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Collective mechanism for nuclear stopping and transverse flow in heavy-ion collisions

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Abstract. The role of filamentation instability of quark-gluon plasma, in explaining collective phenomena in relativistic heavy-ion collisions, has been analyzed. Using equations of SU(2) two fluid color hydrodynamics it is shown that this instability can significantly enhance nuclear stopping and might contribute to collective sideward flows.

Keywords. Heavy ion collisions; stopping power; collective mechanism.

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Recently certain models [1] of nuclear collisions have shown that collective effects may become very significant in collisions of very heavy nuclei at relativistic energies. Numerical studies have shown that for Pb–Pb collisions, complete stopping may occur and collective sideward flow would be generated. The collision experiments of very heavy nuclei at this energy (~ 200 GeVA) have not yet been performed [2] but one tends to take the predictions of the models seriously because these models have been successful in explaining experimental data and observed partial transparency in collisions of light nuclei. A drawback of these models is that they require detailed and involved microscopic description of nuclear collisions [3] which fails to give one an insight into the observed collective phenomena. As a complementary intuitive picture, it is therefore important to look for a simple QCD based collective mechanism which can exhibit similar specific features. In this paper we consider one such mechanism viz. the filamentation instability.

If we assume in conformity with collisions of light nuclei, that transparency would set in collisions of heavy nuclei at energy ~ 200 GeVA, then the nuclear collision may be regarded as counter streaming color fluxes. Under these conditions instabilities related particularly with plasma streaming [4] are of interest. It is well-known that for relativistic plasma streams, the most important instability is the so-called filamentation instability [4], which leads to stratification of initially homogeneous and oppositely directed plasma fluxes interacting via mean vector fields [4] (gluon field for QGP). The mean fields are mixed waves (longitudinal + transverse) which for strongest instability have the propagation vector in a direction perpendicular to the stream velocity [4]. We have used SU(2) color hydrodynamic [5] (CHD) equations to describe the filamentation instability. Our work differs from earlier works [4] in that it describes both *linear* and *non-linear* aspects of filamentation instability and is therefore able to address questions related to collective stopping and sideward flow generation. We believe this is one of the first studies [6], which includes full non-abelian physics for examining dynamic collective phenomena of QGP in classical limit. Basic CHD equations, which can be obtained from the QCD Lagrangian [7], contain three continuous physical quantities: density n, velocity field $\mathbf{V}$ and color charge vector $I_a(a = 1, 2, 3)$ which are functions of space-time and their dynamics is described by CHD equations [5]

$$\partial_t n_A + \nabla\cdot(n_A \mathbf{V}_A) = 0, \tag{1}$$

$$\partial_t \mathbf{V}_A + \mathbf{V}_A\cdot\nabla\mathbf{V}_A = (g/m_A)\sqrt{1 - V_A^2}\, I_{Aa}[\mathbf{E}_a + \mathbf{V}_A \times \mathbf{B}_a - \mathbf{V}_A(\mathbf{V}_A\cdot\mathbf{E}_a)], \tag{2}$$

$$\partial_t I_{Aa} + \mathbf{V}_A\cdot\nabla I_{Aa} = -g\varepsilon_{abc}[A_b^0 - \mathbf{V}_A\cdot\mathbf{A}_b] I_{Ac}, \tag{3}$$

In (1)–(3) the suffix A and m_A denote species label of color particles and mass of species A respectively. For light relativistic quarks mass m_A is replaced by (enthalpy density/number density) for the relevant quark fluid [5]; this then reproduces well-known perturbative QCD results. ε_{abc} is the Levi-Civita tensor and $A_a^\mu(\mu = 0, 1, 2, 3)$ are SU(2) gauge potentials which satisfy Yang-Mills equations $\partial_\mu F_a^{\mu\nu} + g\varepsilon_{abc} A_{\mu b} F_c^{\mu\nu} = j_a^\nu$. Here, j_a^ν represents matter current which can be expressed in terms of CHD variables as $j_a^0 = g\Sigma_A n_A I_{Aa}$ and $\mathbf{j}_a = g\Sigma_A n_A \mathbf{V}_A I_{Aa}$. Equations (1)–(3) together with Yang-Mills equations form a closed set of Lorentz and gauge covariant equations [8] which describe the evolution of the QGP self consistently. It should be noted that among the CHD variables n_A and $\mathbf{V}_A$ are gauge invariant quantities while the color vector I_{Aa} transforms gauge covariantly [8].

We consider the *simplest* SU(2) color hydrodynamic *model* for the nuclear collision: two species $A = 1, 2$ having the same mass are counter streaming with velocity $\pm V_0(\sim 1)$ in the z-direction and their anti-particles are assumed to form an overall static colour neutralizing background. This is a simplified model for the true dynamics and its justification is given below. Both species are assumed to have homogeneous equilibrium densities and colour charge vectors which are assumed to be equal. We consider a filamentation mode propagating along x with electric field components along x and z and magnetic field components along y. After linearizing the equations and taking perturbations of the form $\exp[-i(kx - \omega t)]$ and making the gauge choice $A_a^0 = 0$, we obtain the dispersion relation $\omega_\pm^2 = k^2/2[1 \pm \sqrt{(1 + 8\omega_p^2 k^{-2})}]$ where, $\omega_p^2 = g^2 \rho_0 I_{0a}^2/m$ and ρ_0 is the equilibrium number density in the rest frame of color stream. Clearly the negative root of the dispersion relation gives rise to an instability. As ω_-^2 increases with k, the most unstable modes have $k^2 \gg \omega_p^2$ and in the lowest order in ω_p^2/k^2 one gets $\omega_-^2 \cong -2\omega_p^2$; this result is in agreement with linear kinetic theory result obtained by Mrowczynski [4]. If all four species had been taken as mobile, the space charge contributions simply add up and we get the same dispersion relation as above with ω_p^2 replaced by $2\omega_p^2$. Thus we see that our simpler model problem does not sacrifice significant physics. However, it gives us a considerable reduction in the number of nonlinear differential equations to be solved and it is our justification for using the simpler model. The linear analysis also shows that the color electric field is always opposite to the stream current direction. This provides a collective mechanism for deceleration of stream velocity as was also noted by Ivanov [9] in the context of hadron plasma. The minimum time in which the instability can grow is $t_{\min} \sim |\omega_-|^{-1}$. If one uses finite temperature perturbative QCD value for ω_p for temperature range 160–200 MeV one finds $t_{\min} \sim 0{\cdot}5$–$0{\cdot}7$ fm/c. This estimate is very conservative as smallness of g is assumed in the perturbative calculations. However, non-perturbative estimate of ω_p may be higher [8] and $t_{\min}$ can be further reduced. We compare $t_{\min}$ with the total interaction time between the two nuclei [4], $t_{\mathrm{int}} \sim 2r_0 A^{1/3}\sqrt{\frac{2M_N}{E_{\mathrm{Lab}}}}$ where, A is the atomic number and $r_0 = 1{\cdot}2\,\mathrm{fm}$. For $U+U$ collisions at $E_{\mathrm{Lab}} \simeq 200\,\mathrm{GeVA}$ one gets $t_{\mathrm{int}} \sim 1{\cdot}5\,\mathrm{fm/c}$. As $t_{\min} < t_{\mathrm{int}}$ the filamentation mode may indeed occur in RHIC (relativistic heavy ion collision) and drive up initial fluctuations. Since the number of e-foldings $(t_{\mathrm{int}}/t_{\min})$ is not very large, it is important to ascertain the initial fluctuation level. This can be done by comparing the ratio of field energy E_w to thermal energy E_{th} with the plasma parameter $\beta_p \sim 1/n\lambda_D^3$. Estimating these quantities we obtain

$$\frac{E_w}{E_{\mathrm{th}}} \sim \frac{\omega_p^2 A^2}{T^4} \sim \frac{g^2 A^2}{T^2} \sim \beta_p$$

and using typical values of λ_D, n and T one gets $gA/\omega_p \sim 10^{-1}$. Thus within a time interval of a few $t_{\min}$ the perturbations will grow to a value where the analysis of the non-linear state generated by the instability can become important for RHIC.

For the study of the non-linear state the basic equations constitute a set of eighteen coupled non-linear partial differential equations which are very difficult to solve in their generality. Therefore we look for special solutions of these equations which are non-linear plane stationary waves [8]. Thus we assume that all the quantities depend just on a single variable $\zeta = x + \beta t$. Physically this means that we sacrifice our desire to reproduce the evolution of a linear instability into its saturated nonlinear state and instead focus on the question 'are there nonlinear superpositions of various wavelengths which give a state that is stationary in a moving frame?'. The implication is that if such nonlinear solutions exist, then the unstable waves are likely to saturate into these states. The parameter β expresses the ratio of the phase velocity of the nonlinear plane wave to the velocity of light and may take any value greater than 1. For a dilute plasma a value close to 1 may be assumed, although our calculations show that results are not very sensitive to its choice. With this assumption it is possible to integrate all the differential equations of hydrodynamics except those which correspond to color dynamic equations of one specie. If we introduce dimensionless variables $T = \Omega_p\zeta$, $A_{xa} = a_0^{-1}A_a^x$, $A_{za} = a_0^{-1}A_a^z$, $I_a = i_0^{-1}I_{1a}$, $V_{Ax} = \theta^{-1}V_A^x$ and $V_{Az} = \theta^{-1}V_A^z$ where i_0, a_0, θ and Ω_p are some normalizing factors, then the resulting equations can be written as

$$\dot{I}_a = \frac{\theta\alpha}{\beta}N_1\varepsilon_{abc}[V_{1x}A_{xb} + V_{1z}A_{zb}]I_c, \tag{4}$$

$$\beta^2\ddot{A}_{xa} - \alpha\varepsilon_{abc}A_{zb}\dot{A}_{zc} + \alpha^2[(A_{zb})^2A_{xa} - (A_{xb}A_{zb})A_{za}] = j_{xa}, \tag{5}$$

$$(\beta^2-1)\ddot{A}_{za} + \alpha\varepsilon_{abc}[2A_{xb}\dot{A}_{zc} + \dot{A}_{xb}A_{zc}] + \alpha^2[(A_{xb})^2A_{za} - (A_{xb}A_{zb})A_{xa}] = j_{za}, \tag{6}$$

where $N_1 = \beta/(\beta + \theta V_{1x})$ and dot denotes differentiation with respect to T. We have introduced three dimensionless parameters $\theta = gi_0a_0/m$, $\Omega_p = \omega_p(1 - V_0^2)^{-1/4}$ and $\alpha = ga_0/\Omega_p$. The parameter θ essentially measures the ratio of canonical momentum to mass and hence is related to the velocity which could have value $\sim 10^{-1}$. The parameter α characterizes the strength of non-abelian terms in (6)–(8) and may be expressed in terms of other physical parameters $\alpha = gi_0a_0/\omega_p = g\sqrt{mn_0}(\theta/\omega_p^2)$ which also have values in the same range as θ. Expressions for $V_{\mathrm{A}x}$, $V_{\mathrm{A}z}$ in terms of A_x, A_z

etc. have been obtained but are too lengthy to be reproduced here. Equations (4)–(6) are a set of nine coupled non-linear ordinary differential equations which have been solved numerically, using a fourth order variable step size Runge-Kutta method of sufficient accuracy. Energy and color charge conservation laws derived from the equations have been used as a check on the numerical scheme.

Typical results obtained from (4)–(6) are depicted in figures 1 to 3. Figure 1 depicts variation of V_{2z} with T. It clearly shows that for a choice of the non-abelian parameter $\alpha = 0{\cdot}5$, the mean flow in the nonlinear saturated state is $\simeq -0{\cdot}3$, well below the value $V_{2zO} = -9{\cdot}0$ that we start with at $T = 0$. One may see the stream deceleration as the nonlinear state sets up more clearly in figure 2, which depicts the auto-correlation of V_{2z}. Though the autocorrelation function oscillates a little, it shows an overall decay which shows that the flow velocity fields become chaotic in the nonlinear state. Figure 3 depicts the plot of V_{2x} with T. The figure shows that the transverse flow velocity is also chaotic in the nonlinear state but has acquired a mean value around 0·3. It is to be noted that the generation of mean transverse flow is a consequence of momentum conservation and growth of mixed wave propagating transversely. Looked at another way, the excited nonlinear waves generate a mean force in the transverse direction $[(\tilde{V}_z \times \tilde{B}_y)$ terms etc.] which produces a mean transverse flow. It must be emphasized that this feature of generation of transverse flows is completely independent of choice of initial conditions. Our non-linear theory assumes a given wave propagation direction and generates anisotropic flows. In an experiment, however, the transverse waves and flows will be isotropically distributed. The rearrangement from the initial conditions to set up the nonlinear state leads to

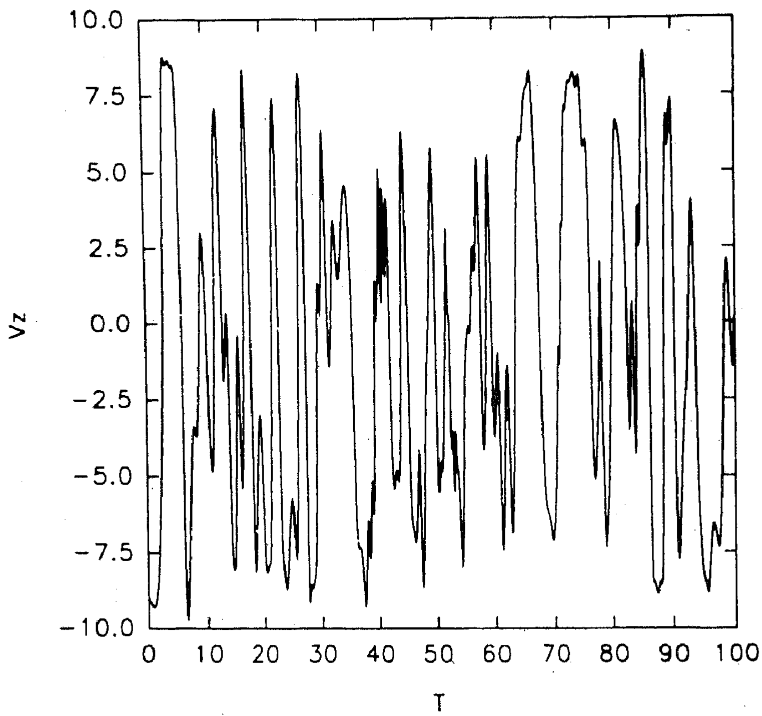

Figure 1. z-component of the flow velocity of specie 2 is plotted as function of T. The values of parameters are $\beta = 1{\cdot}1$, $\theta = 0{\cdot}1$ and $\alpha = 0{\cdot}5$. The intial velocity $V_0 = -9$ (in units of θ). The initial conditions are $I_{1a} = 1$; $(a = 1, 2, 3)$, $A_{xa} = A_{za} = 0$ and $\dot{A}_{x1} = 1{\cdot}4$, $\dot{A}_{x2} = 0{\cdot}2$, $\dot{A}_{x3} = 0{\cdot}3$, $\dot{A}_{z1} = 0{\cdot}2$, $\dot{A}_{z2} = -0{\cdot}3$, $\dot{A}_{z3} = 1{\cdot}1$. Mean velocity $= -3{\cdot}0$.

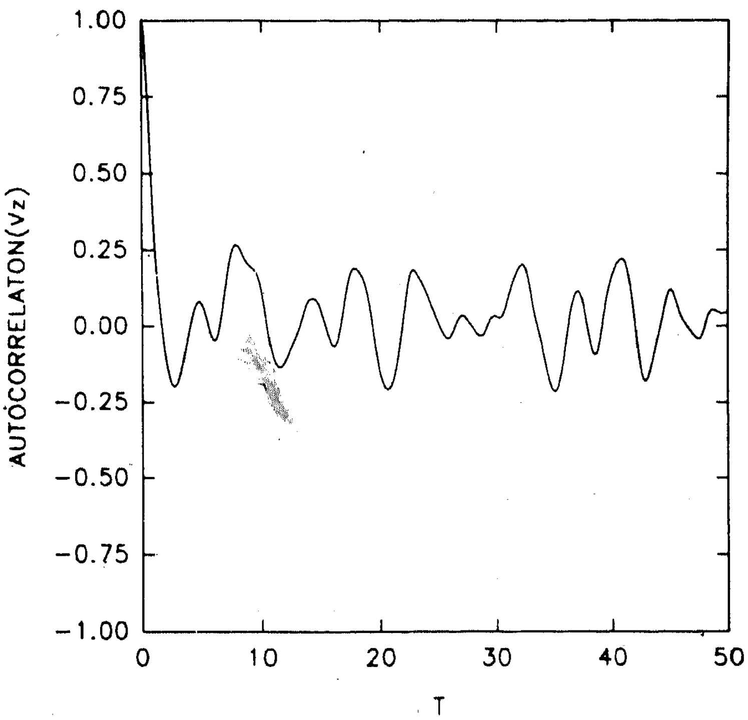

Figure 2. Auto-correlation of the velocity profile shown in figure 1.

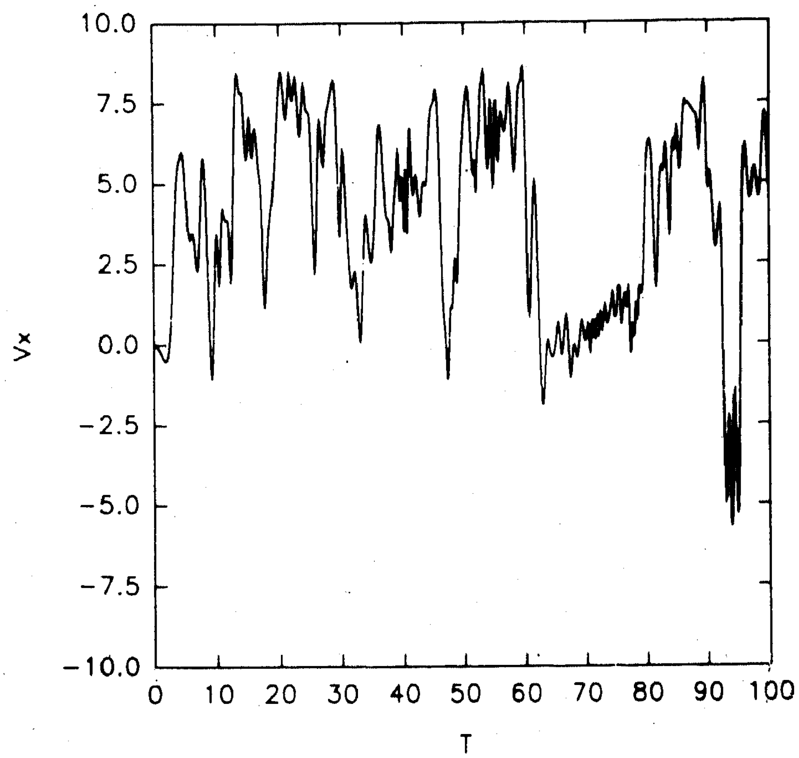

Figure 3. Transverse component of the flow velocity of specie 2 as a function of T. The mean velocity $= 2{\cdot}7$ (in units of θ). Values of parameters and initial conditions are same as those in figure 1.

a conversion of about 90 per cent of the energy in coherent flows into chaotic oscillations and is characterized by a few plasma periods. Taking a typical plasma frequency $\sim 200\,\mathrm{MeV}$, this corresponds to a time-scale of order 1–2 fm/c which is smaller than a plasma life time of order several fm/c. This is consistent with thermalization time estimated by Shuryak [3]. Thus, (i) the linear growth times and nonlinear interaction times needed for setting up a chaotic nonlinear state are well within the plasma life-time, (ii) in the nonlinear plane wave states, the mean kinetic energy along z is considerably reduced and a net mean V_x is generated. Detailed investigations have shown that these numerical results are relatively independent of the choice of parameter β (i.e. phase velocity of nonlinear wave) and other initial conditions. They do, however, critically depend on the non-abelian parameter α. For $\alpha = 0$, we get prototype abelian plasma equations which when numerically integrated show negligible reduction of the initial V_z in the nonlinear state, and does not show generation of significant mean V_x. This seems to be related to the fact that in this case, solutions tend to be coherent and $\tilde{V} \times \tilde{B}$ forces phase average to a negligible value. Thus the collective stopping and transverse flow generation seems to be a specifically non-abelian effect. It may be stressed that had we used four mobile species instead of two, the coupled non-linear equations would still have exhibited similar chaotic behaviour.

In conclusion, we have shown that starting with an assumption of transparency one ends up in a non-linear state showing significant stopping due to non-abelian collective effects. This indicates that one fluid description might be justified for collisions of very heavy nuclei. Our results also indicate the presence of mean collective flow in the transverse direction. Our analysis also indicates that due to the mechanism we have considered baryon rich matter could be created in the central rapidity region.

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