

Dynamic Security-Constrained Rescheduling of Power Systems Using Trajectory Sensitivities

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Abstract—In the deregulated environment of power systems, the transmission networks are often operated close to their maximum capacity to achieve transfer of power. Besides, the operators must operate the system to satisfy its dynamic stability constraints under credible contingencies. This paper provides a method using trajectory sensitivity to reschedule power generation to ensure system stability for a set of credible contingencies while satisfying its economic goal. System modeling issue is not a limiting concern in this method, and hence the technique can be used as a preventive control scheme for system operators in real time.

Index Terms—Optimal power flow, trajectory sensitivity, generation rescheduling, preventive control, dynamic security.

I. INTRODUCTION

Preventive rescheduling of power systems subject to stability constraints for contingencies has been investigated for a number of years. The earliest work [1] investigated the maximum loadability problem followed by an investigation into interface flow across tie lines [2]. Both used the transient energy margin concept and its sensitivity to change in generation schedules. Since then a number of papers have appeared along these lines by extending the criteria to include optimal power flow (OPF) which is logical [3]-[5]. The other approach is to include the stability constraints as part of the OPF problem by converting the differential equations into algebraic constraints [6], [7]. In this case, the system dynamics are represented as classical swing models. The transient energy function (TEF) approach is limited to cases where a closed form expression for the energy is available and is difficult in the case of hybrid systems such as systems with tap changers or systems containing flexible ac transmission systems (FACTS) devices.

In this paper we propose a new approach based on trajectory sensitivities and has no limitation in terms of model complexity. The trajectory sensitivities for each contingency are computed along with the state of the system dynamics. For each contingency the machine which is vulnerable is identified and generation is shifted to the least vulnerable generator. There is a choice as to where to shift the generation whether to a single one or a group of them. The overall idea is to make the system secure. The technique is illustrated for a 3-machine as well as the 10-machine case. The technique could also be useful in congestion management of systems.

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II. MATHEMATICAL MODEL

A. System and Sensitivity Models

In simulating disturbances, switching actions take place at certain time instants. At these time instants, the algebraic equations change, resulting in discontinuities of the algebraic variables. In general the power system can be cast in the form of a differential-algebraic discrete (DAD) model incorporating discrete events as in [8]. A special case is the model described by differential-algebraic equations of the form

$$\dot{\underline{x}} = f(x, y, \lambda) \quad (1)$$

$$0 = \begin{cases} g^-(x, y, \lambda) & s(x, y, \lambda) < 0 \\ g^+(x, y, \lambda) & s(x, y, \lambda) > 0 \end{cases} \quad (2)$$

A switching occurs when the switching function $s(x, y, \lambda) = 0$. For example, if a device is switched into service at time $t = t_{sw}$, then the switching function in this case can simply be defined as $s(x, y, \lambda) = t - t_{sw}$.

In the above model, x are the dynamic state variables such as machine angles, velocities, etc.; y are the algebraic variables such as load bus voltage magnitudes and angles; and λ are the system parameters such as line reactances, generator mechanical input power, or fault clearing time. Note that the state variables x are continuous while the algebraic variables can undergo step changes at switching instants.

The initial conditions for (1)-(2) are given by

$$x(t_0) = x_0, y(t_0) = y_0 \quad (3)$$

where y_0 satisfies the equation

$$g(x_0, y_0, \lambda) = 0 \quad (4)$$

For compactness of notation, the following definitions are used

$$\underline{x} = \begin{bmatrix} x \\ \lambda \end{bmatrix}, \underline{f} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

With these definitions, (1)-(2) can be written in a compact form as

$$\underline{\dot{x}} = \underline{f}(\underline{x}, y) \quad (5)$$

$$0 = \begin{cases} g^-(\underline{x}, y) & s(\underline{x}, y) < 0 \\ g^+(\underline{x}, y) & s(\underline{x}, y) > 0 \end{cases} \quad (6) \quad \text{subjected to} \quad \text{Minimize } C(P_g) \quad (14)$$

Note that the parameters are absorbed in the differential equations as state variables with the derivatives equal to zero. The initial conditions for (5)-(6) are

$$\underline{x}(t_0) = \underline{x}_0, y(t_0) = y_0 \quad (7)$$

Trajectory sensitivity analysis studies the variations of the system variables with respect to the small variations in initial conditions \underline{x}_0 and parameters λ (or equivalently \underline{x}_0).

Away from discontinuities, the differential-algebraic system can be written in the form

$$\dot{\underline{x}} = f(\underline{x}, y) \quad (8)$$

$$0 = g(\underline{x}, y) \quad (9)$$

Differentiating (8) and (9) with respect to the initial conditions \underline{x}_0 yields

$$\dot{\underline{x}}_{\underline{x}_0} = \underline{f}_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + \underline{f}_y(t)y_{\underline{x}_0} \quad (10)$$

$$0 = g_{\underline{x}}(t)\underline{x}_{\underline{x}_0} + g_y(t)y_{\underline{x}_0} \quad (11)$$

where $\underline{f}_{\underline{x}}, \underline{f}_y, g_{\underline{x}},$ and g_y are time varying matrices and are calculated along the system trajectories. $\underline{x}_{\underline{x}_0}(t)$ and $y_{\underline{x}_0}(t)$ are the trajectory sensitivities.

Initial conditions for $\underline{x}_{\underline{x}_0}$ are obtained by differentiating (7) with respect to \underline{x}_0 as

$$\underline{x}_{\underline{x}_0}(t_0) = I \quad (12)$$

where I is the identity matrix.

Using (12) and assuming that $g_y(t_0)$ is nonsingular along the trajectories, initial conditions for $y_{\underline{x}_0}$ can be calculated from (11) as

$$y_{\underline{x}_0}(t_0) = -[g_y(t_0)]^{-1} g_{\underline{x}}(t_0) \quad (13)$$

Therefore, the trajectory sensitivities can be obtained by solving (10) and (11) simultaneously with (8) and (9) using (7), (12), and (13) as the initial conditions. At the discontinuity where $s(\underline{x}, y) = 0$, the trajectory sensitivities $\underline{x}_{\underline{x}_0}, y_{\underline{x}_0}$ typically undergo a jump. Computation of these jump conditions is discussed in [8].

B. Stability Constrained Optimal Power Flow Formulation

An OPF problem can be formulated as [6]

$$P_g - P_L - P(V, \theta) = 0 \quad (15)$$

$$Q_g - Q_L - Q(V, \theta) = 0 \quad (16)$$

$$S(V, \theta) - S^M \leq 0 \quad (17)$$

$$V^m \leq V \leq V^M \quad (18)$$

$$P_g^m \leq P_g \leq P_g^M \quad (19)$$

$$Q_g^m \leq Q_g \leq Q_g^M \quad (20)$$

In the above formulas, $C(.)$ is a cost function; (15) and (16) are the active and reactive power balance equations respectively; P_g, Q_g are the vectors of generator active and reactive power output respectively; P_L, Q_L are the vectors of real and reactive load respectively; V and θ are the vectors of bus voltage magnitudes and angles respectively; $P_g^m, P_g^M, Q_g^m, Q_g^M, V^m, V^M$ are the vectors of lower and upper limits of generator real, reactive power outputs, and bus voltage magnitudes respectively; $S(V, \theta)$ is the vector of apparent power flowing across the transmission lines and S^M is the vector of thermal limits of those lines. Note that the variables in this problem are $P_g, Q_g, V,$ and θ .

We use the relative rotor angles to detect the system stability/instability. To check the stability of the system for a credible contingency, relative rotor angles are monitored at each time step during dynamic simulation. The sensitivities are also computed at the same time. Although sensitivity computation requires extensively computational effort, efficient method to compute sensitivities is available by making effective use of the Jacobian which is common to both the system and sensitivity equations [9]. We propose that when the relative rotor angle $\delta_{ij} = \delta_i - \delta_j > \pi$ for a given contingency, the system is considered as unstable. This is an extreme case as pointed out in [6], and one can choose an angle difference less than π depending on the system. Here i and j refer to the most and the least advanced generators respectively. The sensitivities of the rotor angles at this instant are used to compute the amount of power needed to be shifted from the most advanced generator (generator i) to the least advanced one (generator j) according to the formulas

$$\Delta P_{i,j} = \frac{\delta_{ij} - \delta_{ij}^0}{\frac{\partial \delta_{ij}}{\partial P_i}} \bigg|_{\max \delta_{ij} = \pi} \quad (21)$$

$$P_i^{new} = P_i^0 - \Delta P_{i,j} \quad \text{and} \quad P_j^{new} = P_j^0 + \Delta P_{i,j} \quad (22)$$

where P_i^0 , P_j^0 , and δ_{ij}^0 are the base loading of generators i and j , and the relative rotor angle of the two at the solution of the OPF problem stated in (14)-(20); $\frac{\partial \delta_{ij}}{\partial P_i}$ is the sensitivity of relative rotor angle with respect to the output of the i th-generator. P_i is the parameter λ discussed in part A of this section.

After shifting the power from generator i to generator j according to (22), the system is secure for that contingency but it is not an optimal schedule. We propose to improve the optimality by introducing new power constraints as discussed in the next section. The OPF problem with new constraints is then re-solved to obtain the new operating point for the system.

III. SOLUTION METHODOLOGY

The steps to solve the dynamic security constrained OPF are shown in the flow chart of Fig. 1. The steps are as follows

- 1) Perform the OPF to obtain the optimal operating point according to (14)-(20). Set $k = 1$.
- 2) Apply contingency k from the specified list of credible contingencies.
- 3) Perform a dynamic simulation and compute the trajectory sensitivities using models specified by (8)-(11), and monitor the maximum of relative rotor angles at each time step.
- 4) If $\max(\delta_{ij}) < \pi$ for entire simulation interval t_f

If not at the end of the contingency list, set $k = k+1$, and go to step 2.
Else, stop.

Else, go to step 5.

- 5) Calculate the power needed to be shifted from the i th-generator to the j th-generator, P_i^{new} , and P_j^{new} according to (21) and (22).

- 6) If $P_i^{new} \geq P_i^m$, then set $P_i^M = P_i^{new}$, else set $P_i^M = P_i^m$. If $P_j^{new} \leq P_j^M$, then set $P_j^m = P_j^{new}$, else set $P_j^m = P_j^M$.

- 7) Go back to step 1.

Note that in step 6 if one generator hits its output limit and the other does not as a result of the shift, there will be a mismatch between generations and loads after the shift. This mismatch will be taken care of by the result of the OPF in the rescheduling. Also, in step 6 if the least advanced generator j is at its maximum limit, the shift will be carried out from generator i to the next-to-least advanced generator instead. Keep in mind that (21) is still computed at $\max \delta_{ij} = \pi$. If a solution obtained by applying the proposed algorithm exists, this solution is the globally secure and sub-optimal dispatch over the specified set of contingencies. Of course, there is no guarantee about the existence of the globally secure solution. There may be situations where this solution does not exist. If this is the case, the system loads need to be studied further so

that the system operators will have appropriate actions when contingencies were to occur.

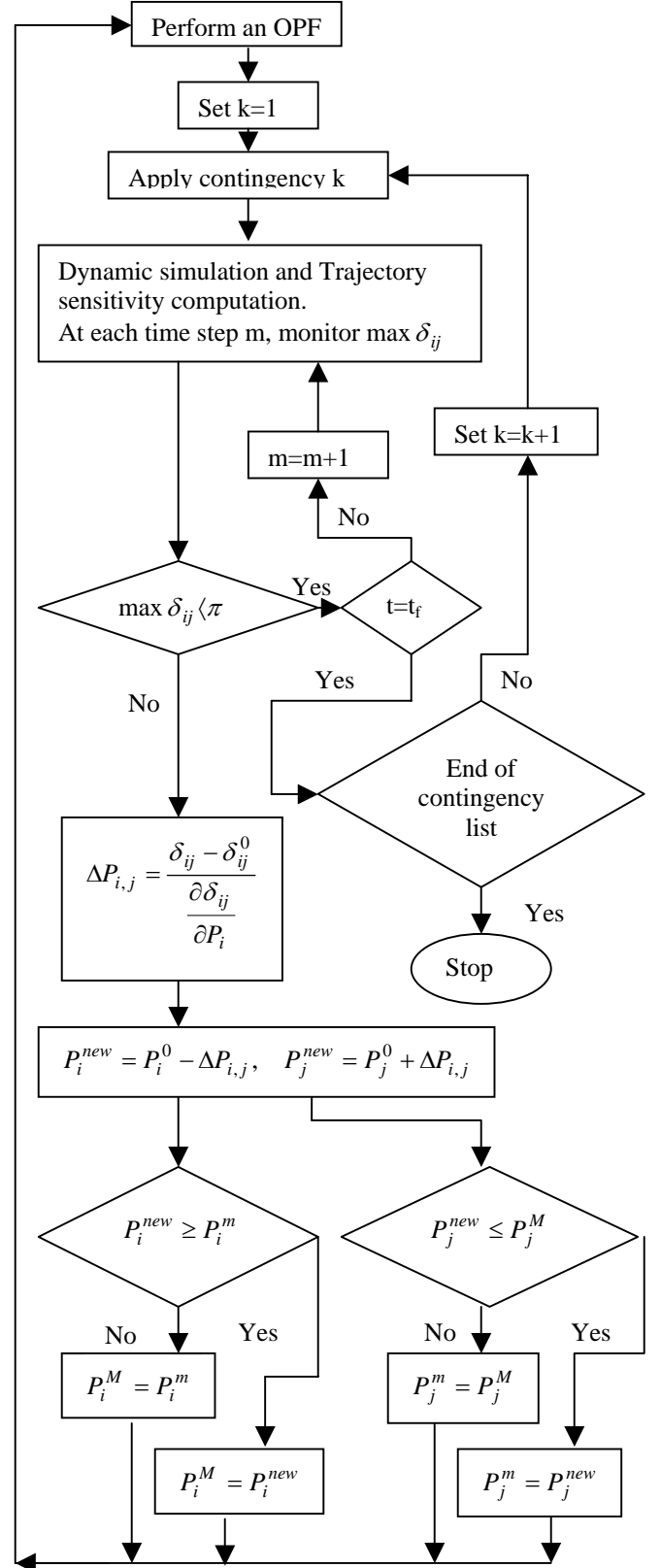


Fig. 1. Flow chart for dynamic security constrained OPF.

IV. NUMERICAL EXAMPLES

The 3-machine, 9-bus system [10], and the 10-machine, 39-bus system [11] are used as numerical examples to illustrate the algorithm stated in section 3. The two-axis model is used for synchronous machines with the IEEE type-1 exciter and constant mechanical input. Loads are modeled as constant-impedance type. Results of the OPF are obtained by using the MATPOWER package [12].

A. The 3-Machine, 9-Bus System

A single line diagram of this system is shown in Fig. 11 of the appendix. The OPF is performed for the base case and the optimal schedule is shown in Table I along with the cost functions for generators and their ratings.

TABLE I
GENERATOR DATA AND THE OPTIMAL SCHEDULE FOR THE BASE CASE

Generators	Rating (MW)	Cost Function (\$/h)	Optimal Loading (MVA)	Total Cost (\$/h)
1	200	$0.0060P^2 + 2.0P + 140$	$106.19 + j24.26$	1132.59
2	150	$0.0075P^2 + 1.5P + 120$	$112.96 + j0.37$	
3	100	$0.0070P^2 + 1.8P + 80$	$99.20 - j11.62$	

1) Fault at bus 7:

The fault is simulated at bus 7 and cleared by tripping line 7-5 at $t_{cl} = 0.35$ s, which is greater than the critical clearing time. The critical generators are G2 and G3, but δ_{21} reaches the threshold π earlier than δ_{31} does.

Case A: Shifting output power from generator 2 to generator 1. Using (21) the amount of power needed to be shifted is found to be $\Delta P_{2,1} = 64.02$ MW. Applying the algorithm of section III, the new schedule for the generators is G1: $170.20 + j27.31$ MVA, G2: $48.94 - j0.08$ MVA, and G3: $98.74 - j9.86$ MVA. The total cost for this case is \$1191.56. The system is dynamically stable in this loading condition for the same fault. The OPF solution is sub-optimal but survives the contingency dynamically if it were to occur. The relative rotor angles after the shift are shown in Fig. 2.

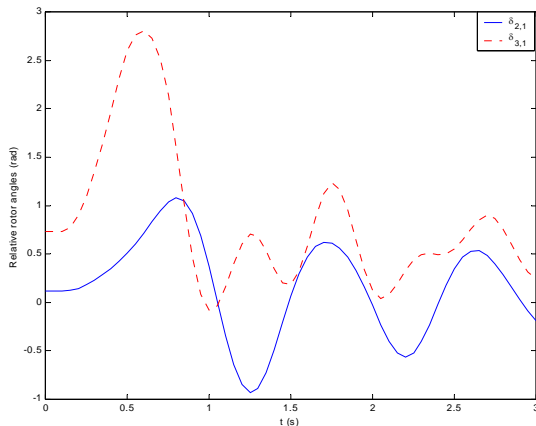


Fig. 2. Relative rotor angles for case A.

2) Fault at bus 9:

The fault is simulated at bus 9 and cleared by tripping line 9-6 at $t_{cl} = 0.30$ s, which is greater than the critical clearing time. The critical generator is G3 and the least advanced one is G1.

Case B: Shifting output power from generator 3 to generator 1. The amount to shift is $\Delta P_{3,1} = 58.19$ MW obtained by using (21). The new loading in this case is G1: $164.38 + j25.96$, G2: $112.44 + j0.95$, and G3: $41.00 - j10.64$, and the system is dynamically stable. The total cost is \$1179.95. With this loading condition, the relative rotor angles are shown in Fig. 3.

Case AB: Now the set of contingencies, which comprises contingencies as in cases A and B, is considered. The globally secure and sub-optimal schedule obtained by applying the algorithm of section 3 is G1: $200 + j30.70$ MVA, G2: $74.73 + j0.77$ MVA, and G3: $43.39 - j9.04$ MVA. The cost in this case is \$1225.26. With this loading condition, the system will be dynamically stable for either contingency specified in cases A and B as shown in Figs. 4 and 5.

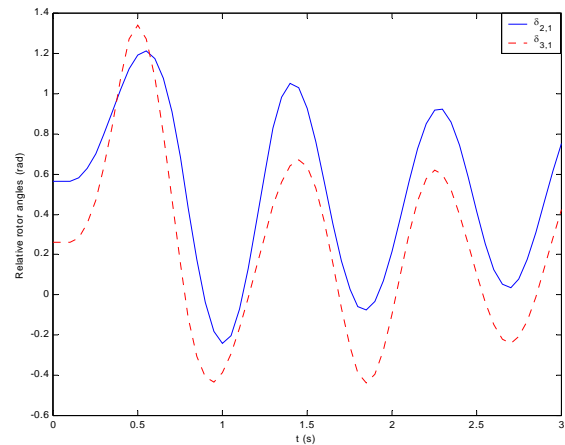


Fig. 3. Relative rotor angles for case B.

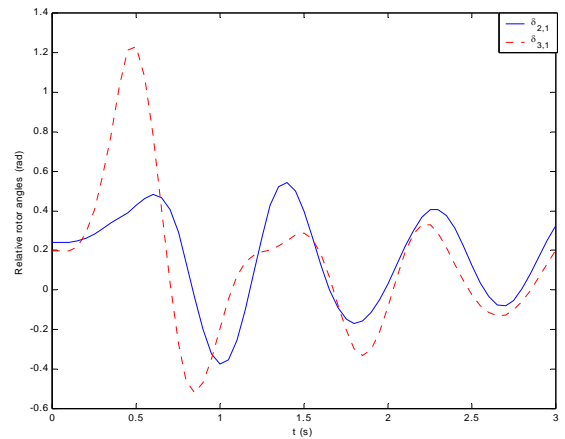


Fig. 4. Relative rotor angles for case AB, fault at bus 9.

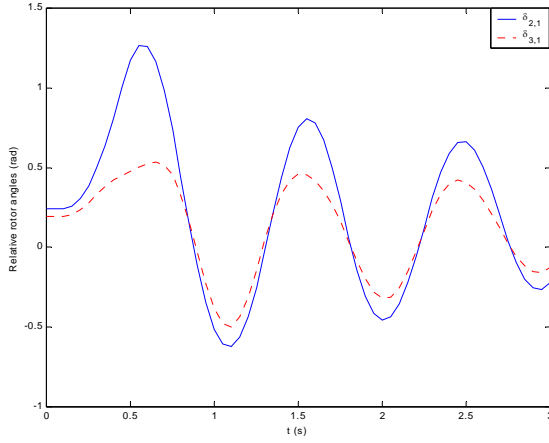


Fig. 5. Relative rotor angles for case AB, fault at bus 7.

The optimal loadings for the base case, the constrained cases A, B, and AB, and the total cost in each case are summarized in Table II. Note that the cost for the dynamically constrained cases A, B, and AB is always higher than for the base case as expected. The highest cost is in case AB where the globally secure sub-optimal schedule is obtained.

TABLE II
OPTIMAL SCHEDULES FOR THE BASE CASE AND THE 3 CASES A, B, AND AB

Case	Optimal schedule			Cost (\$/h)
	G1 (MVA)	G2 (MVA)	G3 (MVA)	
Base	106.19+j24.26	112.96+j0.37	99.20-j11.62	1132.59
A	170.20+j27.31	48.94-j0.08	98.74-j9.86	1191.56
B	164.38+j25.96	112.44+j0.95	41.00-j10.64	1179.95
AB	200.00+j30.70	74.73+j0.77	43.39-j9.04	1225.26

B. The 10-Machine, 39 Bus System

A single line diagram of this system is shown in Fig. 12 of the appendix. The cost functions and the rating of generators are given in Table III. The OPF is performed to obtain the optimal schedule for the base case which is also shown in Table III.

TABLE III
GENERATOR DATA AND THE OPTIMAL SCHEDULE FOR THE BASE CASE

Generators	Rating (MW)	Cost Function (\$/h)	Base case Optimal Loading (MVA)
1	350	0.0193P ² +6.9P	243.63+j167.39
2	650	0.0111P ² +3.7P	567.90+j151.88
3	800	0.0104P ² +2.8P	642.49+j147.64
4	750	0.0088P ² +4.7P	628.97+j43.16
5	650	0.0128P ² +2.8P	507.69+j136.47
6	750	0.0094P ² +3.7P	650.80+j231.72
7	750	0.0099P ² +4.8P	558.44+j197.46
8	700	0.0113P ² +3.6P	534.84+j15.58
9	900	0.0071P ² +3.7P	827.24+j46.70
10	1200	0.0064P ² +3.9P	981.84+j218.29
Total Cost (\$/h)			60,992.88

1) Fault at bus 17:

With this optimal operating point, a fault is simulated at bus 17 and cleared by tripping line 17-18 at $t_{cl} = 0.20$ s, which is greater than the critical clearing time. The advanced generators are 2-9. However, the relative rotor angle $\delta_{5,10}$ is detected to cross the threshold π first.

Case D: Power is shifted from generator 5 to generator 10. Applying (21), power needed to be shifted from generator 5 to generator 10 is $\Delta P_{5,10} = 264.11$ MW. The algorithm proposed in section III is used to obtain the new loading of generators after the shift. With this new loading, the system is found to be dynamically stable as shown in Fig. 6. The cost in this case is 62,261.28 \$/h. The new schedule and the total cost for this case are shown in Table IV. As expected, the cost for the dynamically constrained case D is higher than for the base case.

2) Fault at bus 4:

A fault is simulated at bus 4 for the base case and is cleared by tripping line 4-5 at $t_{cl} = 0.25$ s, which is greater than the critical clearing time. The most advanced generators are 2 and 3, and they cross the threshold π at the same time. The least advanced generator in this case is 10.

Case E: Power is shifted from generator 2 to generator 10. Applying the algorithm in section III, the amount to be shifted is $\Delta P_{2,10} = 215.48$ MW. After the shift, the system is stable as shown in Fig. 7, and the cost is 61,826.53 \$/h.

Case F: Power is shifted from generator 3 to generator 10. Using (21), power needed to be shifted is $\Delta P_{3,10} = 273.87$ MW. The system is stable after the shift, and the cost is 62,113.36 \$/h. Relative rotor angles for this case are shown in Fig. 8.

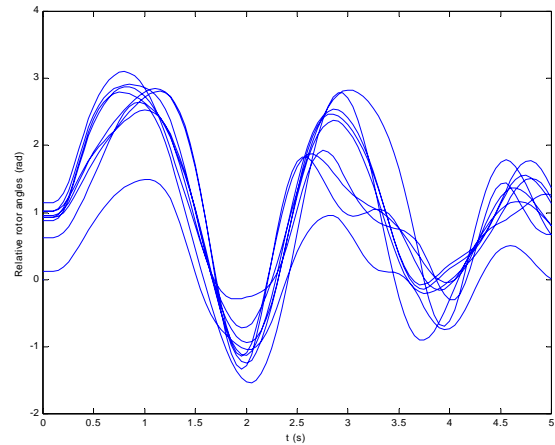


Fig. 6. Relative rotor angles for case D.

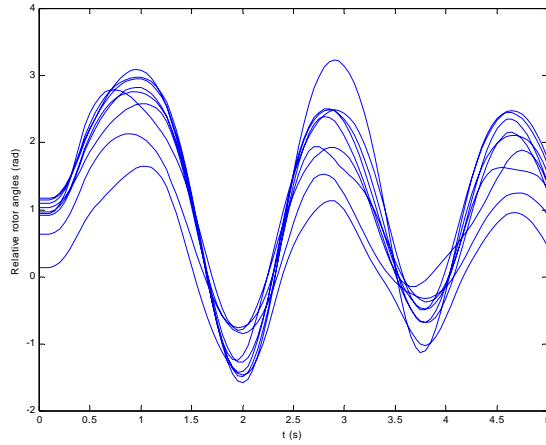


Fig. 7. Relative rotor angles for case E.

Case DE: The set of contingencies, which comprises contingencies specified in case D and E is considered in this case. By applying the proposed algorithm, the globally secure and sub-optimal schedule is found. With this optimal dispatch the system is dynamically stable for either contingency in the set. This is verified by simulation as shown in Figs. 9 and 10. The cost in this case is \$62,559.60.

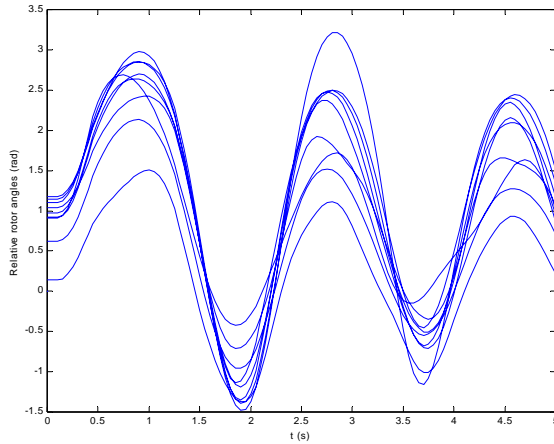


Fig. 8. Relative rotor angles for case F.

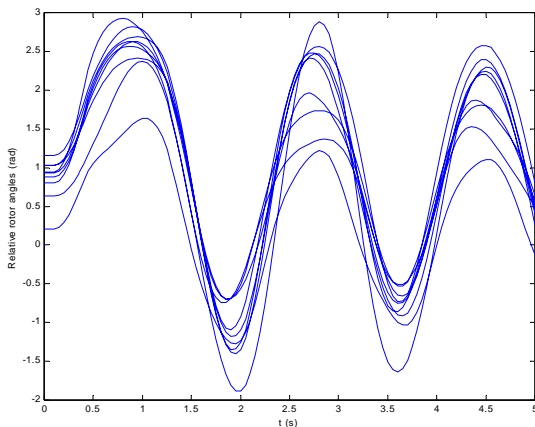


Fig. 9. Relative rotor angles for case DE, fault at bus 4.

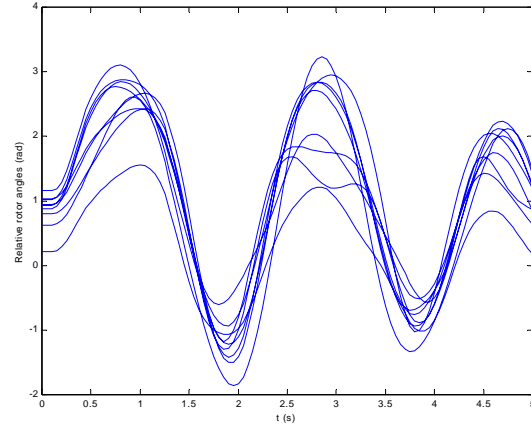


Fig. 10. Relative rotor angles for case DE, fault at bus 17.

The new schedules and the total cost for cases E and F are shown in Table IV. For the fault at bus 4, the system is stable after the power shift of the computed amount from either generator 2 or generator 3 as shown in cases E and F. However, the cost for case F is higher than for case E. In this case, the choice of generator to shift from is based on the operator's knowledge of the system. Table V shows the globally secure and sub-optimal schedule and the total cost for case DE. As expected, the cost for this globally secure case is highest among the dynamic constrained cases. The cost for dynamic constrained cases E and F is also higher than the cost for the base case.

TABLE IV
THE NEW SCHEDULES FOR THE CASES D, E, AND F

Generators	Case D Optimal Loading (MVA)	Case E Optimal Loading (MVA)	Case F Optimal Loading (MVA)
1	243.61+j160.90	242.40+j164.45	246.51+j165.69
2	568.34+j147.42	352.42+j118.06	567.78+j142.19
3	643.81+j143.68	643.93+j142.01	368.62+j104.56
4	644.57+j40.10	628.93+j42.51	637.04+j43.90
5	243.58+j126.82	507.66+j136.17	513.21+j137.06
6	658.27+j228.45	650.76+j231.06	658.67+j233.55
7	565.44+j196.21	558.41+j197.09	565.80+j199.09
8	538.17+j19.41	534.40+j17.28	540.42+j17.47
9	833.19+j48.99	826.84+j47.23	835.77+j49.81
10	1200.00+j199.18	1197.32+j201.61	1200.00+j200.55
Cost (\$/h)	62,261.28	61,826.53	62,113.36

TABLE V
THE NEW SCHEDULES FOR THE CASE DE

Generators	Case DE Optimal Loading (MVA)
1	342.02+j165.72
2	469.93+j130.71
3	645.06+j141.27
4	644.54+j40.16
5	243.58+j126.85
6	658.24+j228.51
7	565.41+j196.24
8	537.72+j19.22.33
9	832.78+j50.14
10	1200.00+j200.15
Cost (\$/h)	62,559.60

A technique using trajectory sensitivities to provide a preventive rescheduling scheme in dynamic security constrained power systems taking into account the economic aspect is proposed. It is a model-independent technique, so it can be applied to systems with any detailed modeling level. The numerical results for the test systems have shown that the technique indeed corrects the dynamically unstable or marginally stable systems to the stable systems for a set of contingencies. The good candidates for rescheduling are the most and the least advanced generators. When the generation is shifted, the OPF is performed with the new limits on the generation resulting in a sub-optimal solution. For a marginally stable system, the algorithm will make it stable but at a higher operating cost. However, more research is needed to make the algorithm robust. Note that if market considerations are taken into account, the choice of less advanced generator may be different. This is a matter for future research [13].

VI. APPENDIX

The single line diagrams for the 3-machine, 9-bus system and the 10-machine, 39-bus system are shown in Fig. 11 and Fig. 12 respectively.

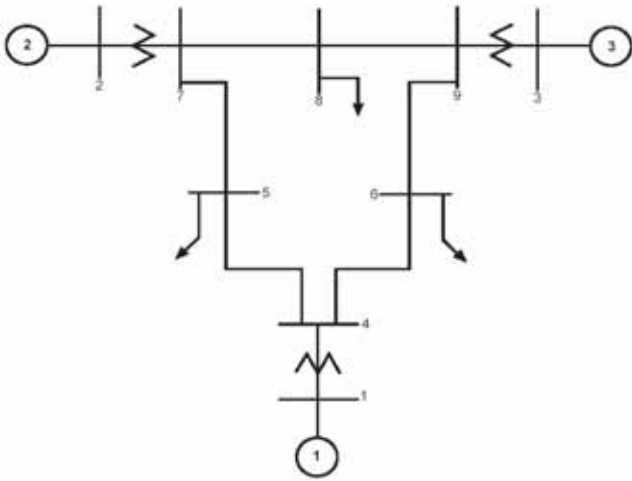


Fig. 11. The 3-machine, 9-bus system.

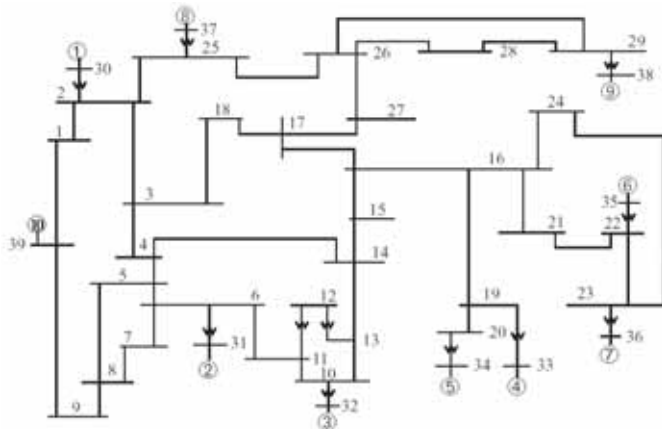


Fig. 12. The 10-machine, 39-bus system.

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VIII. BIOGRAPHIES

Tony B. Nguyen received his BS and MS degrees in Electrical Engineering at the University of Illinois at Urbana-Champaign in 1998 and 1999 respectively. He is currently a PhD student at the University of Illinois at Urbana-Champaign.

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