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Hot carrier diffusion induced instabilities of electromagnetic waves in plasmas and semiconductors*

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The stability of a plasma/semiconductor for small fluctuations in the intensity of plane electromagnetic beam has been studied when a non-linearity in the dielectric constant appears on account of the diffusion of non-uniformly heated carriers (due to fluctuations in the intensity of the beam). Fluctuations of optimum size and long duration have been found to grow at moderate powers. The effect of absorption is, however, to suppress this effect.

1. Introduction

The non-linear propagation of electromagnetic waves in plasmas and dielectrics has been extensively investigated in recent years [1-5]. The field dependent conductivity and dielectric constant of a medium leads (amongst other interesting effects) to the self-focusing [4, 6, 7] of intense beams. Kaw, Schmidt, and Wilcox [9] have recently studied the problem of growth of a small fluctuation in the intensity distribution of an intense uniform electromagnetic beam in a collisionless plasma, where the non-linearity appears through the ponderomotive force. In this paper we have examined the stability of a plasma/semiconductor for small fluctuations in the intensity of a plane electromagnetic beam when the non-linearity of the dielectric constant enters through the non-uniform heating of the carriers and their subsequent diffusion.

The behaviour of the fluctuations can be physically understood as follows. It has been shown in several papers [4, 8, 10, 11] that a beam of non-uniform intensity distribution (typically a Gaussian distribution) in a non-linear medium has a natural tendency of self-focusing around the region of high intensity; the characteristic length for focusing may be given by [1, 10] $R_n = a\sqrt{(\epsilon_0/\epsilon_{nl})}$, where a is the initial width of the beam, ϵ_0 is the linear part of the dielectric constant and ϵ_{nl} is its non-linear part. In a similar manner, in the case of a uniform beam, a small increment (due to fluctuations) in the intensity of the beam causes an increase in the dielectric constant which, owing to

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its focusing nature, attracts more and more power from its surroundings. Thus the perturbation grows as the beam propagates in the medium. However, it should be noted that only those fluctuations will grow which have a duration greater than the energy relaxation time of the medium, otherwise the inhomogeneity in the dielectric constant of the medium will not be set up. This analysis, therefore, is valid only for sufficiently long duration fluctuations.

2. Non-linear dielectric constant

We consider the propagation of an intense electromagnetic wave of uniform intensity distribution along its wavefront in a plasma. The electrons of the plasma acquire a momentum due to the electric vector of the wave. On account of the finite (real part of) conductivity of the medium, the carriers absorb power from the wave and their temperature rises above the thermal equilibrium value. A steady state is reached in the energy relaxation time ($\tau_\epsilon \sim M/2m\,\nu$) when the power gained by the electrons from the field is balanced by that lost to scatterers in collisions. The steady state rise in electron temperature can be given by [1]

$$T_e - T_0 = \alpha\, T_0\, EE^*\,, \tag{1}$$

where

$$\alpha = e^2 M/6m^2 k_0 T_0 \omega^2\,, \quad \nu^2 \ll \omega^2\,,$$

$-e$, m and ν are the electronic charge, mass and collision frequency, M is the mass of scatterer, k_0 is the Boltzmann constant and $\boldsymbol{E} = \boldsymbol{E}' \exp[i\,\omega\, t]$ is the electric vector of the wave. The free carrier current density of the medium [1], is given by

$$\boldsymbol{J} = \sigma \boldsymbol{E}\,, \quad \sigma = \frac{N_0 e^2}{m\,\omega^2}(\nu - i\,\omega)\,,$$

where N_0 is the carrier concentration.

It is clear that for uniform heating by a plane beam, the out of phase part of the current density does not show any non-linearity, though the inphase part (being very small) may show some non-linear behaviour if ν is velocity dependent. Hence, the phenomenon of non-linear refraction does not occur under such situations. However, when a fluctuation $\Delta(EE^*)$ in the intensity occurs in any part of the beam, the electrons in that part are heated more, i.e.,

$$\Delta T_e = T_0\, \alpha\, \Delta(EE^*)\,, \tag{2}$$

while in the rest of the plasma electron temperature is T_e. This non-uniformity gives rise to the redistribution of electrons and ions (ions are dragged by the space charge field which is generated due to the redistribution of electrons) and the electron concentration in the region of fluctuation may be obtained as [8, 11]

$$\Delta N_e \simeq \Delta N_i = -\frac{\Delta T_e}{T_e + T_0} N_0\,. \tag{3}$$

The effective dielectric constant of the plasma can, now, be written as

$$\epsilon = 1 - \frac{4\pi N_e e^2}{m\omega^2}\left(1 + i\frac{\nu_{eff}}{\omega}\right) = \epsilon_0 - i\epsilon_i + \epsilon_2 \Delta(EE^*), \tag{4}$$

where

$$\epsilon_0 = 1 - \frac{\omega_p{}^2}{\omega^2}, \quad \epsilon_i = \frac{\nu_{eff}}{\omega}\frac{\omega_p{}^2}{\omega^2},$$

$$\epsilon_2 = \frac{\omega_p{}^2}{\omega^2}\frac{\alpha}{(2 + \alpha EE^*)}, \quad \omega_p = \left(\frac{4\pi N_0 e^2}{m}\right)^{\frac{1}{2}}, \quad \nu_{eff}{}^2 << \omega^2$$

and ν_{eff} is the effective electron collision frequency.

The above analysis can be easily extended to parabolic semiconductors having equal numbers of electrons and holes. The resulting expression for the non-linear dielectric constant can be given as Equation 4 with

$$\epsilon_0 = \epsilon_L - \frac{\omega_{p_e}{}^2}{\omega^2} - \frac{\omega^2{}_{p_h}}{\omega^2}, \quad \epsilon_i = \frac{\nu_{eff}}{\omega}\frac{\omega^2{}_{p_e}}{\omega^2} + \frac{\nu_h}{\omega}\cdot\frac{\omega^2{}_{p_h}}{\omega^2},$$

$$\epsilon_2 = \frac{\omega^2{}_{p_e}}{\omega^2}\cdot\frac{\alpha_e}{(2 + \alpha_e EE^* + \alpha_h EE^*)} + \frac{\omega^2{}_{p_h}}{\omega^2}\cdot\frac{\alpha_h}{(2 + \alpha_e EE^* + \alpha_h EE^*)},$$

$$\alpha_{e,h} = \frac{e^2 M}{6m^2{}_{e,h} k_0 T_0 \omega^2}, \quad \omega^2{}_{p_{e\,h}} = \frac{4\pi N_0 e^2}{m_{e,h}},$$

$M = k_0 T_0/S^2$ (effective mass of an acoustical phonon), S is the velocity of sound in the semiconductor and ϵ_L is the lattice dielectric constant.

The case of n-type or p-type semiconductors (single type of carrier) is very different. Due to the strong space charge fields, the carriers are not redistributed and hence the present considerations are not valid. However, if the energy bands of carriers in the semiconductor are non-parabolic, then the non-linearity can arise through the energy dependent carrier mass. For example, in n-InSb, the effective average mass of electrons varies with temperature as [10]

$$m = \frac{3k_0 T_e + \mathcal{E}_g}{L},$$

where $\mathcal{E}_g$ is the band gap as L is a constant ($\sim 10^{31}$). Consequently, the phenomenon of non-linear refraction becomes important even when the intensity distribution of the beam is uniform. The non-linear dielectric constant of n-InSb can be written as Equation 4 with

$$\epsilon_0 = \epsilon_L - \frac{\omega_p^2}{\omega^2\left(1 + \dfrac{\alpha EE^*}{1+\epsilon_g}\right)},$$

$$\epsilon_i = \frac{\nu_{eff}}{\omega} \cdot \frac{\omega_p^2}{\omega^2\left(1 + \dfrac{\alpha EE^*}{1+\epsilon_g}\right)},$$

$$\epsilon_2 = \frac{\omega_p^2}{\omega^2} \cdot \frac{\alpha}{(1 + \alpha EE^*/(1+\epsilon_g))^2 (1+\epsilon_g)},$$

$$\omega_p^2 = \frac{4\pi N_0 e^2}{m_0}, \quad m_0 = \frac{3k_0 T_0 + \mathscr{E}_g}{L}, \quad \epsilon_g = \mathscr{E}_g/3k_0 T_0,$$

$$\alpha = e^2 \nu_{eff} \tau_\varepsilon / 3m_0 k_0 T_0 \omega^2,$$

and τ_ε is the energy relaxation time (due to polar optical mode scattering).

3. Growth rate of a perturbation

Consider a plane beam propagating along the z-direction, i.e.

$$E(z, t) = E_0 \exp[i(\omega t - kz)], \tag{5}$$

where E_0 is the amplitude of the electric vector (polarized in the y-direction) at $z = 0$, k is the unperturbed propagation constant given by,

$$k = \frac{\omega}{c} (\epsilon_0 - i\,\epsilon_i)^{\frac{1}{2}}, \tag{5a}$$

and c is the velocity of light in vacuum. Equation 5 is not valid in the case of n-InSb because ϵ_0 is a function of EE^*. However, if $\nu_{eff}/\omega \ll 1$, i.e. the attenuation of the wave is negligible, Equation 5 holds with $k = \omega/c \,.\, \epsilon_0^{\frac{1}{2}}(E_0^2)$. Before discussing the role of absorption, we consider the nature of growth rate of fluctuation in the limit of $\nu_{eff}/\omega \ll 1$. The fluctuation in the electric vector may be taken to be of the form

$$\boldsymbol{E}_f = \boldsymbol{E}_1(x, y, z) \exp[i(\omega t - kz)] \tag{6}$$

where $\boldsymbol{E}_1$ is the complex quantity. Then Equation 4 can be rewritten as

$$\begin{aligned}\epsilon = \epsilon_0(E_0^2) + \epsilon_2(E_0^2)\, \{(\boldsymbol{E} + \boldsymbol{E}_f) \,.\, (\boldsymbol{E}^* + \boldsymbol{E}_f^*) - E_0^2\} \\ \simeq \epsilon_0(E_0^2) + \epsilon_2(E_0^2) \,.\, \{\boldsymbol{E}_0 \,.\, (\boldsymbol{E}_1 + \boldsymbol{E}_1^*)\},\end{aligned} \tag{7}$$

where the square of the perturbation quantities has been neglected.

To study the behaviour of $\boldsymbol{E}_1$ inside the non-linear medium, we start from the wave equation,

$$\nabla^2 \boldsymbol{E}_T - \nabla(\nabla \,.\, \boldsymbol{E}_T) + \frac{\omega^2}{c^2} \epsilon \boldsymbol{E}_T = 0, \tag{8}$$

where $\boldsymbol{E}_T$ is the total electric field in the medium.

Using the relation $\nabla \cdot \boldsymbol{D} = 0$, Equation 8 becomes

$$\nabla^2 \boldsymbol{E}_{\mathrm{T}} + \nabla \frac{(\boldsymbol{E}_{\mathrm{T}} \cdot \nabla \epsilon)}{\epsilon} + \frac{\omega^2}{c^2} \epsilon E_{\mathrm{T}} = 0 . \tag{9}$$

Equation 5 is the zeroth order solution of this equation. For E_1, Equation 9 after linearization, gives

$$\nabla^2 \boldsymbol{E}_1 - 2ik \frac{\partial}{\partial z} \boldsymbol{E}_1 + \nabla \left(\frac{E_0}{\epsilon_0} \frac{\partial \epsilon}{\partial y} \right) + \frac{\omega^2}{c^2} \epsilon_2(E_0^2) \{ \boldsymbol{E}_0 \cdot (\boldsymbol{E}_1 + \boldsymbol{E}_1^*) \} E_0 = 0 . \tag{10}$$

It can be seen from Equations 7 and 10 that if E_1 is perpendicular to E_0 (i.e. the perturbation is polarized in the x-z plane), ϵ is independent of y and the third and the fourth terms in Equation 10 become zero. Then the solution for x and z components are the same as Equation 5 and nothing meaningful results. However, if the perturbation is polarized in the direction of E_0 (i.e. along the y-direction), Equation 10 assumes the following form

$$\nabla^2 E_{1y} - 2ik \frac{\partial}{\partial z} E_{1y} + \frac{\epsilon_2(E_0^2)}{\epsilon_0(E_0^2)} E_0^2 \frac{\partial^2}{\partial y^2} (E_{1y} + E_{1y}^*) + k^2 \frac{\epsilon_2(E_0^2)}{\epsilon_0(E_0^2)} E_0^2 (E_{1y} + E_{1y}^*) = 0 . \tag{11}$$

Expressing E_{1y} as $E_{1y} = E_{1r} + i\, E_{1\mathrm{i}}$, Equation 11 gives

$$\nabla^2 E_{1r} + 2 \frac{\epsilon_2 E_0^2}{\epsilon_0} \frac{\partial^2}{\partial y^2} E_{1r} + 2k \frac{\partial E_{1\mathrm{i}}}{\partial z} + 2k^2 \frac{\epsilon_2 E_0^2}{\epsilon_0} E_{1r} = 0 \tag{12a}$$

and

$$\nabla^2 E_{1\mathrm{i}} - 2k \frac{\partial E_{1r}}{z} = 0 . \tag{12b}$$

To obtain the solution of coupled Equations 12a and 12b, we employ the complex notation by considering the variations of E_{1r} and $E_{1\mathrm{i}}$ as $E_{1r,\mathrm{i}}$ Re exp$[- i(\beta_{\|} z + \beta_{\perp} \boldsymbol{r})$ where $\boldsymbol{r} = \boldsymbol{i}x + \boldsymbol{j}y$ and $\beta_{\|}$ and $\beta_{\perp}$ are three unknown coefficients to be determined. Using this variation in Equations 12a and 12b, the condition for the non-trivial solution comes out to be

$$\left[\beta_{\|}^2 + \beta_{\perp}^2 + \frac{2\, \epsilon_2 E_0^2}{\epsilon_0} \beta_y^2 - 2k^2 \frac{\epsilon_2 E_0^2}{\epsilon_0} \right] \beta^2 = 4k^2 \beta_{\|}^2 . \tag{13}$$

On making a valid assumption $\beta_{\|}^2 \ll \beta_{\perp}^2$ (i.e. the perturbation scale length in the x-y plane is small) Equation 13 can be simplified as

$$\beta_{\|}^2 = \frac{\beta_{\perp}^2}{4k^2} \left[\beta_{\perp}^2 + \frac{2\, \epsilon_2 E_0^2}{\epsilon_0} \beta_y^2 - 2k^2 \frac{\epsilon_2 E_0^2}{\epsilon_0} \right] \tag{14}$$

It is obvious from this relation that whenever

$$k^2 \frac{\epsilon_2 E_0^2}{\epsilon_0} > \frac{\beta_{\perp}^2}{2} + \frac{\epsilon_2 E_0^2}{\epsilon_0} \beta_y^2 , \tag{15}$$

$\beta_{\|}$ becomes imaginary and hence the perturbation grows as it advances in the z direction.

The growth rate of fluctuation is given by

$$\Gamma = i\,\beta_{\|} = \frac{\beta_\perp}{2k}\left[-\,\beta_\perp^2 - \frac{2\,\epsilon_2\,E_0^2}{\epsilon_0}\,\beta_y^2 + 2k^2\,\frac{\epsilon_2\,E_0^2}{\epsilon_0}\right]^{\frac{1}{2}}. \tag{16}$$

To discuss the behaviour of growth rate we consider two special cases.

I. $$\beta_y = 0\,,\quad \beta_\perp = \beta_x\,.$$

In this case the wave vector of the perturbation is at right angles to the electric vector and the perturbation propagates as a TE wave. The growth rate from Equation 16 comes out to be,

$$\Gamma = \frac{\beta_x}{2k}\left[-\,\beta_x^2 + 2k^2\,\frac{\epsilon_2}{\epsilon_0}\,E_0^2\right]^{\frac{1}{2}}. \tag{17}$$

Γ is a function of β_x and shows a maximum corresponding to

$$\beta_{x\,\mathrm{opt}} = k(\epsilon_2\,E_0^2/\epsilon_0)^{\frac{1}{2}} \tag{18a}$$

and the corresponding growth rate is given by

$$\Gamma_{\max} = \frac{k}{2}\left(\frac{\epsilon_2\,E_0^2}{\epsilon_0}\right). \tag{18b}$$

II. $$\beta_x = 0\,,\quad \beta_\perp = \beta_y\,.$$

This case can be discussed in the similar manner; $\beta_{y\,\mathrm{opt}} \simeq \beta_{x\,\mathrm{opt}}$ and $\Gamma_{\max}$ is again given by Equation 18b.

Now we proceed to discuss this effect in the case of a plasma when the absorption is linear. The results can be easily extended to parabolic semiconductors. The Equations 5 and 6 still hold with ϵ_0 and ϵ_i being field independent. The total dielectric constant can be written as

$$\epsilon = \epsilon_0 - i\,\epsilon_i + \epsilon_2(E'_0)^2\,\{\boldsymbol{E}_0\,.\,(\boldsymbol{E}_1 + \boldsymbol{E}_1^*)\}\,.\,\exp[-\,2kiz]\,, \tag{19}$$

where

$$E'_0 = E_0\exp[-\,k_i\,z]\,,\quad k = k_r - i\,k_i = \frac{\omega}{c}\,(\epsilon_0 - i\,\epsilon_i)^{\frac{1}{2}}\,.$$

Following the above treatment, the wave equation can be solved for the perturbation $E_{1y} = E_{1r} - i\,E_{1i}$. On considering the variations of E_{1r} and E_{1i} as

$$E_{1r,i}\ \mathrm{Re}\exp[-\,\mathrm{i}(\beta_{\|}\,z + \boldsymbol{\beta}_\perp.\,\boldsymbol{r})]\,,$$

the expression for $\beta_{\|}$ comes out to be

$$\beta_{\|} = -\,\frac{i\,k_i}{4k_r^2}\left\{2\beta_\perp^2 - \frac{2\,\epsilon_2\,E_0^2}{\epsilon_0}\exp[-\,2k_i\,z]\left(\frac{\omega^2}{c^2}\,\epsilon_0 - \beta_y^2\right)\right\}$$
$$+\,\frac{\beta_\perp^2}{2k_r}\left\{\beta_\perp^2 - \frac{2\,\epsilon_2\,E_0^2}{\epsilon_0}\exp[-\,2k_i\,z]\left(\frac{\omega^2}{c^2}\,\epsilon_0 - \beta_y^2\right)\right\}^{\frac{1}{2}}. \tag{20}$$

The condition for $\beta_{\parallel}$ having imaginary solution is

$$\frac{2\epsilon_2 E_0^2}{\epsilon_0} \exp[-2k_i z]\left(\frac{\omega^2}{c^2}\epsilon_0 - \beta_y^2\right) > \beta_{\perp}^2 \tag{21}$$

and the corresponding growth rate of the perturbation in the absorbing medium is

$$\Gamma = |\beta_{\parallel}| - k_i . \tag{22}$$

This expression is meaningful only when absorption is not predominant. The expression for the optimum value of growth rate can be obtained as

$$\Gamma_{\max} = \frac{k}{2}\left(\frac{\epsilon_2 E_0^2}{\epsilon_0} \exp[-2k_i z]\right) - k_i .$$

4. Discussions and conclusions

The hot carrier non-linearity in the dielectric constant of a plasma/semiconductor is very effective in making the medium unstable for small fluctuations in the intensity of an electromagnetic beam. The perturbation grows with the advancement of the beam. The optimum size of the perturbation is $A_{\text{opt}} \simeq \lambda/2\pi\, \epsilon_0^{1/2} . (\epsilon_0/\epsilon_2 E_0^2)^{1/2}$ and the corresponding length for its growth is $R_{\text{ne}} \simeq 2A_{\text{opt}}(\epsilon_0/\epsilon_2 E_0^2)^{1/2}$. For a perturbation of larger dimension, the characteristic length for the growth increases because the inhomogeneity in the transverse direction is small to focus the beam. When the perturbation size is smaller

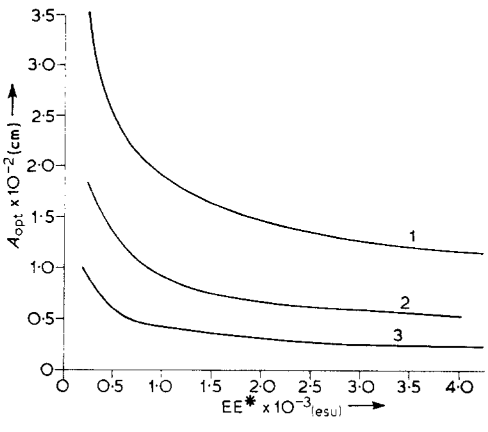

Figure 1 Graph showing the variation of optimum size of the perturbation A_{opt} with the intensity of the beam (EE^*)

1. Plasma

$$\omega \sim 10^{12}\ \text{Hz}\,,\ N_0 \sim 10^{14}\ \text{cm}^{-3},\ T_0 = 77\ \text{K},\ \frac{\omega_p^2}{\omega^2} \sim 0.40\,,\ \alpha \simeq 400\,.$$

2. Parabolic semiconductor (Germanium)

$$\omega \sim 10^{12}\ \text{Hz}\,,\ N_0 \sim 10^{14}\ \text{cm}^{-3},\ T_0 \sim 77\ \text{K}\,,\ \frac{\omega_p^2}{\omega^2} \sim 2.5\,,\ \alpha \sim 110\,.$$

3. Nonparabolic semiconductor (InSb)

$$\omega \sim 10^{12}\ \text{Hz}\,,\ N_0 \sim 10^{14}\ \text{cm}^{-3}\,,\ T_0 \sim 77\ \text{K}\,,\ \frac{\omega_p^2}{\omega^2} \sim 25\,,\ \alpha \sim 600\,,\ \epsilon_g \sim 8.5\,,\ \mathscr{E}_g = 0.17\ \text{eV}\,.$$

than A_{opt}, the diffraction effects play an important role and the effective length for the growth is enhanced. A plot of A_{opt} with E_0^2 for the cases of a plasma and parabolic/nonparabolic semiconductor is shown in Fig. 1.

Besides, the self-focusing effect of non-linearity is suppressed by the absorption effects.

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