

Propagation of Electron Magnetohydrodynamic structures in a 2-D inhomogeneous plasma

Sharad Kumar Yadav,^{*} Amita Das,[†] and Predhiman Kaw

Institute for Plasma Research, Bhat , Gandhinagar - 382428, India

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Abstract

The fully three dimensional governing equations in the electron magnetohydrodynamic (EMHD) regime for a plasma with inhomogeneous density is obtained. These equations in the two dimensional (2-D) limit can be cast in terms of the evolution of two coupled scalar fields. The nonlinear simulations for the two dimensional case are carried out to understand the propagation of EMHD magnetic structures in the presence of inhomogeneity. A novel effect related to trapping of dipolar magnetic structures in the high density plasma region in the EMHD regime is observed. The interpretation of this phenomena as well as its relevance to the problem of hot spot generation in the context of fast ignition is presented.

^{*}Electronic address: sharad@ipr.res.in

[†]Electronic address: amita@ipr.res.in

I. INTRODUCTION

The studies related to plasma response at fast electron time scales are becoming a topic of considerable research interest lately. The topics of laser plasma interaction [1], reconnection in electron current layers [2], fast Z pinches [3], fast plasma based switching devices [4, 5] and also fast ignition physics [6, 7, 8] primarily involve a study of plasma phenomena occurring at electron response regime [5]. One particular simplified fluid model description characterizing the electron dynamics against the background stationary charge neutralizing ions is the Electron Magnetohydrodynamics (EMHD) [5, 9]. The model has so far been primarily studied in the context of plasma with uniform homogeneous density. In realistic situations, however, the plasma typically has an inhomogeneous density. The role of density inhomogeneity was outlined briefly earlier in a paper by Kingsep [5]. The paper, however, discussed the effect in the absence of any electron inertia related terms. In the present manuscript the EMHD model has been generalized for the case of a plasma with inhomogeneous density profile including effects associated with finite electron inertia. The governing equations for the generalized EMHD are shown to reduce to two coupled scalar field evolution in the 2-D limit. For homogeneous density these equations reduce to the well known form studied extensively earlier [9, 10]. The EMHD equations for the homogeneous plasma permits a variety of coherent localized solutions which are of a stationary monopolar form or a travelling (with constant velocity) dipolar form [11, 12]. These solutions are extremely robust and have been observed to form spontaneously in simulations [13]. Here, we study the propagation and evolution of such coherent solutions when they encounter an inhomogeneous plasma. This is done by employing our generalized EMHD (G-EMHD) equations for the numerical evolution of these solutions specified as an initial condition.

In the next section we outline the details of the derivation of the governing equations for the G-EMHD model corresponding to the spatially inhomogeneous plasma density. The 2-D limit of the equation is then obtained and it is shown that the governing equations are a coupled evolution equation for two scalar fields. These scalar fields represent the component of the magnetic field and the vector potential along the symmetry direction. In section III we briefly recapitulate the coherent solutions of the EMHD equations and their well known propagation characteristics in the context of a homogeneous plasma. The physical similarity of the dipole structure to a forward moving current pulse accompanied by spatially separated

reverse shielding currents is also shown in this section. In section IV we then show with the help of our numerical simulations how the density inhomogeneity influences the propagation of these structures. The restrictions on total grid points constrains us to consider a ratio of maximum to minimum plasma density in our simulation domain to be at most a factor of 10 only. This constraint arises from the need of adequately resolving the plasma skin depth scales in the high density regime. In Section V we present a novel observation illustrating the trapping of dipolar EMHD structures in the high density plasma region. The consequence of this novel observation on the physics of fast ignition is also pointed out. Finally, in Section VI we summarize the salient points of the paper.

II. GOVERNING EQUATIONS

In this section we obtain the governing equations for describing dynamical phenomena occurring at fast electron time scales in the presence of plasma density inhomogeneity. The time scales are fast such that the ion response can be ignored. However, it is considered to be slower compared to the minimum value of the local plasma frequency ω_{pe} and/or $\omega_{pe}^2/\omega_{ce}$ (here ω_{ce} is the electron gyrofrequency) whichever is smaller. This ensures that the displacement current can be ignored and hence electron density fluctuations are considered to be negligible. This essentially implies that the electron continuity equation drops out from the evolution. Thus the system of equations which we derive here can be looked upon as the generalized Electron Magnetohydrodynamics (G-EMHD) model for inhomogeneous plasma.

The curl of electron momentum equation can be written as

$$\frac{\partial \vec{G}}{\partial t} = \nabla \times [\vec{v}_e \times \vec{G}] \quad (1)$$

where $\vec{G} = \nabla \times (m_e \vec{v}_e - e\vec{A}/c)$. The electron velocity $\vec{v}_e = -(c/4\pi en_e)\nabla \times \vec{B}$ from the Ampere's law, as the displacement current can be ignored and ions being stationary the current is determined by the electron flow velocity alone. Also, here \vec{A} and \vec{B} and n_e denote magnetic vector potential, magnetic field vector and the plasma density respectively. We choose to normalize the electron density with the minimum value of the plasma density n_{00} in the region of interest, and the length by the corresponding skin depth $d_{e0} = c/\omega_{pe0}$ ($\omega_{pe0} = 4\pi n_{00}e^2/m_e$) corresponding to this density. The magnetic field is normalized by

B_0 , the typical magnitude of the magnetic field and the time by the corresponding electron gyrofrequency $\omega_{ce0} = eB_0/m_e c$. The normalized equation can then be written as

$$\frac{\partial \vec{g}}{\partial t} = \nabla \times [\vec{v} \times \vec{g}] \quad (2)$$

$$\vec{v} = -\frac{1}{n} \nabla \times \vec{B}; \quad \vec{g} = \frac{\nabla^2 \vec{B}}{n} - \nabla \left(\frac{1}{n} \right) \times (\nabla \times \vec{B}) - \vec{B} \quad (3)$$

Equation(2,3) represents the G-EMHD model written in normalized fields and variables.

For the case when the variations of the fields are confined in two dimensional $x - y$ plane, the G-EMHD model (Eq.(2,3)) can be represented by a coupled set of evolution equations for two scalar fields b and ψ . These two scalar fields define the magnetic field of the system through the following relationship:

$$\vec{B} = b\hat{z} + \hat{z} \times \nabla\psi \quad (4)$$

Thus b represents the magnetic field component along the symmetry direction \hat{z} and the magnetic field along x and y directions are $-\partial\psi/\partial y$ and $\partial\psi/\partial x$ respectively. The normalized electron velocity, the expression for g , etc., in terms of these two scalar fields can then be written as

$$\begin{aligned} \vec{v} &= \frac{\hat{z} \times \nabla b}{n} - \frac{\nabla^2 \psi}{n} \hat{z} \\ \vec{g} &= \frac{1}{n} \left[\nabla^2 b \hat{z} + \nabla^2 (\hat{z} \times \nabla\psi) \right] + \nabla \left(\frac{1}{n} \right) \times [\hat{z} \times \nabla b - \nabla^2 \psi \hat{z}] - [b\hat{z} + \hat{z} \times \nabla\psi] \end{aligned} \quad (5)$$

Equations (2,3) (which describe the evolution of \vec{g}) may be separated in terms of components g_z and \vec{g}_\perp from which we may derive the following coupled set of evolution equations for b and ψ fields.

$$\frac{\partial}{\partial t} \left\{ b - \nabla \cdot \left(\frac{\nabla b}{n} \right) \right\} + \hat{z} \times \nabla b \cdot \nabla \left[\frac{1}{n} \left\{ b - \nabla \cdot \left(\frac{\nabla b}{n} \right) \right\} \right] + \hat{z} \times \nabla\psi \cdot \nabla \left(\frac{\nabla^2 \psi}{n} \right) = 0 \quad (6)$$

$$\frac{\partial}{\partial t} \left\{ \psi - \frac{\nabla^2 \psi}{n} \right\} + \frac{\hat{z} \times \nabla b}{n} \cdot \nabla \left\{ \psi - \frac{\nabla^2 \psi}{n} \right\} = 0 \quad (7)$$

Equations(6,7) represent the Generalized EMHD equations in two dimensions. When the plasma density n is a constant, the above coupled set of equations reduce to the EMHD equations in 2-D.

The sum of magnetic and the kinetic energy of the system is given by the expression

$$E = \int \int \left[b^2 + \frac{(\nabla b)^2}{n} + (\nabla\psi)^2 + \frac{(\nabla^2 \psi)^2}{n} \right] dx dy$$

and is an invariant for the above coupled set of G-EMHD equations. However, when the in plane component of the magnetic field is zero, i.e. when $\psi = 0$, an additional square invariant

$$Q = \int \int \frac{1}{n} \left[b - \nabla \cdot \frac{\nabla b}{n} \right]^2 dx dy$$

also exists for the system. Note that for the homogeneous case $Q-E$ represents the enstrophy (space integral of mean square vorticity), which is the second invariant for the system.

In the subsequent sections we will discuss the results of the numerical simulation of Eqs.(6,7) for several distinct choices of plasma inhomogeneity. Equations (6) and (7) are evolved for the generalized vorticities $\Omega_b = L_b b = (b - \nabla \cdot (\nabla b/n))$ and $\Omega_\psi = L_\psi \psi = (\psi - \nabla^2 \psi/n)$ using the flux corrected algorithm developed by Boris *et. al* [14]. (Here L_b and L_ψ are used to denote the operators, which upon operating on b and ψ produce Ω_b and Ω_ψ respectively.) The value of the b and ψ at each time step are then extracted from the generalized vorticities Ω_b and Ω_ψ respectively by constructing matrices corresponding to the inverse operators L_b^{-1} and L_ψ^{-1} for the chosen spatial inhomogeneity of the plasma density. It should be noticed here that the dimension of this matrix being extremely large ($N \times N$, where $N = Nx \times Ny$, where Nx and Ny represent the number of grid points along x and y directions respectively) it puts a severe constraint on the resolution. To adequately resolve the skin depth at the maximum plasma density and to simultaneously also have the box dimension longer than several electron skin depths (corresponding to the minimum value of plasma density) we were constrained to choose the ratio of maximum to minimum plasma density to be at most a factor of 10 only. Alternatively, to achieve higher resolutions a relaxation algorithm to evaluate $L_b^{-1}\Omega_b$ and $L_\psi^{-1}\Omega_\psi$, needs to be implemented [15] which utilizes the standard solvers for Helmholtz equation where the RAM requirements would not be so extensive. We are in the process of implementing this for our future studies on G-EMHD model.

III. COHERENT EMHD SOLUTIONS FOR HOMOGENEOUS PLASMA

The G-EMHD equations (Eqs.(6,7)) reduce to the following form for a homogeneous density $n = n_0$.

$$\frac{\partial \Omega_b}{\partial t} + [b, \Omega_b] = [\psi, \Omega_\psi] \quad (8)$$

$$\frac{\partial \Omega_\psi}{\partial t} + [b, \Omega_\psi] = 0 \quad (9)$$

Here, the uniform plasma density has been chosen for the density normalization. The symbol $[A, B]$ used in the equations represents a Poisson bracket between the field A and B . It is then clearly evident from the above equations that for the homogeneous system all radially symmetric (monopolar) structures are exact stationary solutions of the equations. Furthermore, it has been observed that monopolar structures localized within a spatial extent of electron skin depth are fairly robust and stable and they are spontaneously created during simulations with arbitrary initial configurations. A collection of monopoles which are spatially separated by a distance larger than the electron skin depth also constitute a stationary configuration of the above set of equation. This is so as in this case the self term of the Poisson bracket vanishes on account of radial symmetry and the cross term between two structures vanishes as there is no spatial overlap amidst them. It is then interesting to see how these structures behave in the presence of density inhomogeneity. This is explored by us by numerically simulating the evolution of these structures in a given inhomogeneous density profile. The results of such studies are presented in the next section.

The other interesting class of solutions permitted by Eqs.(8,9) have dipolar form [11]. These solutions are not stationary but have a steady translational velocity with respect to the static ion frame of the EMHD model. The dipolar solutions can be obtained by seeking stationarity in a moving frame $\xi = y - ut$ (assuming the translational velocity to be along the y coordinate). Equation(9) can then be written as the vanishing of the Poisson bracket $[\Omega_\psi, b - ux] = 0$. This implies that $\Omega_\psi = f_\psi(b - ux)$, where f_ψ is a function of $b - ux$. Using this expression for Ω_ψ as well as the stationarity criteria in the ξ frame, Eq.(8) can be expressed as

$$[\Omega_b + f'_\psi \psi, b - ux] = 0;$$

Here $'$ indicates a differential with respect to the argument $(b - ux)$ of the function f_ψ . This suggests that $\Omega_b + f'_\psi \psi = f_b(b - ux)$ (f_b being another function of $b - ux$). Thus a travelling solution can be obtained by seeking solutions of the following coupled set:

$$\begin{aligned} \nabla^2 \psi - \psi &= f_\psi(b - ux) \\ \nabla^2 b - b + f'_\psi \psi &= f_b(b - ux) \end{aligned} \quad (10)$$

The general solutions would correspond to any choice of the functions f_b and f_ψ . However,

analytical form of the dipole solutions are typically obtained by choosing the vorticity source functions f_b and f_ψ as linear functions of their argument ($b - ux$) in the inner spatial region of radii $r \leq r_0$ around the centre of the structure. The differential equation can be separated in r and θ coordinates in the two dimensional $x - y$ plane. It can be shown that an explicit appearance of $x \sim r \cos(\theta)$ (as a coefficient of u) produces a $\cos(\theta)$ dependence and hence a dipolar form of the solution. Furthermore for the radial part in this inner region, the solutions can be represented in terms of a Bessel function of the first kind denoted by J . To achieve localization the vorticity source functions f_b and f_ψ are chosen to be zero in the outer spatial region $r > r_0$. The solutions for the fields b and ψ in the outer region are thus modified Bessel function K of the radial coordinate. The matching of the field and its derivative in the inner and outer region yields a localized solution.

It might appear that the specific linear choice made for the inner region is too restrictive and may in general not correspond to reality. However, it has been shown recently (for the system with $\psi = 0$) that a linear choice of f_b is consistent with a self organization paradigm based on the enstrophy minimization subject to the constancy of the energy of the system [13]. The approach of the system to such a self organized state has been confirmed by numerical simulations which show the spontaneous formations of structures from an arbitrary initial condition satisfying Eq.(10) for a linear functional form of f_b [13]. Similar studies in the presence of finite ψ field are underway and the results will be reported elsewhere. In the present manuscript we report the propagation characteristics of the dipolar solutions for both cases (viz. $\psi = 0$ and ψ finite, with f_b and f_ψ both as linear functions of the argument $b - ux$) as they encounter the inhomogeneous plasma density profile.

It should be noted that a monopolar structure is like a circularly rotating current similar to that of a solenoid. The propagating dipoles on the other hand can be viewed as an electron current pulse translating in space. The central spatial region within the positive and the negative peaks of the magnetic structure carries electron current in the direction of the propagation of the dipole, whereas the outer edge region carries a reverse current as shown in the schematic plot of Fig.1. Thus the dipole structure can be viewed as a current pulse containing a spatially separated forward and the reverse shielding electron current pulse. The current configuration of a dipole is thus quite similar to the one encountered during the ignition phase of the fast ignition experiments. The forward energetic electrons generated at the critical density surface moves towards the dense target core. The background plasma in

this case provides for the return shielding current. The two currents get spatially separated through Weibel instability and form current channels in which the inner region carries the forward current and the outer region the return shielding current. The numerical study of the propagation of the dipole magnetic structure through an inhomogeneous plasma and its behaviour in the high density region is therefore an important issue for investigation. In the subsequent sections the result of such a simulation will be presented.

IV. NUMERICAL STUDIES

In this section we study the propagation of both monopolar and dipolar solutions through an inhomogeneous plasma. The plasma density inhomogeneity is chosen to have the following spatial profile

$$n(x, y) = a - b \tanh \left\{ \frac{\sqrt{y^2} - w}{\sigma} \right\} \quad (11)$$

The parameters a and b were chosen appropriately to define either a density hump in the region $|y| \leq w$ or a density cavity. For the above chosen density profile the constant density contours form straight lines parallel to the x axis.

A. Propagation of monopoles

The monopolar structures are the exact stationary equilibrium solutions of the homogeneous EMHD equations. In order to study the influence of plasma inhomogeneity on their evolution we place a monopolar structure initially at the boundary region (around $y \sim w$), where the plasma density inhomogeneity is typically high. The subplots of the Fig.2 show the evolution of this structure. For this case we had chosen the simulation box of size $L_x = L_y = 10$ and x and y coordinates range from -5.0 to 5.0 . For the plasma density we had chosen $a = 5.5$, $b = 4.5$, $w = 2.5$ and $\sigma = 1.0$. The maximum and minimum value of density is therefore $n_{max} = 10$ and $n_{min} = 1$ respectively. The local electron skin depth therefore ranges from $0.3 \leq d_e \leq 1.0$. The high density plasma region here is confined within $|y| \leq w$ for all x . The density falls sharply within a length $\delta y = \sigma$ from 10 to unity beyond $|y| \gtrsim w$. The density scale length at the boundary is typically of the order of the electron skin depth as $\sigma = 1.0$.

From the figure it is clear that the monopolar structure moves transverse to the density

gradient. The monopoles being stationary in the homogeneous plasma the propagation velocity is clearly an artifact of the presence of plasma density inhomogeneity. The direction as well as the magnitude of the propagation velocity is observed to match with the following drift velocity.

$$\vec{V}_n = \frac{b\hat{z} \times \nabla n}{n^2} \quad (12)$$

which can be obtained from the Eq.(6) upon ignoring electron inertia related terms. For the density profile of Eq.(11) n is a function of y alone and we have the propagation along x and the magnitude of the velocity is

$$V_{nx} = b \frac{\partial}{\partial y} \left(\frac{1}{n} \right)$$

From the subplots of the Fig.2, the value of V_{nx} evaluated by observing the distance propagated by the structure along x is 0.0307 which is close to that estimated from the above expression for the electron drift velocity, as b typically ranges from 0.0233 to 0.1997 in the monopolar structure and $\partial(1/n)/\partial y$ ranges from 0.1131 to 0.448 over the structure. This implies that the value of V_{nx} from the expression can be about 0.0026 to 0.089. The observed value lies within this range. In fact the average of V_{nx} evaluated over the y extent of the structure (through which the structure would translate) turns out to be very close 0.0369 to the observed velocity. This clearly indicates that the monopole is essentially propagating with the drift velocity of $\langle V_{nx} \rangle$.

The other density gradient dependent terms arising through the finite electron inertia related terms are typically smaller in magnitude and they generally contribute as a source causing modification of the spatial profile of the magnetic structure.

B. Propagation of dipoles

We next study the behaviour of dipoles as they encounter the region of plasma density inhomogeneity. Since the dipolar structures are known to propagate along their axis, we start our simulation by placing an exact nonlinear dipole structure at some distance away from the density gradient region, i.e. at a location of a uniform low density plasma where $n = 1$ at a positive $y > w$. The sign of the lobes in this case are chosen in such a way that the dipole propagates towards the high density region. We observe that the dipolar structure crosses past the inhomogeneous density region to enter the high plasma density

region. The subplots of Fig.3 clearly illustrate the penetration of the dipolar structure in the high density plasma region. (For the plots of this figure we again have $a = 5.5$, $b = 4.5$, $w = 2.0$ and $\sigma = 0.4$. The box length in this case is $L_x = L_y = 4\pi$.) In fact we observe that at the inhomogeneous density region the axial translational velocity of the dipole increases considerably. The two lobes get squeezed towards each other forming a shock like structure in the direction transverse to the density gradient. This behaviour appears to be in stark contrast to the propagation characteristics of the monopolar structures, which merely show a movement transverse to the density gradient direction.

The observations on dipole propagation can, however, be understood readily. For the dipole structure approaching the high density plasma region (along decreasing y , in Fig.3) the left lobe corresponds to positive values of b whereas the right lobe has negative b values. Clearly, when the two lobes of the dipole encounter the density inhomogeneity the left lobe has a drift velocity due to the density inhomogeneity towards right (positive x direction) whereas the right one drifts towards the negative x direction. This squeezes the two lobes of the dipoles closer in x . As a result the size of the lobes as well as their separation gets significantly reduced. This also causes an enhancement in the magnitude of $|b|$ of the two lobes. The reduced distance between the lobes as well as the enhanced amplitude of $|b|$ results in an increased axial propagation velocity of the dipole. This accelerates the penetration of the dipolar structure in the high density plasma region. It should be noted that for the case where the dipole approaches a density cavity the effect is entirely different. The sign of ∇n being opposite, in this case the lobes separate with the drift speed. The separation results in a reduced axial velocity of the dipole, which ultimately diminishes to zero as the separation distance between the two lobes exceeds the electron skin depth distance. The two lobes then separately move as monopolar structures transverse to the density gradient direction. Thus, the dipole is unable to penetrate the region of lower plasma density. Our simulations indeed show this effect as the subplots of Fig.4 clearly illustrate. Here the central region $|y| \leq w$ corresponds to a low density plasma region with $n = 0.0.2$. Here $a = 0.6$, $b = -0.4$ and other parameters are same as that of Fig.3.

Let us now study in detail the behaviour of the dipole as it enters the high density plasma region. Though the shape of the dipole is considerably distorted while it traverses the inhomogeneous plasma region, but once it is inside the high density homogeneous plasma region it regains the familiar dipolar form. The scale length of the dipole, in the high density

region changes by the same factor as the ratio of the skin depth of the high and low density regions. For instance the initial dipole was chosen to have $r_0 = 1.0$ and at $t = 690$ when it is completely inside the high density region the value of $r_0 = 0.47$ (a reduction by a factor of approximately $1/3$). We thus observe that the dipolar structures are fairly robust. Even after encountering a strong density inhomogeneity, once in the region of homogeneous plasma they adjust smoothly to the new value of the density.

In the next section we illustrate an interesting consequence of the above observations, namely that of novel trapping behaviour of EMHD dipoles in a high density plasma region.

V. TRAPPING OF EMHD DIPOLES IN HIGH DENSITY PLASMA

The propagation characteristics of the dipolar structure discussed in the preceding section has an interesting consequence. It suggests that the EMHD magnetic structures of dipolar form can enter a high density plasma region. However, once inside a high density plasma region they would remain trapped there.

We choose a density profile $n(x, y)$ which has a high density region with a finite transverse extent as well. A circular region in the $x - y$ plane is chosen to have a high density of the plasma. The functional form of the plasma density is

$$n(x, y) = a - b \tanh \left\{ \frac{\sqrt{x^2 + y^2} - w}{\sigma} \right\} \quad (13)$$

The choice of parameters for simulation with this density profile is $a = 5.5; b = 4.5; w = 2.0; \sigma = 0.4$. For this particular density profile a dipole is placed with its centre on the line $x = 0$ at the positive value of $y = 4.0$. The axis of the dipole is parallel to the y axis as can be seen from the subplot at $t = 0$ of Fig.5. The dipole velocity is directed along the negative y axis so that it approaches the high density plasma region. It can be seen from the subsequent subplots that due to the individual density inhomogeneity related drift velocity of the two lobes, the two lobes of the dipole approach each other. This enhances the axial dipolar velocity and the dipole structure enters the high density region. Once inside the homogeneous high density region it translates along its axis which is along the diameter of the circular high density region. Upon reaching the other end the dipole again encounters the inhomogeneous plasma density region. However, the direction of the density gradient is now opposite to the one that it encountered while entering the high density region. Thus in

this region the two lobes of the dipole separate from each other. As the separation between the lobes exceeds the skin depth distance the lobes act like individual monopolar structures and move transverse to the density gradient. In this case the density gradient being along the radial direction, the two structures move along the perimeter of the circle. They thus again come in close contact at the topmost point of the circle from where they had entered the high density region. At this place they again form a dipolar structure and translate along the diameter of the high density region. The simulations clearly show the repetition of this cycle. It is thus clear that the dipole structure remains trapped inside the high density plasma region.

These numerical results showing penetration of the dipolar structures inside the high density region and its trapping within the high density region suggests an important implication for the problem of fast ignition [6]. In this scheme one relies on energetic electrons generated at the critical density surface by the ignitor laser pulse to penetrate the high density pre-compressed target core and deposit its energy there for the creation of hot spot. Here, we have shown that a current pulse in the form of a dipolar structure (similar in form to a forward moving electron current and a spatially separated reverse shielding current) can easily penetrate a high density plasma region. It has also been observed that once inside the compressed target the structure is unable to come out of the compressed high density region. This suggests that there will be ample time for them to dump their energy in the high density compressed region of the core of the fusion target plasma.

Another interesting thing to note is the fact that as the dipole enters the high density region the squeezing of its lobes results in a shock formation along the direction transverse to the density gradient. Such a shock formation is an artifact of the thermoelectric like source term $\nabla P \times \nabla n$ in the magnetic field evolution equation where the self consistent magnetic field B^2 , itself acts as a pressure term. This nonlinearity is responsible for the shock formation. The sheared current layer in the magnetic shock structure would lead to the Kelvin Helmholtz (KH) like instability [16]. The instability converts the electron flow energy into fine scale vortices. In the three dimensional case such vortex flows cascade energy towards finer scales [17] and eventually dissipate into heat by electron Landau damping effects. We would show through simulations as well as analytical modelling in a subsequent publication that the shock width adjusts in such a fashion that the energy dissipation in such a shock structure is essentially independent of the value of the dissipation mechanism.

This again is very important in the context of fast ignition scheme as it suggests that the energy dissipation would occur no matter how small is the value of the classical collisional damping process in the dense target core.

Furthermore, in some recent fast ignition experiments it has been observed that if a metallic wire is attached to the gold cone where the ignitor laser pulse is incident, the energetic electrons generated at the critical density surface choose a preferential path provided by the ionized wire to traverse from the low density plasma corona towards the compressed target core [18]. This result is important as it indicates that the energetic electrons can be guided in the low density plasma corona region to reach the dense target core. An understanding of this phenomena has, however, been lacking. We feel that this can again be understood on the basis of the novel trapping mechanism outlined above. The ionized wire in this case forms a high density plasma path amidst the low density coronal plasma. The high density region can then trap the current pulse structure formed by the combination of the forward energetic electrons and the background reverse shielding current.

VI. SUMMARY

A generalized Electron Magnetohydrodynamic (G-EMHD) model has been introduced here which accounts for the fluid model description of fast electron time scale phenomena in an inhomogeneous plasma. The electron density fluctuations have, however, been ignored by restricting to times scales which are slower than electron plasma period $\omega < \omega_{pe}$ and/or $\omega < \omega_{pe}^2/\omega_{ce}$ whichever is smaller. This is the same condition which is invoked for the derivation of the EMHD model and justifies ignoring the displacement current. A reduced G-EMHD model for 2-D case has also been obtained. The model is represented in terms of a coupled set of evolution equation for two scalar fields. The two scalar fields correspond to the component of the magnetic field and the vector potential along the symmetry direction.

The G-EMHD equations in 2-D were used to study numerically the evolution and propagation of nonlinear coherent solutions of the EMHD equations in the presence of density inhomogeneity. The two varieties of coherent solutions (viz., the stationary monopolar solutions and the travelling dipolar solutions) show interesting novel aspect of propagation as they encounter a region of plasma density inhomogeneity. The monopolar structures are no longer stationary but are observed to propagate with a diamagnetic drift velocity. The

dipoles on the other hand display a very novel behaviour. They show penetration inside the high density plasma region from a lower density side. However, once inside the higher density region they get trapped inside this region. It has been shown that this behaviour of trapping can have interesting favourable implications in the context of the energetic electron propagation and their stopping inside the dense target core in the context of fast ignition experiments.

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FIGURE CAPTIONS

Fig.1 A schematic plot showing the dipole structure (subplot(a)) represented by constant contour lines in the $x - y$ plane for the magnetic field b along the symmetry direction. The subplot(b) shows a plot of the magnetic field amplitude as a function of x at the mid y plane of the dipolar structure. In subplot (c) the corresponding electron current is shown. The central region represents the forward electron current (electron current along the propagation direction of the dipole) and the reverse electron current flows at the edges.

Fig.2 (Color Online) The propagation of the monopolar structure (color contours) in a inhomogeneous plasma density is depicted by showing the location of the structure at various times in the different subplots of the figure. The thick black lines represent the plasma density contour. In this case the plasma density is chosen to be a function of y only. The central y region of width $w = \pm 2.0$ corresponds to a high density (10 times the density at the edge ($| y | \geq 2.0$)). The monopole structure is seen to propagate transverse to the density gradient.

Fig.3 (Color Online) Various stages of the propagation of a dipolar structure through an inhomogeneous density plasma has been shown. The inhomogeneity in plasma density is similar to that of Fig.2 in this case. The figure clearly shows the penetration of the dipole through the plasma density inhomogeneity to enter the high density region. The lobes of the dipole structure are squeezed towards each other as they pass through the inhomogeneous region. However, once inside the high density homogeneous region they again acquire a balanced dipolar form.

Fig.4 (Color Online) In this figure the dipole is shown to approach a density cavity (lower density plasma region). It can be observed that the dipole is unable to penetrate the lower density plasma. The two lobes of the dipole get separated transverse to the density gradient direction and subsequently they evolve as separate monopolar structures.

Fig.5 (Color Online) The trapping of the dipolar structure in a high density plasma has been illustrated in this figure. A high density plasma with a circular profile in the $x - y$

plane represented by the thick black contour lines are depicted on the various subplots. A dipole structure can be seen to penetrate the high density region. However, once inside the high density region it continues to remain rapped in this region.

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