

Dynamic color screening in a classical quark plasma

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Abstract. Screening of a moving infinite color sheet source is examined in a quark plasma at finite temperature. The classical chromohydrodynamic equations for quarks are integrated, to obtain profiles for quark current density, which in turn are used to solve the SU(2) Yang-Mills equations numerically. This provides a classical but non-perturbative treatment for the screening of a moving source in quark plasma.

The results show two interesting features. We observe that if the test source is at rest the screening does not depend on the color dynamics and the behavior is very similar to that in Coulomb plasma. When the test source is moving with non-relativistic velocity the non-abelian features manifest themselves by weakening the screening and also by exhibiting an oscillatory profile with distance.

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1. Introduction

The screening of a color electric field in quark gluon plasma (QGP) is of considerable interest since it is a reflection of collective behaviour of the system and also because it leads to signatures for detecting the formation of QGP in the laboratory. For example, it was proposed that if the plasma is produced in heavy ion collisions, then due to screening effects charge anti-correlations [1] between pions of similar rapidity and production [2] of $J/\Psi(c\bar{c})$ mesons would be suppressed.

Early estimates of the static screening length have been based on perturbative QCD [3] and classical kinetic theory of abelian plasmas [4]. One obtains standard results at bare 1-loop level in QCD and the linearized calculational level in classical theories. Static screening phenomenon has been studied non-perturbatively using lattice methods [5] and one finds the usual form of the screened potential. Screening of *moving* charges in a quark-gluon plasma is a problem of great interest because of its direct relevance to signatures (J/Ψ suppression) from heavy-ion collisions. However, such calculations have so far been done only in the abelian limit [6] by using the methods of classical Coulomb plasma physics [7]. It is therefore of considerable interest to extend these classical calculations into the non-abelian non-perturbative regime. This is especially important because presently there are no known quantum field theoretic techniques which can be used to study this problem.

In this paper we analyze the *non-abelian* screening of a moving test source in a *classical* quark plasma. We restrict our attention to test particles travelling with non-relativistic velocities $V_0^2/c^2 \ll 1$. This is justified for considering J/ψ screening in

current experiments (such as NA38 experiment) where the transverse momenta of the particles is peaked around $p_T^2 \simeq 0.6(\text{GeV})^2/c^2 \ll m_{J/\psi}$ [8]. We have also not included effects due to the energy loss dE/dx of the test particles. As shown by Gyulassy and Thoma [9] recently, this effect is also negligible for a slowly moving heavy particle. We may now emphasize at the outset that for studying the screening problem it is not essential to use kinetic equations. In fact in Coulomb plasmas it is standard practice to use fluid equations [7]. Moreover to simplify the calculations further one often uses slab geometry [7]. We basically follow this approach in order to carry out a non-perturbative study of screening in quark matter. For this purpose we have extended the color hydrodynamic (CHD) equations for quarks derived by Kajantie and Montonen [10] by adding a pressure gradient term in the momentum balance equation and coupling it to an equation of state for the massless quark gas. The self-consistent Yang-Mills equations in the presence of the test source are then solved in the slab geometry. We find that non-perturbative color precession effects produce a significant weakening of the screening of a moving color sheet source.

It ought to be mentioned that in a classical non-abelian theory there is an inherent difficulty in studying the screening of a test charge, because the charge of the test source can flow into gauge fields and vice-versa. This question has not been adequately addressed in earlier classical studies of screening in QGP. Consequently, in this paper, we use the expression for an effective Debye length to define a gauge invariant charge to study the non-abelian screening.

Section 2 contains a description of the basic equations used for studying the screening. In § 3 we discuss the screening for a static and a moving source, and present numerical results for a few sample cases, from amongst a large number of investigations that we have carried out. It must be pointed out that the broad qualitative features of oscillations and of weakened screening of a moving source, are present in all our numerical studies having different boundary conditions and values of parameters. Finally, § 4 contains a brief summary and conclusions.

2. Basic equations for screening

The CHD equations [10] describe the quark matter fluid in terms of three dynamical quantities, the gauge invariant number density $n(\mathbf{x}, t)$, the gauge invariant velocity field $\mathbf{V}(\mathbf{x}, t)$ and gauge covariant color charge $I_a(\mathbf{x}, t)$. Gauge transformation properties of these hydrodynamic variables and the CHD equation have been discussed in [11]. As mentioned earlier we have extended these equations by including a pressure gradient term in the force equation, and an equation of state relating pressure with temperature. The equation of state is chosen in such a manner that it account for the ultrarelativistic internal motions of the plasma particles. The basic equations are,

$$\frac{\partial n_A}{\partial t} + \nabla \cdot (n_A \mathbf{V}_A) = 0 \quad (1)$$

$$m_A \left[\frac{\partial \mathbf{V}_A}{\partial t} + \mathbf{V}_A \cdot \nabla \mathbf{V}_A \right] = g I_{Aa} [\mathbf{E}_a + \mathbf{V}_A \times \mathbf{B}_a] - \frac{1}{n_A} \nabla P_A \quad (2)$$

$$\left[\frac{\partial I_{Aa}}{\partial t} + \mathbf{V}_A \cdot \nabla I_{Aa} \right] = -g \varepsilon_{abc} [A_b^0 - \mathbf{V}_A \cdot \mathbf{A}_b] I_{Ac} \quad (3)$$

where m_A and P_A denote the mass and the pressure of particles of specie A and the

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components of color electric and magnetic fields E_a and B_a are respectively defined by $E_a^i = F_a^{i0}$ and $B_a^i = -\frac{1}{2}\epsilon^{ijk}F_a^{jk}$. Here $F_a^{\mu\nu}$ is the color field tensor

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\epsilon_{abc}A_b^\mu A_c^\nu \quad (4)$$

with Lorentz indices $\mu, \nu = 0, 1, 2, 3$ and (SU(2)) color indices $a = 1, 2, 3$. In (2), there would be a term involving collisions between particles belonging to different species. This is neglected in the present work and the justification for it is given in Appendix A.

The color fields are governed by the Yang-Mills equation

$$\partial_\mu F_a^{\mu\nu} + g\epsilon_{abc}A_{\mu b}F_c^{\mu\nu} = j_a^\nu \quad (5)$$

where the quark (four) current j_a^ν is expressed in terms of the quark fluid variables according to

$$j_a^0 = g \sum_A n_A I_{Aa}, \quad (6)$$

$$\mathbf{j}_a = g \sum_A n_A \mathbf{V}_A I_{Aa}. \quad (7)$$

Using (1), (3), (6) and (7) one can readily show that the covariant current continuity equation is identically satisfied [8]. In the absence of any perturbations, the plasma is assumed to satisfy a color neutrality condition $\sum_A n_A I_{Aa0} = 0$. This strictly follows work of Kajantie and Montonen [10] and physically assumes that each color is separately giving color neutrality because of several compensating species in equilibrium. This color neutrality permits a wide class of non-abelian phenomena [10, 11]. It is true that a more general form of color neutrality may be essential in a baryon rich QGP. We have not investigated this more general class of non-abelian problem due to their complexity.

For the equation of state (eos) and number density we choose the ideal relativistic gas equations,

$$P_A = h_s N_d T_A^4 \quad (8)$$

$$n_A = d_s N_d T_A^3 \quad (9)$$

where N_d is the number of degrees of freedom and h_s and d_s are factors which depend upon the statistics. For an SU(2) massless quark gas of a single flavor $N_d = 4$,

$$h_s = \frac{7}{8} \left[\frac{\pi^2}{90} \right] \text{ and } d_s = \frac{3}{4} \left[\frac{\zeta(3)}{\pi^2} \right].$$

The above equations have been written for the quark component of QGP. It has recently been pointed out by Elze and Heinz [12] that gluons can also have classical dynamic equations similar to those of quarks (i.e. Eqs (1)–(3)). If such equations are included in the treatment, the contribution of gluons will simply add to that from quarks and define a new plasma frequency and a new screening length. We thus expect that for the non-abelian problem also the addition of new species (viz. gluons) is not going to change the qualitative conclusions significantly.

The basic problem we wish to consider is the screening of a moving test charge that is introduced in the QGP. For simplicity, we consider a moving infinite sheet

charge as the test source; this reduces the problem to 1-d (z) variations (for similar simplifications in Coulomb plasmas see ref. (7)). The test charge polarizes the plasma and we solve the self consistent Yang-Mills field equations coupled with the plasma response to analyze the screening effects. We consider a plasma consisting of two species (particles and antiparticles) and go to a frame in which the test source is at rest but the plasma is moving. In this frame when no perturbation is introduced, the two species move with equal velocity V_0 in the z -direction and have equilibrium density n_0 . The color neutrality condition in equilibrium is $I_{1a0} = -I_{2a0}$ and hence the equilibrium current is zero. We consider a steady-state ($\frac{\partial}{\partial t} = 0$) and drop the coupling to magnetic sector ($A_a^1 = A_a^2 = 0$) since the velocity of the test source is assumed to be non-relativistic ($V_0 \ll 1$). Finally, we make the gauge choice $A_a^3 = 0$ and write the equations for screening as

$$\frac{d^2 A_a^0}{dz^2} = -g \sum_A n_A I_{Aa} - 4\pi g K_a \delta(z), \quad (10)$$

$$g \varepsilon_{abc} A_b^0 \frac{dA_c^0}{dz} = g \sum_A n_A V_A I_{Aa}, \quad (11)$$

$$\frac{d}{dz} (n_A V_A) = 0, \quad (12)$$

$$g I_{Aa} \frac{dA_a^0}{dz} = -\frac{1}{n_A} \frac{dP_A}{dz}, \quad (13)$$

$$V_A \frac{dI_{Aa}}{dz} = -g \varepsilon_{abc} A_b^0 I_{Ac}. \quad (14)$$

In (10) K_a is the color charge component of the test sheet source. An interesting non-abelian feature is seen in (11) (Ampere's law) where, unlike the Coulomb (abelian) plasma, the steady state currents, affect the static potential in the non-abelian plasmas. In writing (13), we have ignored the mean energy of each specie compared to the thermal energy of that specie i.e. $m_A V_A^2 \ll T_A$. This is justified as the thermal velocity of the plasma particle is comparable to the velocity of light (due to the choice of eos).

The covariant current conservation equation for the present case takes the form

$$\frac{d}{dz} j_a + g \varepsilon_{abc} A_b^0 j_c^0 = 0$$

Note from (10) and (11) that $j_a^0 = -d^2 A_a^0 / dz^2$ and $j_a = g \varepsilon_{abc} A_b^0 dA_c^0 / dz$ (remembering that the test charge K_a , being at rest in the frame of calculation, contributes to j_a^0 but not to j_a). Substitution in the above equation shows that it is identically satisfied.

Equation (12) may be readily solved to yield

$$n_1 V_1 = n_0 V_0 = n_2 V_2 \quad (15)$$

Combining (15) and (11) one can find

$$I_{1a} + I_{2a} = \frac{1}{n_0 V_0} \varepsilon_{abc} A_b^0 \frac{dA_c^0}{dz}. \quad (16)$$

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We can now express I_{2a} , T_A and n_A in terms of I_{1A} and A_a^0 and obtain the final equations for screening in the dimensionless form

$$\frac{d^2 A_a}{dZ^2} = I_a(I_b A_b) + \frac{1}{3} \frac{\beta d_s}{4h_s} I_a(I_b A_b)^3 - \alpha \varepsilon_{abc} A_b \frac{dA_c}{dZ} \left[1 + \frac{\beta d_s}{4h_s} I_s A_b \right]^3 - S_a \quad (17)$$

$$\frac{dI_a}{dZ} = -\alpha \left[1 - \frac{\beta d_s}{4h_s} I_b A_b \right]^3 \varepsilon_{abc} A_b I_c. \quad (18)$$

We define

$$\Lambda_D^{-2} = \left[\frac{3N_d d_s^2}{2h_s} \right] g^2 i_0^2 T_0^2 \quad (19)$$

which can be identified as the linear ('perturbative') Debye length and the scaled variables are $Z = z/\Lambda_D$, $A_a = a_0^{-1} A_a^0$, $I_a = i_0^{-1} I_{1a}$ where a_0 , i_0 , T_0 are normalizing factors and S_a is the source term. We also introduce the dimensionless parameters $\alpha = \frac{g a_0 \Lambda_D}{V_0}$ and $\beta = \frac{g i_0 a_0}{T_0}$. The dimensionless parameter β essentially measures the ratio of 'average' potential energy per particle to average kinetic energy and as in Coulomb plasmas is given by $\beta = \frac{g i_0 a_0}{T_0} \simeq (n \Lambda_D^3)^{-1}$. The parameter α characterizes the strength of the non-abelian terms in Eqs (17 and 18) and may also be written as $\alpha = \frac{g a_0 \Lambda_D}{V_0} = g \beta \left[\frac{\sigma \Lambda_D^4 \varepsilon_{th}}{\varepsilon_{kin}} \right]^{1/2}$ where σ is the mass density, $\varepsilon_{kin} = m n_0 \frac{V_0^2}{2}$ and ε_{th} the thermal energy of plasma particles. For a typical QGP that might be produced in heavy ion collisions $\Lambda_D \sim 1/4 - 1/3$ fm, $\varepsilon_{th} \sim 2 - 5$ GeV/fm³ and with $V_0 \leq 0.4$, we find $\beta \sim 10^{-1} - 10^{-2}$ and $\alpha \geq g 10^{-2}$.

3. Screening effects for static and moving sources

First we consider the screening of a moving test charge. Equations (17) and (18) may be combined to generate an equation for $I_a A_a$, viz.

$$\frac{d^2(I_a A_a)}{dZ^2} = \left[I_a^2 - 6\alpha \theta I_a \varepsilon_{abc} A_b \frac{dA_c}{dZ} \right] \left[1 + \frac{\theta^2}{3} (I_b A_b)^2 \right] (I_a A_a) - S \quad (20)$$

where $\theta = \frac{\beta d_s}{4h_s}$. The 'potential' energy $I_a A_a$ is a gauge invariant quantity in the static ($\frac{\partial}{\partial t} = 0$) case and the equation above shows that it is screened by a non-abelian Debye length Λ_{DNA} , which is a dynamical quantity given by the gauge invariant expression

$$\Lambda_{DNA}^{-2} = \left[I_a^2 - 6\alpha \theta I_a \varepsilon_{abc} A_b \frac{dA_c}{dZ} \right] \left[1 + \frac{\theta^2}{3} (I_b A_b)^2 \right]. \quad (21)$$

From (23) we may also define a gauge invariant charge $Q_{inv}^2 = I_a^2 - 6\alpha \theta I_a \varepsilon_{abc} A_b (dA_c/dZ)$

which takes into account exchange of color charge with the gauge fields. This expression is similar to that obtained in earlier studies [13] of color screening in classical Yang-Mills theories with an external source.

Next we demonstrate that for static charges ($V_1, V_2 \rightarrow 0$) we recover the usual abelian screening results. From (15) we note that since n_1 and n_2 are finite even when $V_0 \rightarrow 0$, V_0/V_A must also stay finite in this limit. If we now multiply (16) by V_1 or V_2 and then take limit $V_1, V_2 \rightarrow 0$, we get the constraint conditions,

$$\varepsilon_{abc} A_b \frac{dA_c}{dZ} = 0. \quad (22)$$

In this limit A_a and (dA_a/dZ) are vectors parallel in color space. Consequently, the non-abelian term disappears from (20) and the right hand side will become function of $I_a A_a$ only. Thus, in solving (20) the dynamics of I_a (Eq. (18)) is redundant and the screening behavior is similar to that of a Coulomb plasma. It should be noted that the covariant continuity equation for quark current would give, in the limit $V_0 \rightarrow 0$, $\varepsilon_{abc} A_b I_c = 0$. However, as we have used both Gauss and Ampere's equations, the constraints arising from the continuity equation are not necessary. Finally on integration, (20) gives (for $V_0 \rightarrow 0$) the well-known Debye screening result with the Debye length Λ_D given by the 'perturbative' expression, Eq. (19).

In order to study (17 and 18) further, we first look for conservation laws. We find

$$\sum_a I_a^2 = \text{constant}, \quad (23)$$

$$\sum_a \left[\frac{dA_a}{dZ} \right]^2 - (I_b A_b)^2 - \frac{\theta^2}{6} (I_b A_b)^4 = E, \quad (24)$$

$$\sum_a \alpha^2 M_a^2 - \frac{\alpha}{3\theta} I_b M_b = M, \quad (25)$$

where $M_a = \varepsilon_{abc} A_b (dA_a/dZ)$ and M is a constant. Equation (23) is an obvious reflection of SU(2) color algebra and allows precession of color charge \mathbf{I} keeping its magnitude constant. Equation (24) may be interpreted in terms of field energy and thermal energy of the particles. Unlike an oscillatory solution where there is a conservation law for the sum of field and particle energies we get for a screened system, an invariance of the difference in energies. It implies that the particle and the field energies are simultaneously large or small so as to preserve the difference. As regard (25) we note that M_1, M_2 and M_3 are related to color charge fluctuations of the Yang-Mills fields. The second term of right hand side of (25) is a consequence of the exchange of color charge between the fields and particles.

We would like to emphasize that all the important quantities described above and subsequently used below in numerical calculations viz. $I_a E_a, I_a A_a, Q_{\text{inv}}^2$ and the conserved quantities are gauge invariant under the class of static gauge transformation $U = U(x)$ as A^0 transforms covariantly. The gauge invariance of Q_{inv}^2 has been explicitly demonstrated for general case in [13].

To fully investigate the novel qualitative and quantitative features of non-abelian screening we must solve eqs (17) and (18) numerically. Before proceeding with the numerical solutions we must specify the boundary conditions on the variables $A_a, I_a, (dA_a/dZ)$ etc. However, all these variables are gauge dependent. To study the

screening in a gauge invariant manner, we construct useful gauge invariant quantities such as potential energy, the force $I_a E_a$, the gauge invariant charge Q_{inv}^2 and use these and the three invariant given by eqs (23–25) to specify the boundary conditions. As the fields are screened at large distances, we expect the non-abelian field strength to vanish at distances large compared to Debye length and to have $I_a E_a \rightarrow 0$, $I_a A_a \rightarrow 0$ and $Q_{inv}^2 \rightarrow I_a^2$ as $Z \rightarrow \infty$.

In figures 1 and 2 we have plotted the gauge invariant force $\left(-I_a \frac{dA_a}{dZ} \equiv IE\right)$ on a color charged quark fluid element as a function of distance from the test source. This way of studying the screening has advantage over the usual potential vs. distance plots. This is because two different sets of boundary conditions related say by a global gauge transformation can give very different behaviour in the potential vs. distance plots [14] while IE vs. distance plots remain invariant. For this reason the non-abelian screening can be studied uniquely in terms of physically meaningful gauge invariant quantities.

In order to study the effect of the velocity of test source on the screening, we vary the parameter α . Clearly a change in the value of α will alter the values of the gauge invariant charge (eq. (21)) and the conserved quantity M (eq. (25)) at the boundary. To ensure that the six gauge invariant physical quantities mentioned above remain fixed at the boundary we vary the boundary values of gauge dependent quantities A_a , and (dA_a/dZ) . Indeed, it is found that under the transformations $\alpha \rightarrow \alpha' = \alpha + d$ and $A_a \rightarrow A_a + d$ where,

$$\tilde{A}_a = -\frac{d}{\alpha + d} \frac{I_b A_b}{IE} \frac{dA_a}{dZ} + \frac{\alpha}{\alpha + d} A_a \quad (26)$$

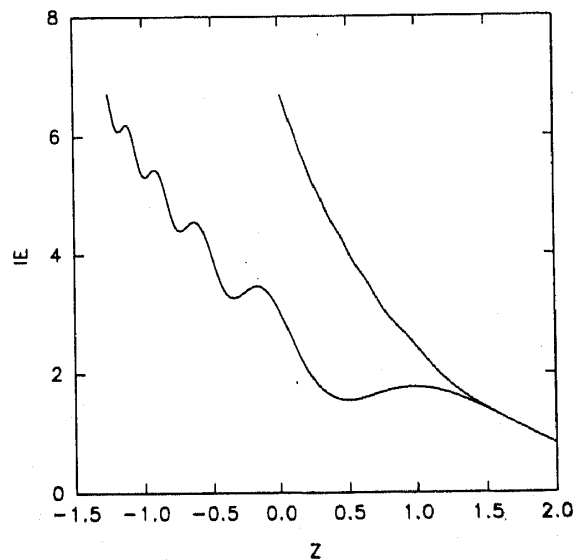


Figure 1. The screening of the force $IE(\equiv I_a E_a)$ acting on a fluid element at a distance z from the infinite sheet test charge. z is in units of “perturbative” Debye length. The value of the parameter β is 0.1.

The upper curve represents $\alpha = 15$ case and the lower curve corresponds to $\alpha = 2$ case with the boundary values of the gauge invariant quantities $I_a A_a = 0.8569683$, $I_a (dA_a/dZ) = -0.85696827$ and $Q_{inv}^2 = 0.9961094$. The values of the conserved quantities are $E = 0.002824$, $M = 11.49416$ and $\sum_a I_a^2 = 0.9961$.

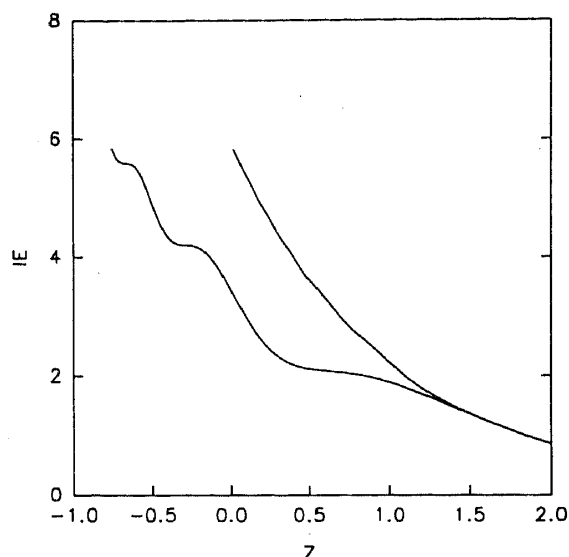


Figure 2. Caption same as figure 1. The upper curve represents the case $\alpha = 15$ and the lower curve corresponds to $\alpha = 2$ case with the boundary values of the gauge invariant quantities $I_a A_a = 0.9901809$, $I_a (dA_a/dZ) = -0.8162973$ and $Q_{inv}^2 = 1.101744$. The values of the conserved quantities are $E = -0.37477$, $M = 15.46753$ and $\sum_a I_a^2 = 1.1$.

all the physical quantities at the boundary remain fixed. Further it clearly shows that when $d \rightarrow \infty$ (i.e. $V_0 \rightarrow 0$), A_a become parallel to (dA_a/dZ) as should be the case.

The numerical approach we adopt is the fourth order Runge-Kutta method, with variable step size. The conserved quantities defined in (23)–(25) are used as checks for accuracy of the numerical integration scheme. In the numerical integration, it is necessary to specify the boundary conditions with high accuracy (i) to make certain that the equations give a screening (not an exponentially increasing) solution and (ii) to ensure that the constancy of conserved quantities is good to several significant places.

Figure 1 shows the force for two values of α viz. $\alpha = 2$ and $\alpha = 15$; higher α corresponds to slower moving particles. In order to compare the abelian screening with the non-abelian one, we integrate the equations backward from a point far away from the charge. The value of $I_a E_a$ at this location ($Z = 2$ in the figure which is two abelian Debye lengths from origin) be the same. As we integrate the equations towards the source we go up to value of Z where $I_a E_a$ again takes the same values. This is because we expect the $I_a E_a$ near a source to have the same magnitude. Thus we notice from the figures that the source for the non-abelian case (lower curve) is located at a *farther* distance from $Z = 2$. First we note that the force shows typical screening behavior. Treating fall-off distance of IE as measure of screening length we find that screening is weaker for higher velocity (lower α) by 38%. Figure 2 shows similar results for a different set of boundary conditions and again shows a significant weakening of screening (62%) as a function of particle velocity V_0 . It is also noted that non-abelian effects lead to a novel oscillatory behavior of IE over and above the mean screening effects (clearly seen in figures 1 and 2). These new features may be understood in terms of non-abelian effects that lead to a precession (see (14)) of the color charge vector of the fluid element. As a result its orientation is not always

opposite to the color vector of the moving charge, and therefore a larger distance is needed to screen the field of a moving test source. These are qualitatively new effects and have not been described before. It is clear that as α increases (and velocity diminishes), the oscillatory behavior should diminish and screening becomes more and more abelian. This is demonstrated by our numerical results.

It should be emphasized that over many different sets of boundary conditions, the numerical solutions have been studied and it was found that the qualitative features of the solutions are similar. Hence, the general features of our results are independent of any specific choice of parameters and the boundary conditions. The numerical results presented are merely representative of a large set of parameter choices and boundary conditions.

4. Summary and conclusions

To summarize, we have investigated the dynamic screening of an infinite color sheet source, moving in quark matter. We find that non-abelian effects lead to a significant modification of the screening of a moving test source. Firstly, there are oscillations in the screening behavior and secondly and more importantly there is significant weakening of the mean screening. The former effect is similar in nature to non-abelian longitudinal plasma oscillations discovered recently by the authors [11]. The latter is a consequence of the precession of the color charge vector of the screening plasma fluid, which weakens its screening effects.

It ought to be made clear that the well-known non-perturbative, non-abelian problem of magnetic mass [15], that would remove the infrared divergences [15], in perturbative QCD at finite temperature has not been examined by us. As pointed out by Nadkarni [16], due to the infrared problem, a non-perturbative study is required even for the static screening of a color source. Thus, our omission of the magnetic effects has prevented us from studying this question. All the same, we have shown that, for moving sources, there are new non-abelian effects in the purely electric sector due to precession of color charges. A full treatment of the screening problem would include these new effects together with other non-abelian features arising from the magnetic sector.

Finally, it is interesting to speculate that, the weakening of screening due to non-abelian effects found by us, may also contribute to the increase of survival probability of J/Ψ with p_T that is experimentally observed [8].

Appendix A

If two species interpenetrate, there would be some transfer of momentum from one specie to another. In this situation, the equation of motion for the two species can be (phenomenologically) written as

$$m \left[\frac{\partial \mathbf{V}_1}{\partial t} + \mathbf{V}_1 \cdot \nabla \mathbf{V}_1 \right] = g I_{1a} [\mathbf{E}_a + \mathbf{V}_1 \times \mathbf{B}_a] - \frac{\nabla P_1}{n_1} - \nu_m (\mathbf{V}_1 - \mathbf{V}_2), \quad (27)$$

$$m \left[\frac{\partial \mathbf{V}_2}{\partial t} + \mathbf{V}_2 \cdot \nabla \mathbf{V}_2 \right] = g I_{2a} [\mathbf{E}_a + \mathbf{V}_2 \times \mathbf{B}_a] - \frac{\nabla P_2}{n_2} + \nu_m (\mathbf{V}_1 - \mathbf{V}_2). \quad (28)$$

Here ν is the collision frequency, which is related to the mean free path and the

thermal velocity $\langle V_0 \rangle$ by the formula $v = \langle V_0 \rangle / l$. In the equations above, one can take the length scale for the pressure gradient term to be of the order of screening length Λ_D . Thus, we may neglect the collision term if $\frac{P}{n\Lambda_D} \gg \frac{m\langle V_0 \rangle}{l} V_h$ where, V_h is the hydrodynamic velocity.

We take the Debye length $\Lambda_D \simeq 1/4 - 1/3$ fm, and the mean free path [17] $l = 0.5$ fm for a QGP with energy density $\varepsilon = 2$ GeV/fm³, so that $\Lambda_D/l \sim 1$. Now for a high temperature plasma, in which the test particle has a large rest mass ($m_q \gg T$) we have $\langle V_0 \rangle / V_h \gg 1$. Thus, for the model studies discussed in this work the neglect of the collision term is justified.

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