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Stability of quark gluon plasma to Nielsen–Olesen mode

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Abstract. Nielsen and Olesen showed that perturbative vacuum with uniform chromomagnetic field in one space and one color direction is unstable. This instability is called Nielsen–Olesen instability (NOI), and leads to formation of a 'spaghetti of flux tubes' as a model for non-perturbative vacuum and confinement. We re-examine this instability in presence of color sources, quarks and gluons, at a finite temperature and find that at sufficiently high temperature NOI is stabilized due to an 'effective mass' of gluons arising through plasma effects. This explains how a QGP with no confinement effects may exist at high temperature. As the temperature is lowered, NOI reappears at a value $T = T_c$, which is very close to confinement-deconfinement transition from hadrons to QGP..

Keywords. QCD vacuum; Nielsen–Olesen instability; quark gluon plasma; fluid equations; stabilization; phase transition.

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1. Introduction

It is widely recognized that the quantum chromodynamics (QCD) vacuum is very complex due to non-abelian nature of fields. Various models such as the Nielsen–Olesen flux tube model (NOM) [1], magnetic superconductor model [2], dual QCD model [3], color dielectric model [4], instanton liquid model [5] so on have been proposed to describe vacuum. In these models, vacuum is described as condensation of flux tubes, chromomagnetic monopoles, gluons, glue balls etc. The models also give one a qualitative picture of the confinement properties of QCD. In this letter we consider certain new features of the Nielsen–Olesen model of *QCD vacuum*. This model has been extensively used for study of vacuum [6] and has also been applied in electroweak theory [7]. In this model vacuum is described as 'spaghetti of flux tubes' which is formed after a series of instabilities starting from a uniform chromomagnetic field in one particular direction in colour as well as coordinate space. It was shown by Savvidy [8] that such uniform fields arise naturally as potential energy minima of vacuum in the presence of quantum fluctuations. Later Nielsen and Olesen pointed out that the Savvidy state was unstable to excitation of certain modes which lower the system energy further by formation of flux tubes. Further work by Ambjorn and Olesen focussed on the arrangement of the flux tubes, vibration and rotation aspects etc. in order to get lowest energy state and arrived at the sphagetti model of flux tubes. Such a random medium is likely to lead to nonpropagation of quarks and gluons as discussed in reference [9] and hence to the property of confinement.

In this paper we study the Nielsen–Olesen instability (NOI) in presence of color sources such as the ones which may arise in a quark gluon plasma. Our objective is to investigate how the instability is modified by the 'effective mass' of gluons which arises because of plasma effects. We find that the instability is stabilized by plasma effects. The critical temperature at which NOI is completely stabilized is very close to the critical temperature T_c for confinement/deconfinement phase transition [10, 5]. It must be emphasized that our method of studying finite temperature effects is different from that of Kapusta [11], Ninomiya and Sakai [12] and others who consider modifications to the Savvidy state due to finite temperature effects. Kapusta investigated finite temperature modification of the Savvidy state by retaining free energy contribution from thermal gluons in an approximate treatment. He concluded that the phase transition may be associated with the relatively sudden migration of Savvidy minimum field to $H = 0$, as temperature is increased. Ninomiya and Sakai pointed out that Kapusta's treatment was incomplete because he did not retain the $n = 0$ NOI mode; however, their treatment incorporating the NOI was unable to give any clear indication of phase transition. In this paper we have re-examined that question of stability of NOI at a finite temperature by including the response of the quarks and thermal gluons to the $n = 0$ mode. Since there is still some controversy on the question of phase transition from the study of the Savvidy state, we take the classic Savvidy value for the minimum energy magnetic field and see how the plasma effects at finite temperature modify the NOI. Once NOI is cured then there is no formation of "spaghetti state" of Nielsen–Olesen, which is the lower energy state than Savvidy state and is believed to be responsible for confinement. This however, does not fully address the question of phase transition. For that one must re-examine the Savvidy state with a stabilized NOI and free energy contributions from quarks and gluons. Such an investigation is not attempted here.

2. General formalism

Since our main aim is to exhibit the stabilization of NOI by quark gluon current sources, we take a simple model in which the SU(2) Yang–Mills equations (with variation in one space and time) are written in the presence of quark sources. The quark response current is obtained by solving appropriate fluid and color dynamic equations for quarks and anti-quarks [13]; the thermal gluon contributions are added in the usual manner while defining the effective plasma frequency in terms of temperature [14]. The resultant coupled equations for the color excitations are then solved for the frequency and exhibit the NOI and its modifications at finite temperature.

The SU(2) Yang–Mills equations are

$$\partial_\mu G_a^{\mu\nu} + g\varepsilon_{abc} A_{\mu b} G_c^{\mu\nu} = j_a^\nu, \tag{1}$$

where the field tensor

$$G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\varepsilon_{abc} A_b^\mu A_c^\nu,$$

a, b, c are color indices which take values 1, 2, 3 and Lorentz indices μ, $\nu = 0, 1, 2, 3$ with metric $(1, -1, -1, -1)$. ε_{abc} is antisymmetric Levi–Civita tensor. In terms of new set of variables

$$A^\mu \equiv A_3^\mu \text{ and } X^\mu \equiv \frac{1}{\sqrt{2}}(A_1^\mu + iA_2^\mu),$$

(1) reduces to

$$\partial_\mu F^{\mu\nu} - ig[\partial_\mu(X^{\mu *} X^\nu) + X_\mu (D^\nu X^\mu)^* - X_\mu (D^\mu X^\nu)^* - [\mathrm{c.c}]] = j^\nu, \qquad (2)$$

$$D_\mu D^\mu X^\nu - D_\mu D^\nu X^\mu - igX_\mu F^{\mu\nu} + g^2[(X,X)X^{\nu *} - (X,X)^* X^\nu] = j^\nu_+, \qquad (3)$$

where

$$D^\mu \equiv \partial^\mu + igA^\mu, \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu, \quad j^\nu_+ \equiv \frac{1}{\sqrt{2}}(j^\nu_1 + ij^\nu_2) \text{ and } j^\nu \equiv j^\nu_3.$$

Next step is to calculate j^ν_a. We consider quark-antiqurak plasma with two species. The set of fluid equations governing the above system is [13],

$$\frac{\partial n_A}{\partial t} + \nabla\cdot(n_A \mathbf{v}_A) = 0, \qquad (4)$$

$$\frac{\partial \mathbf{v}_A}{\partial t} + (\mathbf{v}_A\cdot\nabla\mathbf{v}_A) = \frac{g}{m_A} I_{Aa}(\mathbf{E}_a + \mathbf{v}_A \times \mathbf{B}_a), \qquad (5)$$

$$\left[\frac{\partial}{\partial t} + \mathbf{v}_A\cdot\nabla\right] I_{Aa} = -g\varepsilon_{abc}(A^0_b - \mathbf{v}_A\cdot\mathbf{A}_b) I_{Ac}, \qquad (6)$$

where n_A and $\mathbf{v}_A$ are density and velocity of species A with $A = 1, 2$. I_{Aa} is the dynamical charge. It is a classical, space time dependent color charge which may be obtained by taking the expectation value of color spin operator. Equation (6) describes the hydrodynamics of color charge, which may be obtained by taking the classical limit of Heisenberg equation of motion of color spin operator [15]. Equation (4) is the mass continuity equation and (5) is the force equation. Equation (5) is a non-relativistic fluid equation. This assumption will not change our results qualitatively because, in linear analysis ($\mathbf{v}_A \ll 1$), and fluid relativistic effects drop out [16]. When the system is microscopically relativistic (zero mass case) one may replace the mass parameter, m_A, by enthalphy density/number density [17] in this equation. This prescription reproduces the perturbative QCD results from the hydrodynamic equations, (Eqs (4–6)), in the study of QGP as shown in [16]. In our estimate of temperature, we shall use results of relativistic kinetic theory which also take account of some part of contribution due to thermal gluons. The current components are given by

$$j^0_a = g\sum_A n_A I_{Aa} \text{ and } \mathbf{j}_a = g\sum_A n_A \mathbf{v}_A I_{Aa}.$$

In further discussion, we make the gauge choice $A^0_a = 0$.

3. Stability analysis

To study stability, we need to perturb the system from equilibrium and study the linear response. At equilibrium we have uniform chromomagnetic field in z-direction in coordinate space and third direction in color. That is,

$$\mathbf{B}^a_0 = H\hat{z}\delta_{a,3} \text{ or } \mathbf{A}^a_0 = Hx\hat{y}\delta_{a,3}. \qquad (7)$$

Linearizing the eqs (2)–(3) and Fourier analyzing in t we get

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \omega^2\right] A^\mu = -j^\mu. \qquad (8)$$

$$D^2 X^\mu + 2igF^{\mu\nu}_0 X_\nu = j^\mu_+, \qquad (9)$$

where $D^2 \equiv -\omega^2 - (d^2/dx^2) + g^2 H^2 x^2$. Here all quantities with subscript zero refers to equilibrium values and all other dependent variables are perturbations. Second term in the left hand side of (9), which is due to gluon spin or due to self-interactions of gluons, is instrumental for the instability. Only two equations, with Lorentz indices 1 and 2, have that term and so we consider only those two equations from (9).

To calculate the current, we perturb eqs (4)–(6) and Fourier analyze in time to get

$$-i\omega n_A + n_{A0}\nabla\cdot\mathbf{v}_A = 0, \tag{10}$$

$$-i\omega\mathbf{v}_A = \frac{g}{m_A}[I_{Aa0}\mathbf{E}_a + I_{Aa0}\mathbf{v}_A \times \mathbf{B}_{a0}], \tag{11}$$

$$-i\omega I_{Aa} = g\varepsilon_{abc}\mathbf{v}_A\cdot\mathbf{A}_{b0}I_{Ac0}. \tag{12}$$

Out of three equations we need only (11) to calculate the current. Expressing $\mathbf{E}_a$ and $\mathbf{B}_a$ in terms of new variables A^μ, X^μ and after some algebra we get, for $\mu = 1,2$

$$j^1_+ = -[\omega_1^2 X^1 + \omega_2^2 X^{1*}], \tag{13}$$

$$j^2_+ = -[\omega_1^2 X^2 + \omega_2^2 X^{2*}], \tag{14}$$

where

$$\omega_1^2 = \sum_A \frac{g^2 n_{A0}|I_{A+0}|^2}{m_A}, \quad \omega_2^2 = \sum_A \frac{g^2 n_{A0} I^2_{A+0}}{m_A},$$

with

$$I_{A+0} \equiv \frac{I_{A10} + iI_{A20}}{\sqrt{2}},$$

and similar equation for j^1 and j^2 with X replaced by A. In arriving at the above equation we also assumed $I_{A30} = 0$, for simplicity, which eliminates the coupling between A^μ and X^μ as well as the effects of particle gyration about the equilibrium chromomagnetic field. Putting these currents in (9) and its complex conjugates, and decoupling the final four equations, we get

$$\chi_1'' + (-\omega_1^2 - q_+ - g^2H^2x^2)\chi_1 = 0, \tag{15}$$

$$\chi_2'' + (-\omega_1^2 - q_- - g^2H^2x^2)\chi_2 = 0, \tag{16}$$

and their complex conjugates, where

$$q_\pm = -\frac{\lambda + \lambda^*}{2} \pm \frac{1}{2}[(\lambda - \lambda^* + 4gH)^2 + 4|\omega_2^2|^2]^{1/2} \text{ and } \lambda \equiv \omega^2,$$

and χ_1, χ_2 are some complicated linear combination of X^1, X^2 and their complex conjugates as given in the appendix. Further one can notice that

$$|\omega_2^2|^2 = \omega_1^4,$$

which follows from color neutrality condition. Equations (15)–(16) are of the form of harmonic oscillator Schrödinger equation and the eigen values are related to the parameters of our problem by

$$-\omega_1^2 - q = (2n+1)gH, \tag{17}$$

where $\dot{q}$ refers to q_+ or q_-. It is easy to see from (17) that $\lambda_I = 0$ and for instability to occur $\lambda_R < 0$, where λ_R and λ_I are real and imaginary parts of λ. It follows from (17) that

$$\lambda_R = \omega_1^2 + (2n+1)gH - (4g^2H^2 + \omega_1^4)^{1/2}. \tag{18}$$

Here we see that $n \neq 0$ modes are always stable. But for $n = 0$ $\lambda_R < 0$, or mode is unstable for $\omega_1^2 = 0$. That is, in vacuum $n = 0$ mode is unstable, the well known Nielsen–Olesen instability. Finite ω_1^2 leads to stabilization of above instability since it is a positive quantity. One may easily obtain ω_1^2 required to stabilize the instability as

$$\omega_1^2 = \omega_p^2 = \tfrac{3}{2}gH. \tag{19}$$

Thus we see that the presence of quarks or color sources stabilizes NOI instability and the stabilizing factor is proportional to the plasma frequency. The physical explanation is that gluons acquire an effective mass, which is proportional to plasma frequency, and compete with the gluon anomalous magnetic moment term in determining the stability. Plasma frequency is related to temperature and hence we can get a temperature, T_c, at which NOI is just stabilized.

4. Estimate of T_c

Now we assume that the above relation, eq. (19) is valid even in relativistic QGP with thermal gluons in order to get some numbers related to QGP. We know from linear response theory of QGP that the plasma frequency is related to temperature by the relation [14]

$$\omega_p^2 = \frac{g^2T^2}{18}(N_f + 2N_c), \tag{20}$$

where N_f, N_c and T are number of flavors, number of colors and temperature of the plasma respectively. The factor N_c is due to the thermal gluon response.

The temperature, T_c, is calculated from the equation,

$$\frac{g^2T_c^2}{18}(N_f + 2N_c) = \tfrac{3}{2}gH \approx \tfrac{3}{2}(gH)_{\min},$$

where

$$(gH)_{\min} = \Lambda_s^2,$$

is the strength of the uniform chromomagnetic field, which has the minimum energy, as estimated in [8, 18]. We find that $T_c \approx 0{\cdot}66\Lambda_s \approx 160\,\text{MeV}$ for $\alpha_s = 1$ and $\Lambda_s = 240\,\text{MeV}$ [11]. Above value of T_c is for $N_f = 1$ and $N_c = 2$ and it decreases as the number of flavors or colors in plasma are increased. It is interesting to note that critical temperature of confinement-deconfinement phase transition of QCD also has such a dependence on number of flavors in lattice QCD [10].

5. Conclusions

In conclusion, we have shown that the NOI in quark gluon plasma is stabilized due to the effective mass of gluons due to plasma effects. We find that the temperature at which NOI is fully stabilized is close to the critical temperature for phase transition. Once the the NOI is stabilized there is no question of formation of flux tubes, "spaghetti state" and the resultant confinement effects. Of course, to fully understand the phase transition physics, one must now ask what happens to the Savvidy state at this temperature. Kapusta has already investigated the problem of finite temperature effects on the Savvidy state. However, as pointed out, by Ninomiya and Sakai, he did not include the effect of the Nielsen–Olesen unstable mode. When these latter authors included the effects due to NOI, they were unable to get a clear idea of the phase transition. Our calculations show that it is also important to take account of response currents due to quarks and gluons because these can stabilize the NOI mode. We thus believe that the finite temperature effect on the Savvidy state needs to be re-examined taking account of the NOI and the plasma screening effects discussed in this paper. Such a calculation is in progress.

Appendix

χ_1 and χ_2 of (15)–(16) are defined as

$$\chi_1 = \frac{\omega_2^2}{|\omega_2^2|}\left[\frac{1}{2} + \frac{gH}{[4(gH)^2 + |\omega_2^2|^2]^{1/2}}\right]^{1/2} \frac{(X^{1*} - iX^{2*})}{\sqrt{2}} + \left[\frac{1}{2} - \frac{gH}{[4(gH)^2 + |\omega_2^2|^2]^{1/2}}\right]^{1/2} \frac{(X^1 - iX^2)}{\sqrt{2}}, \tag{21}$$

and

$$\chi_2 = -\frac{\omega_2^2}{|\omega_2^2|}\left[\frac{1}{2} - \frac{gH}{[4(gH)^2 + |\omega_2^2|^2]^{1/2}}\right]^{1/2} \frac{(X^{1*} - iX^{2*})}{\sqrt{2}} + \left[\frac{1}{2} + \frac{gH}{[4(gH)^2 + |\omega_2^2|^2]^{1/2}}\right]^{1/2} \frac{(X^1 - iX^2)}{\sqrt{2}}, \tag{22}$$

where we used the fact that λ is real.

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