. Nucl. Phys. **1**, 749 (1965); K. Kawarabayashi and W. W. Wada, Phys. Rev. **137**, B1002 (1965). We thank Dr. S. Pakvasa for bringing these works to our attention.

The restriction to octet currents and octet-type symmetry violation studied here was made in view of the physical relevance of such a scheme. The same type of result would apply even if the scheme is suitably altered. In the next section we study two examples.

General Perturbation of $SU(3)$ Tensors

(i) 27-Type Perturbations of an Octet Tensor

Let $J_\beta{}^\alpha$ continue to denote a (traceless) octet of dynamical variables in the symmetric limit. Let us consider a perturbation of the unitary symmetry which behaves like the $I=Y=0$ component of a 27-dimensional representation. The 27-type tensor can be displayed as a fourth-rank mixed tensor symmetric in the upper indices, symmetric in the lower indices and traceless. The first-order perturbation correction can now be written in the form

$$\Delta_\beta{}^\alpha=\epsilon\{T_\beta{}^\alpha{}_3{}^3{}_3{}^3-\tfrac{4}{5}T_\beta{}^\alpha{}_\nu{}^\nu{}_3{}^3+\tfrac{1}{10}T_\beta{}^\alpha{}_\nu{}^\nu{}_\mu{}^\mu\}\,, \tag{1.14}$$

which we will denote by the symbolic expression

$$\Delta_\beta{}^\alpha=\epsilon T_\beta{}^\alpha{}_{\cdot}{}^{\cdot}{}_{\cdot}{}^{\cdot}.$$

The baryonic matrix elements have the general form

$$\begin{aligned}\Delta_\beta{}^\alpha=a_1B^\dagger{}_{\cdot}{}^\alpha B_{\cdot}{}^{\cdot}\delta_\beta{}^{\cdot}+a_2B^\dagger{}_{\cdot}{}^{\cdot}B_\beta{}^{\cdot}\delta_{\cdot}{}^\alpha+b_1B^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^\alpha\delta_\beta{}^{\cdot}\\+b_2B^\dagger{}_\beta{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_{\cdot}{}^\alpha+cB^\dagger{}_{\cdot}{}^\nu B_\nu{}^{\cdot}\delta_{\cdot}{}^\alpha\delta_\beta{}^{\cdot}\\+d(B^\dagger B)_{\cdot}{}^{\cdot}\delta_{\cdot}{}^\alpha\delta_\beta{}^{\cdot}+eB^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_\beta{}^\alpha.\end{aligned} \tag{1.15}$$

The tracelessness of $\Delta_\beta{}^\alpha$ yields the constraint

$$a_1+a_2+b_1+b_2+3e=0. \tag{1.16}$$

The requirement that the electric-charge component vanish gives

$$\Delta_1{}^1=0,$$

which entails

$$a_1+a_2=b_1+b_2=a_1+b_2=c=d=e=0.$$

This conclusion would have held even if the perturbation was a mixture of 27 type and 1 type so that

$$\Delta_1{}^1+p(B^\dagger{}_\nu{}^1B_1{}^\nu-\tfrac{1}{3}(B^\dagger B)_\nu{}^\nu)+q(B^\dagger{}_1{}^\nu B_\nu{}^1-\tfrac{1}{3}(B^\dagger B)_\nu{}^\nu)=0\,,$$

which would imply

$$p=q=0.$$

The general kind of perturbation is of the form

$$\begin{aligned}\Delta_\beta{}^\alpha=a'\{B^\dagger{}_{\cdot}{}^\alpha B_{\cdot}{}^{\cdot}\delta_\beta{}^{\cdot}-B^\dagger{}_{\cdot}{}^{\cdot}B_\beta{}^{\cdot}\delta_{\cdot}{}^\alpha\\+B^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^\alpha\delta_\beta{}^{\cdot}-B^\dagger{}_\beta{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_{\cdot}{}^\alpha\}.\end{aligned} \tag{1.17}$$

This term has, however, the wrong charge-conjugation property since charge-conjugation invariance implies

$$a_1-a_2=b_1-b_2=0. \tag{1.18}$$

Thus provided, the electric component continues to behave like a component of an octet no first-class contributions can arise; for second-class contributions a one-parameter family survives; it will imply no second-class contributions to the electric component.

(ii) Octet Perturbations of a 27-Type Tensor

As another example let us consider the perturbations of the 8 type but now consider the matrix elements of a 27-type operator. An operator of the 27 type can be realized by a fourth-rank mixed tensor symmetric in its two upper indices and its two lower indices separately and all whose contractions vanish. If $M_\xi{}^\alpha{}_\eta{}^\beta$ is a tensor satisfying

$$M_\xi{}^\alpha{}_\eta{}^\beta=M_\xi{}^\beta{}_\eta{}^\alpha=M_\eta{}^\beta{}_\xi{}^\alpha,$$

then its 27-type part is given by

$$\begin{aligned}J_\xi{}^\alpha{}_\eta{}^\beta=M_\xi{}^\alpha{}_\eta{}^\beta-\tfrac{1}{5}\{M_\sigma{}^\sigma{}_\eta{}^\beta\delta_\xi{}^\alpha+M_\sigma{}^\sigma{}_\eta{}^\alpha\delta_\xi{}^\beta\\+M_\sigma{}^\sigma{}_\xi{}^\alpha\delta_\eta{}^\beta+M_\sigma{}^\sigma{}_\xi{}^\beta\delta_\eta{}^\alpha\}\\+(1/20)M_\sigma{}^\sigma{}_\tau{}^\tau(\delta_\xi{}^\alpha\delta_\eta{}^\beta+\delta_\eta{}^\alpha\delta_\xi{}^\beta).\end{aligned} \tag{1.19}$$

If we denote this expression by $M_{\cdot}{}^{\cdot}{}_{\cdot}{}^{\cdot}$ and the corresponding 27-type part (with respect to the indices $\alpha\beta\xi\eta$) of the tensor $M_\xi{}^\alpha{}_\eta{}^\beta{}_3{}^3$ by $M_{\cdot}{}^{\cdot}{}_{\cdot}{}^{\cdot}{}_3{}^3$, we can write down the first-order perturbation correction in the form

$$\begin{aligned}\Delta_{\cdot}{}^{\cdot}{}_{\cdot}{}^{\cdot}=a_1B^\dagger{}_{\cdot}{}^3B_{\cdot}{}^{\cdot}\delta_3{}^{\cdot}+a_2B^\dagger{}_{\cdot}{}^{\cdot}B_3{}^{\cdot}\delta_{\cdot}{}^3+b_1B^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^3\delta_3{}^{\cdot}\\+b_2B^\dagger{}_3{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_{\cdot}{}^3+c(B^\dagger B)_{\cdot}{}^{\cdot}\delta_{\cdot}{}^3\delta_3{}^{\cdot}\\+dB^\dagger{}_{\cdot}{}^\nu B_\nu{}^{\cdot}\delta_{\cdot}{}^3\delta_3{}^{\cdot}+eB^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_3{}^3.\end{aligned} \tag{1.20}$$

The requirement that the "electric component" $\alpha=\beta=\xi=\eta=1$ vanish gives the relation

$$\begin{aligned}a_1\{4(B^\dagger{}_3{}^3B_1{}^1+B^\dagger{}_1{}^3B_3{}^1)-(B^\dagger B)_3{}^3\}+a_2\{4(B^\dagger{}_1{}^1B_3{}^3+B^\dagger{}_1{}^3B_3{}^1)-B^\dagger{}_3{}^\nu B_\nu{}^3\}+b_1\{4(B^\dagger{}_3{}^1B_1{}^3+B^\dagger{}_1{}^1B_3{}^3)-(B^\dagger B)_3{}^3\}\\+b_2\{4(B^\dagger{}_3{}^3B_1{}^1+B^\dagger{}_3{}^1B_1{}^3)-B^\dagger{}_3{}^\nu B_\nu{}^3\}+c\{3(B^\dagger B)_1{}^1-(B^\dagger B)_\nu{}^\nu\}+d\{3B^\dagger{}_1{}^\nu B_\nu{}^1-(B^\dagger B)_\nu{}^\nu\}\\-e\{20B^\dagger{}_1{}^1B_1{}^1-4(B^\dagger B)_1{}^1-4B^\dagger{}_1{}^\nu B_\nu{}^1+(B^\dagger B)_\nu{}^\nu\}=0.\end{aligned} \tag{1.21}$$

This enables us to infer that

$$a_1=-a_2=-b_1=b_2;\quad c=d=e=0,$$

so that the first-order perturbations belong to a one-parameter family

$$\Delta_{\cdot}{}^{\cdot}{}_{\cdot}{}^{\cdot}=a'(B^\dagger{}_{\cdot}{}^3B_{\cdot}{}^{\cdot}\delta_3{}^{\cdot}-B^\dagger{}_{\cdot}{}^{\cdot}B_3{}^{\cdot}\delta_{\cdot}{}^3-B^\dagger{}_{\cdot}{}^{\cdot}B_{\cdot}{}^3\delta_3{}^{\cdot}+B^\dagger{}_3{}^{\cdot}B_{\cdot}{}^{\cdot}\delta_{\cdot}{}^3). \tag{1.22}$$

If we now make use of charge-conjugation invariance which demands

$$a_1=a_2,\quad b_1=b_2,$$

we could conclude that the first-order perturbation vanishes.

II. STABILITY OF TENSOR OPERATORS UNDER PERTURBATIONS

The computations in the last two sections point to the existence of a group-theoretical stability principle which assures the vanishing to first order of the perturbation of a tensor operator provided a suitable component is held fixed. We have been able to state and demonstrate such a stability principle: It can then be shown that a variety of results of the type discussed in the earlier part of this paper can be deduced under weaker conditions. The principle can be stated in a fashion which is applicable to any (simple compact) group G though our interest is primarily in the unitary groups.

General Theory of First-Order Perturbations

Let α stand for the complete set of labels for the components of an irreducible representation A of the group G. We shall choose the labels in such a fashion that the symmetry-breaking interaction transforms like a specific component β of an irreducible representation B. Given any two representations A, R we can construct the component α' of the irreducible representation A' occurring in the product of A and R making use of the Clebsch-Gordan (CG) coefficients:

$$\psi_{\alpha'}{}^{A',a}=\sum_{\alpha,\rho} C_\alpha{}^A{}_\rho{}^R{}_{\alpha'}{}^{A',a}\psi_\alpha{}^A\psi_\rho{}^R. \tag{2.1}$$

The same representation A' may occur more than once in the product of A and R; in that case we could choose any linear combination of the relevant CG coefficients. We can choose these coefficients so that the matrices $U(R,a,\rho)$ whose matrix elements are given by

$$U_{\alpha'\alpha}=C_\alpha{}^A{}_\rho{}^R{}_{\alpha'}{}^{A',a} \tag{2.2}$$

constitute a set of trace-orthonormal and complete set of matrices. As an immediate consequence of their completeness and orthonormality any matrix $M_{\alpha'\alpha}$ can be expanded in terms of $U_{\alpha'\alpha}$ in one and only one way.

Consider now a set of operators J_ρ which, in the symmetric limit, behave like the components of a tensor of type R. Under the effect of a perturbation which transforms like the β component of a tensor operator of type B with, possibly, an additional singlet perturbation, to first order in the perturbation, the operator J_ρ may be written

$$\begin{aligned} J_\rho&=S_\rho{}^R+T_\rho{}^R{}_\beta{}^B\\ &=S_\rho{}^R+\sum_{R'r\rho'} C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r}T_{\rho'}{}^{R',r}. \end{aligned} \tag{2.3}$$

If we now compute the matrix element of both sides between states belonging to the representations A and A' we may write

$$\langle A'\alpha'|J_\rho|A\alpha\rangle=\sum_s C_\alpha{}^A{}_\rho{}^R{}_{\alpha'}{}^{A',s}t(s)+\sum_{R'r\rho'} C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r}\times\sum_s C_\alpha{}^A{}_{\rho'}{}^{R'}{}_{\alpha'}{}^{A',s}t(r,R',s). \tag{2.4}$$

This equation is valid (to first order in B) for every component of J_ρ. If we further postulate that the special linear combination

$$\sum_\rho q^\rho J_\rho$$

happens to have matrix elements which coincides with the matrix elements of the same linear combination of a suitable tensor $J_\rho{}^R$ of the type R, with reduced matrix elements $j(s)$ which may or may not be equal to $t(s)$, we then have the relation

$$\sum_\rho q^\rho\{\sum_s U(R,s,\rho)t(s)+\sum_{R'r\rho's} C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r}U(R',s,\rho')t(r,R',s)\}=\sum_\rho q^\rho\sum_s U(R,s,\rho)j(s). \tag{2.5}$$

Because of the linear independence of $U(R,s)$ we now deduce that

$$q^{\rho'}\delta_{RR'}\{t(s)-j(s)\}+\sum_{\rho r} q^\rho C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r}t(r,R',s)=0. \tag{2.6}$$

This is the fundamental equation. It is interesting to note that no immediate reference to the representations A and A' appear in this equation. As is to be expected, the structure of these equations is independent of s. However, indirectly the set of nontrivial representations R' that can contribute depends on A and A'. For the case of $R\neq R'$ we have the simpler equation

$$\sum_{\rho r} q^\rho C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r}t(r,R',s)=0. \tag{2.7}$$

In the case of $SU(3)$ violation β refers to an $I=Y=0$ component so that unless $\rho=\rho'$ the CG coefficient vanishes; hence for all ρ for which $q^\rho\neq 0$ we get

$$\sum_r C_\rho{}^R{}_0{}^B{}_\rho{}^{R',r}t(r,R',s)=0,\quad q^\rho\neq 0. \tag{2.8}$$

Hence, unless for all such ρ the CG coefficients are linearly dependent, we can deduce that

$$t(r,R',s)=0, \tag{2.9}$$

and hence we may drop the corresponding tensor contribution of the first-order breaking. Thus, for example, in the $SU(3)$ case for $R=8$ and $B=8$ we know that the CG coefficient does not vanish for the $\rho=\{I=1, Y=0\}$ component for $R'=27$, 10, 10*. Hence from the isovector components alone we deduce that the corresponding reduced matrix elements are all zero.

For the $R'=R$ cases we have an equation of the form

$$\sum_{\rho,r} q^\rho C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R,r}t(r,R,s)+q^{\rho'}[t(s)-j(s)]=0.$$

We again consider the special case where β is such that $\rho=\rho'$ for the CG coefficient not to vanish. Then for all ρ for which q^ρ did not vanish, we could deduce

$$\sum_r C_\rho{}^R{}_0{}^B{}_\rho{}^{R,r}t(r,R,s)=j(s)-t(s). \tag{2.10}$$

Since the right-hand side is independent of ρ the left-hand side should also be independent of ρ. However this is not in general true; if q^ρ is nonvanishing for more than one value then at least one of these representations should be absent.

For the particular case that B behaves like the adjoint representation, we know that in general one of the CG coefficients reproduce the matrix elements of the generators (antisymmetric in the two representations R) and the other CG coefficient is symmetric in the two representations R. For the $SU(3)$ octet type B these are the usual F- and D-type octets. For the CG coefficient realizing the generator matrices, the CG coefficient vanishes for the $I=Y=0$ component for the (nonstrange) values of ρ for which $q^\rho \neq 0$. Thus it follows that the corresponding reduced matrix element $t(FRs)$ is the only nonvanishing one.

Application to Special Cases

We now examine each case in turn.

(*i*) $R=8$, $B=8$, $q^\rho \neq 0$ *for* $I=1$, $Y=0$ *only.*

This already eliminates all reduced matrix elements except $R'=8$, $r=F$-type CG coefficient. (But charge-conjugation invariance of B now makes this coefficient also vanish for the baryon-baryon vertex.) It is interesting to note that the $I=0$, $Y=0$ component was not constrained. This observation will be important in the application to the stability of the axial-vector octet. The $I=0$, $Y=0$ component could not replace the $I=1$, $Y=0$ component since the $R'=10$, 10^* reduced matrix elements would then not necessarily vanish.

(*ii*) $R=27$, $B=8$, $q^\rho \neq 0$ *ᵃor* $I=1$ $Y=0$ *only.*

All reduced matrix elements should vanish except $R'=27$, $r=F$-type CG coefficient. (Charge-conjugation invariance of B makes this also vanish for the baryon-baryon vertex.) Note that we have not used the $I=Y=0$ component. If instead of the $I=1$, $Y=0$ component we used only the $I=Y=0$ component, the reduced matrix elements for $R'=35$, 35^*, 10, 10^* would not be zero. Of course if we are considering baryon matrix elements of J_ρ the 35 and 35^* contributions would identically vanish.

(*iii*) $R=8$, $B=27$, $q^\rho \neq 0$ *for* $I=1$, $Y=0$.

The only reduced matrix element not eliminated is for the case $R=27$ and one coupling (called 27_2 by deSwart). (Charge-conjugation invariance eliminates this also for the baryon-baryon vertex.)

It is relevant to point out that the results obtained from (2.6) are more general than the explicit calculations of the previous sections, *since they are valid for the entire operator without any restriction on the tensor character of the initial and final multiplets* A *and* A', provided the requirement

$$\sum_\rho q^\rho \{ T_\rho{}^R + \sum_{R'r\rho'} C_\rho{}^R{}_\beta{}^B{}_{\rho'}{}^{R',r} T_{\rho'}{}^{R',r} \} = \sum_\rho q^\rho \mathcal{J}_\rho{}^R \quad (2.11)$$

holds valid when the matrix elements between the multiplets A and A' are taken. Thus, if we require that the pion-baryon octet-baryon decimet vertex is of an octet type it would follow that, to first order in symmetry violation, the octet of pseudoscalar-octet-decimet vertices continue to remain octets. Observation that the magnetic moments of the decimet-octet transitions behaved like an octet would likewise imply that the corresponding weak-magnetism terms would behave like an octet.

Perturbations in Higher Order and Mixtures of Perturbations

We now deal with the question of treating perturbations to higher order and of treating perturbations which are not components of an irreducible tensor but are a linear combination of several such. We have already considered the question of the admixture of a nontrivial singlet term added to the perturbations. We shall see that the two problems are interrelated.

(*i*) *Mixtures of Perturbations*

Let us first consider the case of a perturbation which is a linear combination of various types B' with the component β' labeled in the same fashion. The first-order perturbation equation now becomes

$$J_\rho = S_\rho{}^R + \sum_{B',\beta'} \sum_{R',r,\rho'} C_\rho{}^R{}_{\beta'}{}^{B'}{}_{\rho'}{}^{R',r} T_{\rho'}{}^{R'rB'}. \quad (2.12)$$

The constraint equation may now be written without loss of generality in the form

$$\sum_\rho \sum_{B'\beta'r} q^\rho C_\rho{}^R{}_{\beta'}{}^{B'}{}_{\rho'}{}^{R',r} t(B'rR's) = 0. \quad (2.13)$$

In the case of $SU(3)$ we would restrict B' to the self-conjugate representations and β' to be the $I=Y=0$ component, so that this equation may be simplified to yield

$$\sum_{B'r} C_\rho{}^R{}_0{}^{B'}{}_\rho{}^{R',r} t(B'rR's) = 0, \quad q^\rho \neq 0. \quad (2.14)$$

For each value of ρ for which q^ρ is nonvanishing we have an equation connecting the reduced matrix elements $t(B'rR's)$ for R',s kept fixed and B', r varied. These equations may or may not require the reduced matrix elements to vanish. In general, several of these reduced matrix elements will be nonvanishing, but with definite relations between them.

As an example consider the case where B' runs over the representation 1, 8, 27 with charge-conjugation invariance imposed and where R is an octet. The possible values of R', r are

$$R',r=\begin{cases}1;\,8_1;\,8_2;\,10;\,10^*;\,27 & B'=8\\ 8;\,10;\,10^*;\,27_1;\,27_2;\,35,\,35^*,\,64 & B'=27\\ 8 & B'=1.\end{cases}$$

Among these only the $R'=8$ have both types of matrix elements within the baryon octet; if it is an octet-decimet vertex it has only one matrix element; we may now look up[4] the CG coefficients $C_\rho{}^{8}{}_0{}^{B'}{}_\rho{}^{R',r}$, where ρ has $Y=0$ and $I=0$, 1. Equation (2.14) for $I=1$, $Y=0$ yields

$$\begin{aligned}
&\frac{10}{9\sqrt{7}}t(27,64,s)=0\,,\\
&(\sqrt{\tfrac{3}{10}})t(8,27,s)+\frac{4}{\sqrt{70}}t(27,27_1,s)=0\,,\\
&\frac{1}{\sqrt{5}}t(8,8_1,s)-\frac{2}{3\sqrt{5}}t(27,8,s)=0\,,\\
&\tfrac{1}{2}t(8,10,1)+\tfrac{1}{3}t(27,10,1)=0\,,\\
&\tfrac{1}{2}t(8,10^*,1)-\tfrac{1}{3}t(27,10^*,1)=0\,,\\
&\frac{2\sqrt{5}}{9}t(27,35,s)=\frac{2\sqrt{5}}{9}t(27,35^*,s)=0.
\end{aligned}\tag{2.15}$$

For $I=0$, $Y=0$ they yield

$$\begin{aligned}
&\frac{2}{\sqrt{7}}t(27,64,s)=0\,,\\
&\tfrac{3}{2}(\sqrt{\tfrac{3}{10}})t(8,27,s)-\frac{4}{\sqrt{70}}t(27,27_1,s)=0\,,\\
&t(1,8,s)=\frac{1}{\sqrt{5}}t(8,8_1,s)-\frac{1}{\sqrt{5}}t(27,8,s)=0\,,\\
&-\frac{1}{2\sqrt{2}}t(8,1,s)=0.
\end{aligned}\tag{2.16}$$

These equations do not impose any restrictions on $t(8,8_2,s)$ and $t(27,27_2,s)$. By virtue of the constraint equations we deduce

$$\begin{aligned}
&t(27,64,s)=t(8,27,s)=t(27,27_1,s)\\
&\qquad=t(27,35,s)=t(27,35^*,s)=t(8,1,1)=0\,,\\
&t(27,10,1)=-\tfrac{3}{2}t(8,10,1)\,,\\
&t(27,10^*,1)=+\tfrac{3}{2}t(8,10^*,1)\,,\\
&t(27,8,s)=+\tfrac{3}{2}t(8,8_1,s)=+\frac{3}{\sqrt{5}}t(1,8,s)\,.
\end{aligned}\tag{2.17}$$

Under charge conjugation the two reduced matrix elements $t(8,8_2,s)$, $t(27,27_2,1)$ are both constrained to vanish since the corresponding CG coefficients vanish. No constraint is imposed on $t(27,8,s)$, $t(8,8_1,s)$, $t(1,8,s)$. Charge-conjugation invariance requires

$$\begin{aligned}
&t(8,10,1)=t(8,10^*,1)\,,\\
&t(27,10,1)=-t(27,10^*,1)\,,
\end{aligned}\tag{2.18}$$

which are consistent with the restrictions already imposed on the reduced matrix elements by the constraint Eqs. (2.17). Thus the nonvanishing reduced matrix elements are

$$\begin{aligned}
&t(27,8,s)=\tfrac{3}{2}t(8,8_1,s)=(3/\sqrt{5})t(1,8,s)\,,\\
&t(27,10^*,1)=\tfrac{3}{2}t(8,10^*,1)=\tfrac{3}{2}t(8,10,1)\\
&\qquad\qquad=-t(27,10,1)\,.
\end{aligned}\tag{2.19}$$

For the baryon-baryon vertex we have a three-parameter family (F- and D-type octet matrix elements and the $10+10^*$ matrix elements), while for the baryon-baryon resonance vertex there are a two-parameter family of matrix elements. For the baryon-baryon vertex a direct calculation gives the general change in the current:

$$\begin{aligned}
\Delta_\beta{}^\alpha=&\,a\{(B^\dagger B)_3{}^\alpha\delta_\beta{}^3+(B^\dagger B)_\beta{}^3\delta_3{}^\alpha-2(B^\dagger B)_\nu{}^\nu\delta_3{}^\alpha\delta_\beta{}^3\}\\
&+b\{B^\dagger{}_\beta{}^\nu B_\nu{}^\alpha\delta_\beta{}^3+B^\dagger{}_\beta{}^\nu B_\nu{}^3\delta_3{}^\alpha-2B^\dagger{}_3{}^\nu B_\nu{}^3\delta_3{}^\alpha\delta_\beta{}^3\}\\
&+c\{B^\dagger{}_3{}^3B_3{}^\alpha\delta_\beta{}^3+B^\dagger{}_\beta{}^3B_3{}^3\delta_3{}^\alpha\\
&\qquad-B^\dagger{}_3{}^\alpha B_3{}^3\delta_\beta{}^3-B^\dagger{}_3{}^3B_\beta{}^3\delta_3{}^\alpha\}\,,
\end{aligned}\tag{2.20}$$

in accordance with the above conclusion.

(ii) *High-Order Perturbations of SU(3)*

To treat perturbations of a definite tensor type B in higher order we may proceed as follows: In second order we have

$$J_\rho=S_\rho{}^R+J_\rho{}^R{}_\beta{}^B+J_\rho{}^R{}_\beta{}^B{}_\beta{}^B.$$

We may rewrite

$$J_\rho{}^R{}_\beta{}^B{}_\beta{}^B=\sum_{B'b\beta'}T_\rho{}^R{}_{\beta'}{}^{B',b}C_\beta{}^B{}_\beta{}^B{}_{\beta'}{}^{B',b}=\sum_{B'\beta'}\mathcal{T}_\rho{}^R{}_{\beta'}{}^{B'}\,,\tag{2.21}$$

where the coefficients $C_\beta{}^B{}_\beta{}^B{}_\beta{}'^{B'b}$ are independent of R and ρ and $\mathcal{T}_\rho{}^R{}_{\beta'}{}^{B'}$ are a set of tensors. The problem of second-order perturbation of pure tensor type B is thus essentially the same as the problem of perturbations to first order by a linear combination of perturbations B' (which occur in the product of B with itself). For the case of $SU(3)$ the component β is taken to have $I=Y=0$ so that the sum on β' can be omitted. The same procedure can be continued to arbitrary order in the perturbation; in each case we would couple the tensor product of B with itself to the required order and treat the mixture of representations so generated considered as an effective perturbation to first order.

As an example, consider the perturbation of the octet type in $SU(3)$ taken to second order. The familiar reduction rule $8\times8=1+8+8+10+10^*+27$ together

[4] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).

with the restriction to the $I=Y=0$ component tells us that the effective perturbation is a mixture of 1, 8, and 27. This is therefore essentially the problem discussed above. To study the perturbation of an operator of type R we would now proceed to couple R and B' to obtain R', r and so on. For the special case of $R=8$ we get to second order in an octet perturbation a 3-parameter family of corrections to the baryon-baryon vertex, provided the $I=1$, $Y=0$ and $I=0$, $Y=0$ components are both used to constrain the perturbations.

We can make further use of this analysis to study in which order of what kind of perturbation all sum rules would be lost. We again restrict attention to $SU(3)$. For the baryon-baryon vertex only $R'=1, 8, 10, 10^*, 27$ have nonvanishing contributions. If R is chosen to be an octet current it follows that B' must be 1, 8, 27, or 64 to contribute. A mixture of 1, 8, 27, 64 for B' would therefore imply no restriction. If one or more of these terms are absent they will imply corresponding sum rules. (We point out once again that the singlet perturbation is nontrivial in the sense that it can alter the D/F ratio of the octet current.) An octet perturbation taken to third order will generate all these terms for B' so that no sum rule will survive for octet baryon-baryon vertices; the same will be true of a 27-type perturbation to second order (or a mixture of 27 and 8 taken to second order). A mixture of 1-, 8-, and 64-type perturbation in first order would imply one sum rule: This case is of interest in that the 64-type perturbation cannot contribute to the mass formula and this mixture of perturbations would give a pure octet mass formula in first order. Incidentally, if we were considering a singlet operator like the mass and its perturbation it would follow that B' must be restricted to 1, 8, 27 for baryons and hence the baryon mass formula is destroyed in the second order of an octet perturbation.

For the octet-decimet transition vertex of octet type, B' must be restricted to the same values 1, 8, 27, 64 and hence no sum rules survive in the third order of octet perturbations while in second order one sum rule survives. For the mass operator within the decimet we would be able to have B' run over all these values and hence there is a mass formula to second order. For the decimet-decimet vertex of octet type, B' could be allowed to run over the values 1, 8, 27, 64, 125. Hence in third order of octet perturbation there will be one sum rule (coming from the absence of 125). However, if we consider a 27-type perturbation even to second order this will be destroyed.

III. STABILITY OF VECTOR AND AXIAL VECTOR OCTETS

Following Ademollo, Gatto,[1] Bouchiat and Meyer,[2] one could provide some grounds for considering that the vector current in the leptonic decays of hyperons (or the three-body leptonic decays of mesons) continues to remain an octet in spite of $SU(3)$ violations of the first order. We emphasize that as long as the vector currents are considered to be conserved in the absence of symmetry violation, it is unchanged under an arbitrary perturbation treated to first order. To second order there are perturbations, but there is one sum rule which has been discussed already by Zakharov and Kobzarev,[3]

$$\{V(\Xi^0\Sigma^+)+V(\Sigma^-n)\} + (\sqrt{6})\{V(\Xi^-\Lambda)+V(\Lambda p)\}=0. \quad (3.1)$$

For the decimet-octet transition through a (weak) conserved vector leptonic transition we have the transitions vanishing to the lowest order for the simple reason that the baryon decimet is a $\frac{3}{2}^+$ system and hence the transition to the baryon octet of $\frac{1}{2}^+$ particles can take place only through a Gamow-Teller transition. One may consider, however, the momentum-dependent transition form factors, in particular the magnetic moments for electromagnetic transitions. The assumption that this magnetic moment behaves like an $SU(3)$ octet gives rise to the familiar relations[5]

$$\begin{aligned}\mu(N^{*+}\to p)&=\mu(N^{*0}\to n)=-\mu(Y^{*+}\to\Sigma^+)\\&=2\mu(Y^{*0}\to\Sigma^0)=-\frac{2}{\sqrt{3}}\mu(Y^{*0}\to\Lambda)\\&=-\mu(\Xi^{*0}\to\Xi^0),\end{aligned} \quad (3.2)$$

with the other transition moments $\mu(Y^*\to\Sigma^-)$, $\mu(\Xi^{*-}\to\Xi^-)$ vanishing. If one considers the transition moment to be the $\alpha=\beta=1$ component of an octet of operators whose $\alpha=2$, $\beta=1$ and $\alpha=3$, $\beta=1$ components, respectively, yield the strangeness-conserving and strangeness-violating Gamow-Teller matrix element (for small-momentum transfers) we conclude that similar relations hold for these leptonic decay amplitudes as well. For the strangeness-conserving weak-interaction amplitudes we get

$$\begin{aligned}&W(N^{*0}\to p)\\&=-\sqrt{2}W(Y^{*0}\to\Sigma^+)=\frac{1}{\sqrt{3}}W(N^{*-}\to n)\\&=-W(\Xi^{*-}\to\Xi^0)=-(\sqrt{\tfrac{2}{3}})W(Y^{*-}\to\Lambda)\\&=-\sqrt{2}W(Y^{*0}\to\Sigma^+)=+\sqrt{2}W(Y^{*-}\to\Sigma^0),\end{aligned} \quad (3.3)$$

and for the strangeness-violating decays the relations

$$\begin{aligned}\sqrt{2}W(Y^{*0}\to p)&=W(Y^{*-}\to n)=-(\sqrt{\tfrac{2}{3}})W(\Xi^{*-}\to\Lambda)\\&=-W(\Xi^{*0}\to\Sigma^+)=\sqrt{2}W(\Xi^{*-}\to\Sigma^0)\\&=\frac{-1}{\sqrt{3}}W(\Omega\to\Xi^0).\end{aligned} \quad (3.4)$$

[5] A. J. Macfarlane and E. C. G. Sudarshan, Nuovo Cimento 31, 1176 (1964).

In general we would expect these relations to be lost when an octet perturbation is considered.

We shall restrict the perturbation by the requirement that the magnetic-moment relations (and hence the corresponding strangeness-conserving Gamow-Teller matrix-element relations) are unchanged by the perturbation. The strangeness-violating amplitudes would then be perturbed. It turns out that this perturbation has the curious property of preserving the relations (3.4), but uniformly scaling all the strangeness-violating matrix elements by the same factor (relative to the strangeness-conserving amplitudes). Thus the net result of the perturbation within this framework is to change the Cabibbo angle for the leptonic decays of the decimet.

It is interesting to note that the remarks made in the Introduction about the lack of renormalization of any conserved operator, to lowest order in an arbitrary perturbation, does not hold for the octet-decimet transitions since the perturbations on the decimet could produce an octet or vice versa. Hence, in this case, a conserved operator could have transition matrix elements in first order of perturbation.

However, the leptonic axial-vector matrix elements of baryons also seem to transform like an octet with the same Cabibbo angle.[6] In this case we certainly cannot make use of any conserved current. If we believed that the chiral $SU(3)\times SU(3)$ scheme gives a good description of the baryons the axial-vector current also would be conserved in this limit.[7] But this limit is expected to be rather poor for the physical baryons; in any case it would not be satisfactory to talk about the symmetric limit in the chiral group and first order in $SU(3)$ violation. We must look for an alternative justification.

If, on physical grounds, we could guarantee that the $I=1$, $Y=0$ component of the axial-vector current is not affected by the symmetry violation, we could appeal to our earlier results to show that to first order in the perturbation the entire axial-vector current should remain an octet with the same Cabibbo angle (between the $\Delta S=0$ and $\Delta S=1$ components). We emphasize that the $I=1$, $Y=0$ component alone, which enters strangeness-conserving weak decays, have been used to constrain the amplitude. It follows that to first order in $SU(3)$ violation, the Cabibbo angles for the vector and axial-vector interactions must be equal. If we believe that the pure octet nature of the strangeness-conserving decays remains valid to the second order in the perturbation, the Zakharov-Kobzarev sum rule should hold equally for the axial-vector amplitudes to second order. Related arguments could be applied to discuss the ratios of octet-decimet transitions in reactions initiated by high-energy neturinos.

In searching for a constraint on the $I=1$, $Y=0$ matrix element we note the following circumstance: The Adler-Weisberger relation[8] for the renormalization of the axial vector current in nuclear β decay,

$$\frac{1}{g_A{}^2}=1-\frac{4M^2}{g_\pi{}^2}\int\frac{WdW}{W^2-M^2}\{\sigma^+(W)-\sigma^-(W)\}\,,$$

is in good agreement with the experimental value. The pion total cross sections used here get their major contribution from final states without any kaons. It is likely that the same would be true for the axial-vector renormalization constants of other baryons also. To the extent that the violation of $SU(3)$ invariance is implemented through baryon and meson mass differences it is plausible that the renormalization constants for the strangeness-conserving β interactions are not affected. If we accept this argument it implies that the $I=1$, $Y=0$ components of the axial-vector current remain unchanged by perturbation of $SU(3)$; they are of course renormalized (and the D/F ratio may have been changed) but the renormalization is invariant under $SU(3)$.

Universality of the Cabibbo Angle

The two-body leptonic decays of pions and kaons proceed purely via the axial-vector interaction. In this case we know that only an 8-type tensor could cause the transition. For $I=1$, $Y=0$ particles (the pions) the weak-decay amplitude could be taken as a component of a pure octet coupling. Assuming that the absolute value of this amplitude is unchanged by $SU(3)$ perturbations to first order in $SU(3)$ violation there should be no violation of the octet nature of the meson-decay amplitude. In particular the Cabibbo angle for the meson two-body leptonic decays must also remain unrenormalized to first-order symmetry violation.

If we choose the vector and axial-vector Cabibbo angles to be the same in the $SU(3)$ limit, we have thus deduced to first order in the $SU(3)$ symmetry breaking that the "observed" Cabibbo angles of vector and axial-vector baryon decay amplitudes, the three-body leptonic modes of mesons, and for the three-body leptonic modes of mesons must all be equal. This result is in good agreement with experiment.[6]

While in most of the applications we have restricted attention to $SU(3)$ symmetry and its violations, they could be equally well adapted to the discussion of departures from $SU(6)$ and other such symmetry groups. The tensor analysis on these groups is much more complicated and we shall not be able to deduce the results in such an immediate fashion. We hope to study these questions elsewhere.[9]

[6] N. Cabibbo, in *Proceedings of the XIIIth International Conference on High Energy Physics*, edited by M. Alson-Garnjost (University of California Press, Berkeley, California, 1967).

[7] G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Letters **20**, 64 (1966); J. Schechter and Y. Ueda, Phys. Rev. **144**, 1338 (1966).

[8] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); S. L. Adler, *ibid.* **14**, 1051 (1965).

[9] *Note added in proof*. A term of the type $B^{\dagger}{}_y{}^{\nu}B_\nu{}^x\delta_\beta{}^\alpha$ in Eq. (1.3) has been omitted, since it is expressible as a linear combination of the terms included in this equation. We thank Professor A. J. Macfarlane for pointing this out to us.