Instability of Shear Waves in an Inhomogeneous Strongly Coupled Dusty Plasma

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It is demonstrated that low frequency shear modes in a strongly coupled, inhomogeneous, dusty plasma can grow on account of an instability involving the dynamical charge fluctuations of the dust grains. The instability is driven by the gradient of the equilibrium dust charge density and is associated with the finite charging time of the dust grains. The present calculations, carried out in the generalized hydrodynamic viscoelastic formalism, also bring out important modifications in the threshold and growth rate of the instability due to collective effects associated with coupling to the compressional mode.

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I. INTRODUCTION

Dusty plasmas are of great interest because of their possible applications to a number of fields of contemporary research such as plasma astrophysics of interplanetary and interstellar matter, fusion research, plasmas used for semiconductor etching, arc plasmas used to manufacture fine metal and ceramic powders, plasmas in simple flames etc. [1]. It is now widely recognized that the dust component in these plasmas is often in the strongly coupled coulomb regime with the parameter, $\Gamma \simeq (Z_d e)^2/T_d d$, typically taking values much greater than unity ($(-Z_d e)$ is the charge on the dust particle, $d \simeq n_d^{-1/3}$ is the interparticle distance and T_d is the temperature of the dust component). This leads to many novel physical effects such as the formation of dust plasma crystals [2], modified dispersion of the compressional waves [3,4], the existence of the transverse shear waves [4] etc. Many of these novel features have now been verified by experiments and computer simulations [5].

Recently, an experiment on the self–excitation of the vertical motion of the dust particles trapped in a plasma sheath boundary, has been reported [6]. The physics of this excitation is related to charging of the dust particles by the inflow of ambient plasma currents in the inhomogeneous plasma sheath and the delay resulting because of the finite time required by the charging process to bring the dust charge to its ambient steady state value. In this paper, we demonstrate that the same physical mechanism can be used for the excitation of the transverse shear modes in an inhomogeneous strongly coupled dusty plasma. Using a generalized hydrodynamic viscoelastic formalism [7] to describe the strongly coupled dusty plasma and incorporating the novel feature of time variation of the dust charge through a charge dynamics equation [8], we have derived a general dispersion relation for low frequency shear and compressional modes in the plasma. We find that in a plasma with finite gradients of the equilibrium dust charge density, the two modes are coupled and we show that the shear mode is driven unstable if certain threshold values are exceeded.

Our paper is organized as follows. In the next section we briefly discuss the equilibrium of an inhomogeneous dusty plasma that is confined against gravity by the electric field of a plasma sheath. In such a configuration dust particles of varying sizes and charges arrange themselves in horizontal layers at different heights to form a nonuniform cloud [9,10]. In section 3 we carry out a linear stability analysis of such an equilibrium in the framework of the generalized hydrodynamic equations. The dispersion relation of the coupled shear wave and compression wave is solved analytically (in simple limits) as well as numerically in section 4. The physical mechanism of the shear wave instability is also discussed and the modifications in the threshold and growth rate brought about by the coupling to compressive waves are elucidated. Section 5 is devoted to a summary and discussion of the principal results.

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II. DUST CLOUD EQUILIBRIUM

We consider an inhomogeneous sheath equilibrium in which the dust particles are suspended with electric field forces balancing the gravitational force on the particle and in which the dust charge $(-Z_d e)$ and dust size r_d are both functions of the vertical distance z. Then the force balance equation gives,

$$Z_d(z)eE_0(z) = \frac{4}{3}\pi r_d(z)^3 \rho g, \tag{1}$$

where, ρ , g , E_0 refer respectively to the dust mass density, gravitational acceleration and the sheath electric field. For particle sizes of the order of a few microns, other forces acting on the particle (such as the drag and viscous forces) are about an order of magnitude smaller than the electric and gravitational forces and can be neglected for the equilibrium calculation [10]. Note that for dust particles of a uniform size (monodispersive size distribution) the above equilibrium can only be attained at one vertical point leading to a monolayer of dust. A dispersion in sizes leads to a large number of layers resulting in a nonuniform dust cloud with a gradient in the equilibrium charge $(-Z_d e)$ and the dust size r_d. The electric field E_0 is determined by the sheath condition,

$$\frac{dE_0}{dz} = -4\pi e\left(n_e - n_i + Z_d n_d\right) \tag{2}$$

where $n_{e,i,d}$ are the local electron, ion and dust densities respectively. The charge $(-Z_d e)$ on a dust particle in the sheath region is given by $(-Z_d e) = C_d(\phi_f - \phi)$ where C_d is the capacitance, ϕ_f is the floating potential at the surface of the dust particle and ϕ is the bulk plasma potential. For a spherical dust particle $C_d = r_d$, and the floating potential can be determined by the steady state condition from the dust charging equation, namely, [8]

$$I_e + I_i = 0 \tag{3}$$

where the electron and ion currents impinging on the dust particle are given by [1]

$$I_e = -\pi r_d^2 e\left(\frac{8kT_e}{\pi m_e}\right)^{1/2} n_e \exp\left[\frac{e}{kT_e}(\phi_f - \phi)\right], \tag{4a}$$

$$I_i = \pi r_d^2 e\left(\frac{8kT_i}{\pi m_i}\right)^{1/2} n_i\left[1 - \frac{2e}{kT_i + m_i v_{id}^2}(\phi_f - \phi)\right]. \tag{4b}$$

Here T_e and T_i are the electron and ion temperatures, m_i is the ion mass and v_{id} is the mean drifting velocity of the ions in the electric field of the sheath (it is assumed to be the ion sound velocity at the sheath edge). We also assume that the dust particles have much smaller thermal velocities than the electrons and ions.

Equations (1 - 3) selfconsistently determine the equilibrium of the dust cloud. Such clouds have been experimentally observed in a number of experiments [9,10]. In [10] theoretical modeling along the lines discussed above, agree very well with the experimental observations of clouds formed with polydispersive particle size distribution of dust particles trapped in the plasma sheath region. A typical equilibrium variation of the dust particle size with the vertical distance, when the Child Langmuir law holds for the plasma sheath potential, is given as [10],

$$r_d = \left(\frac{3(\phi_f - \phi)}{4\pi\rho g}\right)^{1/2}\left(\frac{6\pi e n_s C_s}{\mu_i}\right)^{1/3}(\delta - z)^{1/3} \tag{5}$$

where n_s , C_s are the plasma density and the ion sound velocity, δ is the thickness of the sheath and $\mu_i = (e\lambda_{i-n}/m_i)^{1/2}$ with λ_{i-n} representing the mean free path of ions colliding with the background neutrals. Using (5) we can obtain the corresponding z variation for Z_d.

As discussed in detail in [10], this dust cloud equilibrium is confined to the plasma sheath boundary region in the potential well created from the upward electrostatic and downward gravitational forces. Note that the force balance equation (1) does not prevent the particles from oscillating about their mean positions especially if they have significant kinetic energy or temperature. However their mean positions are at various vertical distances and the mean Z_d is a function of z. This is reminiscent of particle gyrations in a magnetic field. If we consider wave motions in which dust oscillation excursions are much smaller than wavelengths, we can use a fluid theory to analyze such behaviour. In the next section, we adopt this view point and carry out a linear stability analysis of the equilibrium discussed above to low frequency wave perturbations.

III. LINEAR STABILITY ANALYSIS

For low frequency perturbations in the regime $kv_{thd} << \omega << kv_{the}, kv_{thi}$, where v_{thd}, v_{the} and v_{thi} are the thermal velocities of the dust, electron and ion components respectively, the electron and ion responses obey the Boltzmann law which can be simply obtained from an ordinary hydrodynamic representation. The dust component on the other hand can be in the strongly coupled regime for which a proper description is provided by the generalized viscoelastic formalism. Using such a description a general dispersion relation for low frequency waves (with typical wavelengths longer than any lattice spacings) was obtained in [4] for longitudinal sound waves and transverse shear waves. The shear modes exist in a strongly coupled dusty plasma because of elasticity effects introduced by strong correlations [4]. Our objective in this work is to look for the effect of dust charge dynamics on these shear modes in the strongly coupled regime. As demonstrated in our earlier work [4], the coupling of the low frequency shear modes to transverse electromagnetic perturbations is finite but negligibly small; we ignore this coupling here. However, introduction of the dust charge dynamics in the inhomogeneous plasma leads to a coupling of the low frequency shear and compressional modes; thus the space charge dynamics and quasineutrality condition play an important role in describing the perturbations. The basic equations for the dust fluid [7] we work with, are the continuity equation,

$$\frac{\partial}{\partial t}\delta n_d + n_{d0}\vec{\nabla}\cdot\delta\vec{u}_d + \frac{n_{d0}}{M}(\delta\vec{u}_d\cdot\vec{\nabla})M = 0, \tag{6}$$

the equation of motion,

$$\Big(1+\tau_m\frac{\partial}{\partial t}\Big)\Bigg[\Big(\frac{\partial}{\partial t}+\nu\Big)\delta\vec{u}_d + \frac{\vec{\bigtriangledown}\delta P}{M n_{d0}} + \frac{Z_d e}{M}\delta\vec{E} + \frac{\delta Z_d}{M}e\vec{E}_0\Bigg] = \frac{1}{M n_{d0}}\Big[\eta\vec{\bigtriangledown}^2\delta\vec{u}_d + \big(\zeta+\frac{\eta}{3}\big)\vec{\bigtriangledown}(\vec{\bigtriangledown}\cdot\delta\vec{u}_d)\Big], \tag{7}$$

and the equation of state, $(\partial P/\partial n)_T \equiv MC_d^2$, given in terms of the compressibility, μ_d, as [4]

$$\mu_d \equiv \frac{1}{T_d}\Big(\frac{\partial P}{\partial n}\Big)_T = 1 + \frac{u(\Gamma)}{3} + \frac{\Gamma}{9}\frac{\partial u(\Gamma)}{\partial\Gamma}, \tag{8}$$

with the excess internal energy of the system given by the fitting formula [11]

$$u(\Gamma) = -0.89\Gamma + 0.95\Gamma^{1/4} + 0.19\Gamma^{-1/4} - 0.81. \tag{9}$$

In the above, M is the dust mass, ν is the dust–neutral collision frequency, δu_d, δn_d and δZ_d are the perturbations in the dust velocity, number density and dust charge, δP, δE are the pressure and electric field perturbations, n_{d0} and Z_d are the equilibrium number density and charge for the dust and E_0 is the unperturbed electric field. η and ζ refer to the coefficients of the shear and bulk viscosities and τ_m is the viscoelastic relaxation time. Note that in the continuity equation we have a contribution from the equilibrium inhomogeneity in the dust mass distribution (arising from the size dispersion of the particles). This term as we shall see later modifies the real frequency of the shear waves.

These equations are supplemented with the dynamical equation for the dust charge perturbations which, for perturbations with phase velocity much smaller than the electron and ion thermal velocities, is given as [8]

$$\frac{\partial}{\partial t}(\delta Z_d) + \delta\vec{u}_d\cdot\vec{\bigtriangledown}Z_d + \eta_c\delta Z_d = -\frac{|I_{e0}|}{e}\Bigg(\frac{\delta n_i}{n_{i0}} - \frac{\delta n_e}{n_{e0}}\Bigg), \tag{10}$$

where, $\eta_c = \Big(e|I_{e0}|/C\Big)\Big(1/T_e + 1/w_0\Big)$ is the inverse of charging time of dust grains and $w_0 = T_i - e(\phi_f - \phi)_0$. Note that the second term on the left hand side of eq.(10) arises because of the inhomogeneity of the mean charge on the dust particles; as shall be shown later, this is the critical term responsible for the instability. It is also obvious that the dust charge variation in space will lead to shielding by electrons and ions with the associated coupling of the perturbation to dust compressional modes. We must thus extend the above set of equations to include the quasi-neutrality condition,

$$\delta n_e + Z_d\delta n_d + n_{d0}\delta Z_d - \delta n_i \simeq 0, \tag{11}$$

and the equation describing the electron and ion density perturbations in terms of the potential, as

$$\frac{\delta n_e}{n_{e0}} = \frac{e\delta\phi}{T_e}; \quad \frac{\delta n_i}{n_{i0}} = -\frac{e\delta\phi}{T_i}. \tag{12}$$

These are the Boltzmann relations which arise whenever the perturbations satisfy $\omega << kv_{the}, kv_{thi}$.

We shall next derive the dispersion relation for the low frequency mode. We may note that the typical time scale for the decay of the charge fluctuations for the dust can be very small [6], with $\eta_c >> \omega$ and we shall work in that limit. We use the local approximation (wave lengths smaller than characteristic equilibrium scale lengths) and choose the propagation vector for the wave perturbation as $\vec{k} = (k, 0, 0)$, the perturbed dust velocity, $\delta\vec{u}_d = (\delta u_1, 0, \delta u_3)$ and the perturbation in the electric field as $\delta\vec{E} = -ik\delta\phi(1, 0, 0)$. Using the continuity equation (6) and the equations (10) – (12), and after some simple algebra, one obtains the fluctuation in the dust charge and the potential as

$$\delta Z_d = \frac{a_1}{D}\left(\frac{k}{\omega}\right)\delta u_1 + \left(\frac{a_2}{D} + \frac{a_3}{(i\omega)D}\right)\delta u_3, \tag{13a}$$

$$\delta\phi = -\frac{Z_d n_{d0}\eta_c}{eD}\left(\frac{k}{\omega}\right)\delta u_1 + \frac{n_{d0}}{eD}\left(Z_d' - \frac{Z_d M'\eta_c}{M(i\omega)}\right)\delta u_3, \tag{13b}$$

where,

$$a_1 = -\frac{|I_{e0}|}{e}\left(\frac{1}{T_e} + \frac{1}{T_i}\right)Z_d n_{d0}; \ a_2 = -Z_d'\left(\frac{n_{e0}}{T_e} + \frac{n_{i0}}{T_i}\right),$$

$$a_3 = -\frac{|I_{e0}|}{e}\left(\frac{1}{T_e} + \frac{1}{T_i}\right)\frac{M'}{M}n_{d0}Z_d$$

$$D = \eta_c\left(\frac{n_{e0}}{T_e} + \frac{n_{i0}}{T_i}\right) + n_{d0}\frac{|I_{e0}|}{e}\left(\frac{1}{T_e} + \frac{1}{T_i}\right), \tag{14}$$

and the primes denote derivatives with respect to z the vertical direction. We then write down the longitudinal and transverse components of the dust momentum equation (i.e. of equation (7)), as

$$(1 - i\omega\tau_m)\Big[(-i\omega + \nu)\delta u_1 + ik\frac{\delta P}{Mn_{d0}} - \frac{Z_d e}{M}(ik\delta\phi)\Big]$$
$$= -\frac{1}{Mn_{d0}}\eta_l k^2\delta u_1 \tag{15a}$$

$$(1 - i\omega\tau_m)\Big[(-i\omega + \nu)\delta u_3 + \frac{\delta Z_d}{M}eE_0\Big] = -\frac{1}{Mn_{d0}}\eta k^2\delta u_3, \tag{15b}$$

where, $\eta_l = \frac{4}{3}\eta + \zeta$. In the limit $\omega\tau_m >> 1$, using equations (13)– (15), we obtain the dispersion relation for the coupled shear–compressional mode, as

$$\Big[\omega^2 + i\omega\nu + i\omega\frac{eE_0}{MD}a_2 + \frac{eE_0}{MD}a_3 - C_{sh}^2k^2\Big]\Big[\omega^2 + i\omega\nu - C_{DA}^2k^2\Big]$$
$$- i\omega k^2\frac{eE_0}{MD}\frac{a_1 Z_d Z_d' n_{d0}}{MD} + k^2\frac{eE_0}{MD}\frac{a_1 M'}{M}(C_d^2 + C_{da}^2) = 0, \tag{16}$$

where $C_{sh}^2 = (\eta/Mn_{d0}\tau_m)$, $C_{da}^2 = (Z_d^2 n_{d0}\eta_c/MD)$ and $C_{DA}^2 = C_d^2 + C_{da}^2 + (\eta_l/Mn_{d0}\tau_m)$. In the above equation the expression in the first set of brackets represents the dispersion relation for the transverse shear wave, the second set of brackets contains the compressive mode dispersion relation and the final two terms denote the coupling between the two branches. We will now study the behaviour of the shear mode in the presence of the charge inhomogeneity and the coupling to the compressive mode.

IV. SHEAR WAVE INSTABILITY

In the limit when the coupling to the compressive wave is weak, so that the last two terms in the dispersion relation (16) can be neglected, we can obtain the roots for the shear branch as,

$$\omega = -\frac{i}{2}\Big(\nu + \frac{eE_0}{MD}a_2\Big) \pm \Big[k^2 C_{sh}^2 - \frac{eE_0}{MD}a_3 - \frac{1}{4}\Big(\nu + \frac{eE_0}{MD}a_2\Big)^2\Big]^{1/2}. \tag{17}$$

In the absence of the inhomogeneities and the collision term, this is the basic shear wave described in [4]. The collisional term introduces wave damping. The inhomogeneous terms introduce two important modifications. The term proportional to the mass (size) inhomogeneity contributes to the real part of the frequency whereas the charge inhomogeneity term can drive the wave unstable if $E_0 a_2 < 0$ (i.e., $E_0 Q_0' < 0$) and the threshold condition $\nu < |\frac{eE_0}{MD}a_2|$ is satisfied. Physically, this instability arises because of delayed charging effect, the same physical mechanism which was used by Nunomura *et al* [6] to explain the observed instability of single particle vertical displacement in their sheath experiments. Specifically, the charge on the vertically oscillating dust particle in the shear wave propagating in the inhomogeneous plasma, is always different from the equilibrium value Z_d because of the finite charging time η_c^{-1}. This perturbation is of order $\delta Z_d \simeq Z_d' \delta u_3/\eta_c$ and leads to an energy exchange between the shear wave and the ambient electric field at a rate $\delta Z_d E_0 \delta u_3^* \approx |E_0 Z_d'||\delta u_3|^2/\eta_c$. When this energy gain by the shear wave exceeds the loss rate due to collisions $\approx \frac{\nu M}{2}|\delta u_3|^2$, we have an instability. This gives us the approximate threshold condition described above. If we express the dust neutral collision frequency, ν in terms of the ambient neutral pressure as $\nu = p\big(\frac{2m_n}{T_n}\big)^{1/2}\frac{\pi a^2}{M}$, our threshold condition is functionally identical to that derived by Nunomura *et al* [6] on the basis of physical arguments. The only substantial difference is their use of exponential charging time which follows from our equation (10) viz. $\delta Z_d \approx (\delta u_3 Z_d'/\eta_c)[1 - exp(-\eta_c t)]$; since we have assumed the frequency of the shear mode $\omega << \eta_c$, we use the asymptotic condition described above.

We now demonstrate that for the collective shear mode being described here, the coupling to the compressional dust acoustic wave due to the last two terms in equation (16) is very crucial; thus the above single particle results are strongly modified by the hydrodynamic treatment. A simple analytic result clearly demonstrating the modification is obtained by neglecting $\omega^2 + i\omega\nu$ compared to $k^2 C_{DA}^2$ in the second bracket of equation (16); this is reasonable when the wave–vector k is not too small. In this limit, the shear modes are described by the root

$$\begin{aligned}\omega = &-\frac{i}{2}\Big(\nu + \frac{eE_0}{MD}\Big(a_2 + a_1\frac{Z_d Z_d'}{MD}\frac{n_{d0}}{C_{DA}^2}\Big)\Big) \\ &\pm \Big[k^2 C_{sh}^2 - \frac{eE_0}{MD}\Big(a_3 - a_1\frac{M'}{M}\frac{(C_d^2 + C_{da}^2)}{C_{DA}^2}\Big) - \frac{1}{4}\Big(\nu + \frac{eE_0}{MD}\Big(a_2 + a_1\frac{Z_d Z_d'}{MD}\frac{n_{d0}}{C_{DA}^2}\Big)\Big)^2\Big]^{1/2}.\end{aligned} \tag{18}$$

We thus note that the threshold condition and the growth rates are significantly modified by the inclusion of coupling to compressional waves. In order to quantitatively illustrate the effect of coupling terms, we now present a detailed numerical investigation of the dispersion relation equation (16). It is generally the case that the bulk viscosity coefficient ζ is negligible compared to the shear viscosity coefficient, η, particularly in the one component plasma (OCP) limit [7] and so we shall drop it in our calculations. Further, the viscoelastic relaxation time, τ_m, is given as [4],

$$\tau_m = \frac{4\eta}{3n_{d0}T_d(1 - \gamma_d\mu_d + \frac{4}{15}u)} \tag{19}$$

with γ_d as the adiabatic index and the compressibility, μ_d defined through (8). We assume the gradient of the equilibriated dust charge to be of the form, $Z_d' = Z_d/L_Z$, the mass gradient to be of the form $M' = M/L_M$ where $L_Z \sim L_M = L$ is a few Debye lengths. In our computations, we choose $L \approx 5$ times the Debye length, which is the typical order of magnitude as observed experimentally [10]. For further computations, we introduce the dimensionless quantities,

$$\begin{gathered}\hat{\omega} = \omega/\omega_{pd}; \quad \hat{\nu} = \nu/\omega_{pd}; \quad \hat{k} = kd; \quad \hat{\tau}_m = \tau_m\omega_{pd}; \\ \hat{\eta} = \frac{\eta}{Mn_{d0}\omega_{pd}d^2}; \quad \hat{C}_\alpha^2 = C_\alpha^2/(\omega_{pd}^2 d^2); \alpha \equiv sh, d, da, DA, \\ e_0 = \frac{eE_0}{MD}\frac{a_2}{\omega_{pd}}; \quad e_1 = \frac{eE_0}{MD}\frac{a_3}{\omega_{pd}^2}; \\ e_{01} = \frac{a_1 Z_d Z_d' n_{d0}}{a_2 MD}\frac{1}{\omega_{pd}^2 d^2}; \;\; e_{11} = \frac{eE_0}{MD}a_1\frac{M'}{M}(\hat{C}_d^2 + \hat{C}_{da}^2)\frac{1}{\omega_{pd}^2},\end{gathered} \tag{20}$$

where ω_{pd} and d are the dust plasma frequency and the inter–grain distance respectively. The dispersion relation for the shear mode (16) can then be written as

$$\left[\hat{\omega}^2 + i\hat{\omega}\hat{\nu} + i\hat{\omega}e_0 + e_1 - \hat{C}_{sh}^2\hat{k}^2\right]\left[\hat{\omega}^2 + i\hat{\omega}\hat{\nu} - \hat{C}_{DA}^2\hat{k}^2\right]$$
$$- i\hat{\omega}\hat{k}^2 e_0 e_{01} + \hat{k}^2 e_{11} = 0, \qquad (21)$$

Equation (21) has been solved numerically for the shear mode roots and some typical results are presented in figures (1) and (2). Figures (1a) and (1b) display a comparison of the dispersion curve for the shear mode ($\hat{\omega}_R$ *vs* $\hat{k}$ and $\hat{\gamma}$ *vs* $\hat{k}$ for fixed values of $e_0 = -0.0008$ and $e_1 = -0.05$), with and without the inclusion of the coupling to the compressional mode. The various fixed parameter values corresponding to these curves are $\hat{C}_{sh}^2 = 0.02$, $\hat{C}_{DA}^2 = 0.4$, $\hat{\nu} = 0.0004$ and $e_{01} = 0.3$, $e_{11} = -0.01$ when the coupling is on. The choice of these numerical values for the dimensionless parameters $\hat{\nu}$, $\hat{k}$, e_0, e_{01}, $\hat{C}_{sh}$ and $\hat{C}_{DA}$ has been guided by the magnitude of these quantities observed in some of the laboratory plasmas [9,10]. It is seen from these plots that there is a substantial influence of the compressional mode coupling, described through the parameter, e_{01}, e_{11}, on the growth rate and the real frequency of the shear wave emphasizing the importance of the collective physics of coupling to the compressional mode. We next plot in figure (2) the gas pressure, p, versus n_{e0} profiles for various values of γ, the imaginary part of ω. Plotting the $\gamma = 0$ curve, we get a threshold relation between p and n_{e0}, where we fix the other parameters as follows – dust radius, r_d=2.5 microns, the inter–grain distance, d=430 microns, $T_e = T_i \simeq 1eV$, $kd = 1$, and dust mass density, ρ_d=2.5 gms/cm^3. We see that the qualitative trend of the curve is similar to that observed in the single particle instability studies of [6] illustrating the commonality of the underlying physical mechanism. However it should be emphasized that the experiment in [6] did not observe any collective excitations and their equilibrium consisted of a monolayer of equal sized particles. The equilibria of [9,10] are more appropriate for observing collective excitations of shear waves and our theoretical results can be usefully employed in such a situation. In Fig.(2) we have once again highlighted the significance of the coupling to the compressive wave, in this case for its effect on the threshold values, by displaying the uncoupled threshold and growth rate curves (dashed curves). Note that the influence of the coupling is to raise the threshold value at low values of n_{e0} (i.e. a higher value of p is needed to excite the instability) whereas it reduces the threshold at the higher end of the n_{e0} scale. The rest of the curves displayed in the figure (2) correspond to the various positive values of γ, which correspond to the situation where the shear mode is excited and saturates at some values. These figures are again qualitatively similar to the curves obtained in [6] for various saturation amplitudes. However a direct comparison is again not appropriate for the reason discussed above and also because our calculations are linear and cannot provide any quantitative results about nonlinearly saturated amplitudes.

V. CONCLUSION AND DISCUSSION

To summarize, in this paper we have investigated the stability of a low frequency shear mode in an inhomogeneous dusty plasma in the strongly correlated regime. The equilibrium dust cloud has both an inhomogeneity in the dust charge distribution and in the dust mass distribution (arising from a distribution in the sizes of the dust particles). The shear mode in such a plasma undergoes two significant modifications. Its real frequency is shifted by a contribution from the mass inhomogeneity and the dust charge inhomogeneity can drive it unstable through the dynamics of dust charge fluctuations in a manner very similar to the instability of the vertical motion of single particles in a plasma sheath as observed in the recent experiment of Nunomura *et al* [6]. The finite charging time, η_c^{-1} of the dust particles plays a critical role in the instability. We also show how collective effects due to coupling with the compressional modes strongly modify the threshold conditions for the instability as well as its growth rate and real frequency. Our calculations have been carried out in the hydrodynamic formalism including viscoelastic effects and we have neglected any kinetic effects. Our results are therefore strictly valid in the low frequency limit. Finite corrections arising from kinetic effects can occur at higher frequencies and wave numbers. This has recently been demonstrated for the compressive dust acoustic mode in a dusty plasma from a kinetic calculation based on the dynamic local field correction (DLFC) method [12]. Such corrections, if any, for the transverse shear mode has not yet been done and needs to be examined.

Finally we would like to remark that the transverse dust shear mode which is a collective mode of the strongly coupled plasma regime has only been observed in computer simulations till now; its detailed experimental investigation is therefore of great current interest. Such waves can be excited in inhomogeneous dust clouds that have been obtained in the experiments carried out with varying grain sizes [9,10]. It would be of interest therefore to look for the wave features discussed in our model calculations in controlled propagation experiments on such equilibria. It is also apparent that free energy sources, such as ion beams, which may readily couple with the compressional waves may also be useful for exciting the more interesting shear waves in the strongly coupled inhomogeneous plasma. Investigation of these and related effects are in progress.

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[1] E. C. Whipple, T. G. Northrop and D. A. Mendis, J. Geophys. Res. 90, 7405 (1985); E. C. Whipple, Rep. Prog. Phys. 44, 1197 (1981); D. A. Mendis and M. Rosenberg, Ann. Rev. Astron. Astrophys. 32,419 (1994); U. de Angelis, Phys. Scr. 45, 465 (1992); C. K. Göertz, Rev. Geophys. 27, 271 (1989); M. Horanyi, H. L. F. Houpis and D. A. Mendis, Astrophys. Space Sci. 144, 215 (1988); V. N. Tsytovich, G. E. Morfill, R. Bingham and U. de Angelis, Comments Plasma Phys. Controlled Fusion 13, 153 (1990); G. S. Selwyn, J. Singh and R. S. Bennet, J. Vac. Sci. Technol A7, 2758 (1989).

[2] H. Ikezi, Phys. Fluids 29, 1764 (1986); H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher and D. Mohlmann, Phys. Rev. Lett. 73, 652 (1994).

[3] M. Rosenberg and G. Kalman, Phys. Rev. E56, 7166 (1997).

[4] P. K. Kaw and A. Sen, Phys. Plasmas 5, 3552 (1998).

[5] J. B. Pieper and J. Goree, Phys. Rev. Lett. 77, 3137 (1996); P. Schmidt, G. Zwickmagel, P. G. Reinhard and C. Toepffer, Phys. Rev. E56, 7310 (1997).

[6] S. Nunomura, T. Misawa, N. Ohno and S. Takamura, Phys. Rev. Lett. 83, 1970 (1999).

[7] S. Ichimaru, H. Iyetomi and S. Tanaka, Phys. Rep. 149, 91 (1987); M. A. Berkovsky, Phys. Lett. A166, 365 (1992); S. Tanaka and S. Ichimaru, Phys. Rev. A 35, 4743 (1987).

[8] M. R. Jana, A. Sen and P. K. Kaw, Phys. Rev. E48, 3930 (1993); J. R. Bhatt and B. P. Pandey, Phys. Rev. E 50, 3980 (1994).

[9] J. H. Chu and Lin I, Phys. Rev. Lett. 72, 4009 (1994).

[10] S. Nunomura, N. Ohno and S. Takamura, Phys. Plasmas 5, 3517 (1998).

[11] W. L. Slattery, G. D. Doolen and H. E. DeWitt, Phys. Rev. A 21, 2087 (1980);ibid, 26, 2255 (1982).

[12] M.S. Murillo, Phys. Plasmas 5, 3116 (1998); M.S. Murillo, Phys. Plasmas 7, 33 (2000).

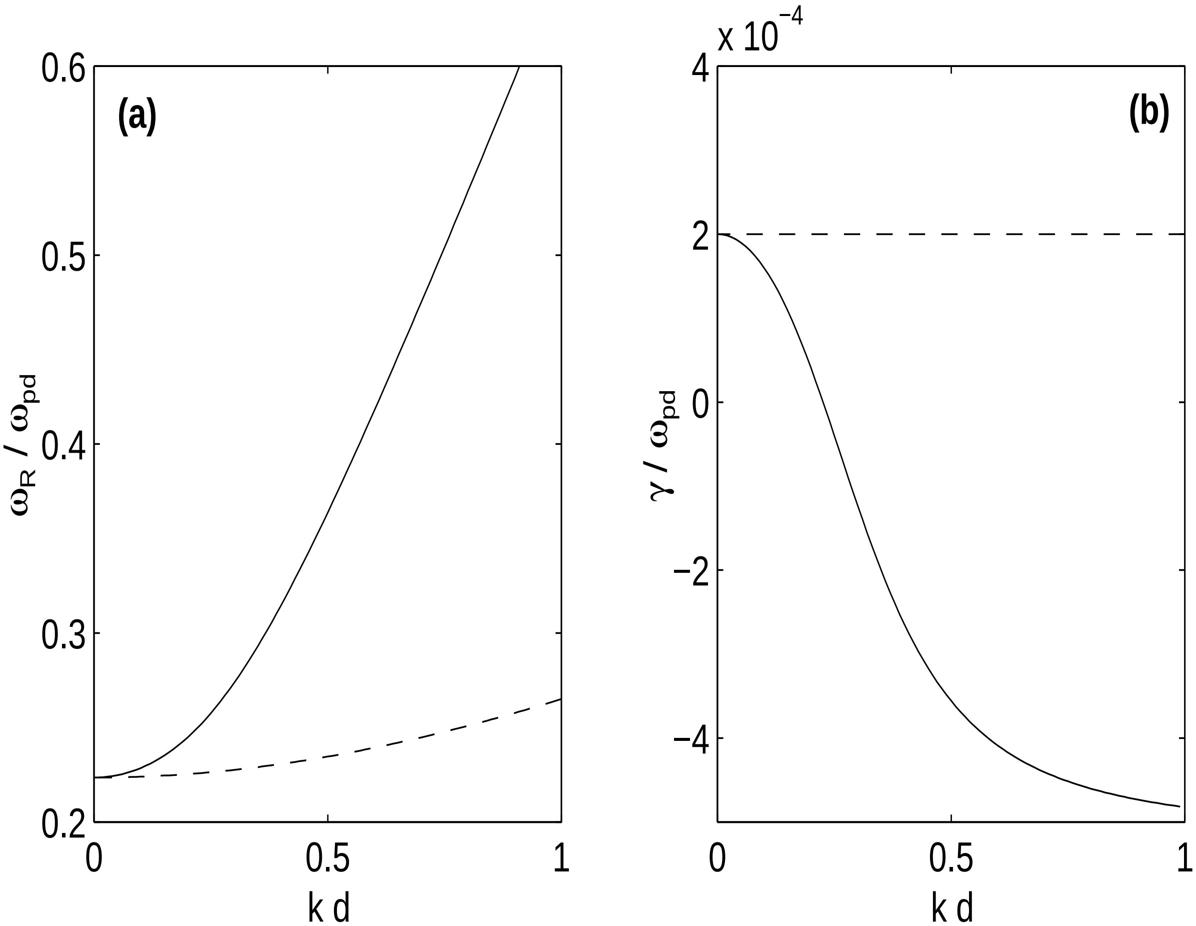

FIG. 1. (a) The normalized real frequency and (b) the normalized imaginary frequency, *vs*. the normalized wave number for the shear mode with $e_0 = -0.0008$, $e_{01} = 0.3$, $e_1 = -0.05$, $e_{11} = -0.01$ (solid curves). The dashed curves are for $e_{01} = e_{11} = 0$ and correspond to the uncoupled shear mode.

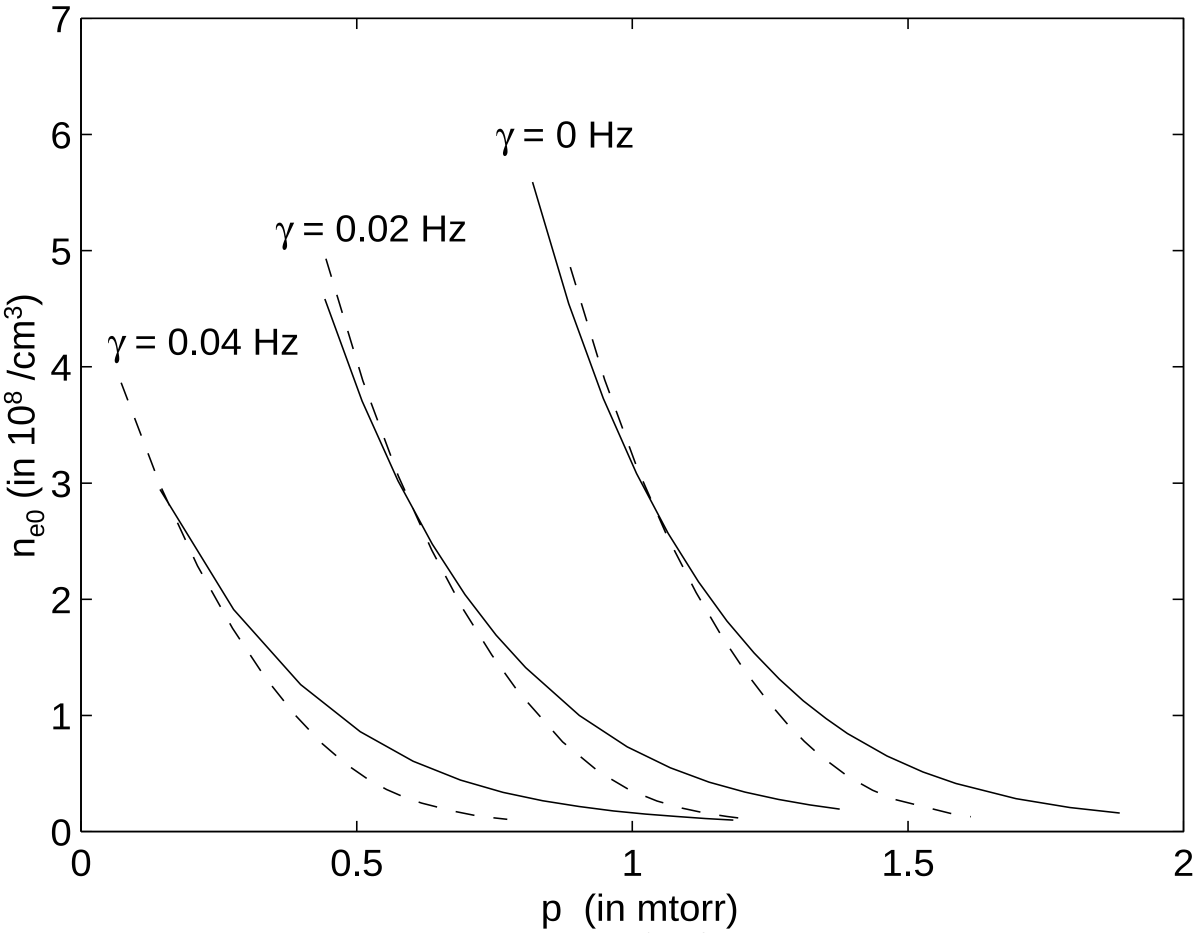

FIG. 2. The electron number density n_{e0} (in units of $10^8/\mathrm{cm}^3$) is plotted as a function of the gas pressure, p (in mtorr) for various values of γ. For comparison, the accompanying dashed curves display the situation when the coupling to the compressional mode is neglected.