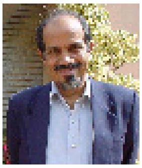
The Challenge of Fluid Flow

2. What One Can and Cannot Do

Roddam Narasimha



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The Navier–Stokes Equations

In Part 1 we saw how diverse flow phenomena can be, even when we are looking at such common fluids as air and water (more complex fluids – from paint to slurry – are another big story by themselves). We saw how in many of those flows both order and disorder are simultaneously present. We have no theories that can handle these mixed order-disorder phenomena well.

But we do believe that the basic laws governing fluid motion are known. They are consequences of Newton's laws of motion and his concept of viscosity. The laws are best written down in the formalism of a continuous field, along the lines that Leonhard Euler introduced in 1755 – exactly 250 years ago. For an incompressible viscous fluid of density ρ and viscosity μ, the flow is governed by the equations of conservation of mass and momentum,

$$\operatorname{div} \boldsymbol{u} = 0$$

and $$\rho \frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} \equiv \rho \left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \,.\, \operatorname{grad})\, \boldsymbol{u} \right) = -\operatorname{grad} p + \mu\, \nabla^2 \boldsymbol{u} + \boldsymbol{F},$$

where $\boldsymbol{u} = \boldsymbol{u}(\boldsymbol{x}, t)$ is the (vector) velocity field, dependent on the position vector $\boldsymbol{x}$ and time t, $p = p(\boldsymbol{x}, t)$ is the pressure field, and $\boldsymbol{F} = \boldsymbol{F}(\boldsymbol{x}, t)$ is a body force (per unit volume of fluid).

Note the presence in the equations of the quadratically nonlinear term $\boldsymbol{u}.\operatorname{grad} \boldsymbol{u}$. This nonlinear term is the crux of the problems of fluid dynamics; if it had not been there the subject would legitimately have become 'classical' by now. It is only in the 20th century that fluid dynamicists began slowly learning how to handle the non-linearity, but the learning process is far from over.

Box 1. The Navier-Stokes Equations

We have written down these equations for an incompressible viscous fluid. They assume that the fluid is continuous (no voids, for example), and that the state of internal stress in the fluid depends linearly on the local rate of strain that the fluid experiences during its motion. The principles of conservation of mass and momentum are then sufficient to obtain the partial differential equations that govern the motion: one expressing continuity or conservation of mass, and three scalar equations (constituting one vector equation) each of which expresses conservation of momentum in an independent direction.

Because fluid velocity and pressure depend in general on both space and time, the governing equations are partial differential equations. The term that is responsible for the difficulty of solving these equations is what may be called the advective acceleration, written as $(\boldsymbol{u}.\text{grad})\ \boldsymbol{u}$ in the text. This term represents the acceleration experienced by a fluid particle, originally at a given position and time, as it moves a short interval of time later to a neighbouring position where the fluid velocity is slightly different. Because the increment in velocity and the distance travelled by the particle both depend on the velocity field, this advective acceleration is nonlinear in the fluid velocity. This nonlinearity has profound consequences, and results in all the complexities that we see in the motion of even such common fluids as air and water.

Prandtl's Idea

The first shot here was fired by Ludwig Prandtl almost exactly a hundred years ago, so I should say a few words about it in this centenary year. In 1904 this remarkable engineer read a paper at the 3rd International Congress of Mathematics held at Heidelberg, and showed how, in the limit of large Reynolds numbers, the equations could be made manageable without sacrificing nonlinearity. His approach can be summarized in the recipe, 'Divide, conquer and unify'. The first big step was to realize that the flow could be divided into different *regions*: let us call them, in the simplest cases, inner and outer (assuming there are only two such regions). The corresponding inner and outer *equations*, each the limiting form of the governing equation in its respective region, are separately conquered, i.e. solved, yielding inner and outer *solutions*. By cleverly specifying their boundary conditions these separate solutions are then 'unified' into a composite solution, which was later shown to be asymptotic to the exact solution *everywhere* in the large Reynolds number limit.

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Prandtl posed the problem for the flow past an aligned flat plate, By example Prandtl brought to an end the war between ancient hydraulics and 19th century hydrodynamics, which till then had scorned each other.

which was trivial in 19th century Eulerian (*inviscid*) hydrodynamics ($\mu = 0$), but was fundamental to the new fluid dynamics he was creating. By an ingenious mixture of extensive visual observation (in a $ 40 water channel), mathematical approximation and numerical calculation, he produced an answer for the drag or resistance of the plate that in principle solved the classical d'Alembert paradox (of no resistance in a non-viscous fluid). In the process of solving this special problem, however, Prandtl did many other things. He showed how, at high Re, fluid flows tend to fold or squash into layers; he invented what later became the more formal method of matched asymptotic expansions for handling singular perturbation problems; by example he brought to an end the war between ancient hydraulics and 19th century hydrodynamics, which till then had scorned each other (hydraulics was dismissed as a science of variable constants, hydrodynamics as the mathematics of dry water); and, in the process, he founded modern fluid dynamics, giving it the tools by which many of the earlier 'paradoxes' that had plagued the subject could be resolved one by one.

Some Things We Know

Thanks to modern fluid dynamics we know enough to build aircraft traveling at Mach 3 or carrying nearly a thousand passengers, make rockets that can shoot us to the moon and bring us back, predict weather a few days in advance, manage flows ...

Thanks to that modern fluid dynamics we now know a lot about many fluid flows – enough to build aircraft traveling at Mach 3 or carrying nearly a thousand passengers (the A380 can take upto 873 people including crew), to make rockets that can shoot us to the moon and bring us back, to predict weather a few days in advance, to manage flows so as to enhance or diminish heat transfer between a solid surface and a fluid or to promote or suppress mixing between two fluids. But we still do not *understand* the central problem of turbulent flows, which remains fundamentally unsolved. (As the word appears repeatedly in this article, I should explain what I mean by *understanding* a phenomenon. In the first place, it implies that there is a quantitative explanation that is derived from accepted first principles – e.g. the Navier–Stokes equations – if necessary by exploiting reasonable approximations using well-tested methods, but not appealing to additional experimental data.

Related to this is a second kind of explanation, consistent with the first but possibly qualitative, where the phenomenon in question may be shown to emerge from others which are already 'understood'.) If the physics underlying the phenomenon is utterly new, it is natural that understanding in the above sense will emerge only slowly. To illustrate the bizarre situation in fluid dynamics, consider the common plumbing problem of estimating the pressure loss suffered by water flowing through a pipe. Man has been pushing water through pipes and channels for thousands of years; our ancestors did that already very well in the Indus valley civilization of some 4000 years ago. Thousands of engineers make confident and successful designs using data codified into diagrams of the type shown in *Figure* 1. But it is only the green line in the diagram, representing laminar flow, that is 'understood'. The rest is known, by a mix of testing and ingenious heuristic argument about turbulent flow, but not really *understood*. To highlight this extraordinary situation, let me note that there is a new analysis of turbulent pipe flow that claims that some of the results of the kind shown in *Figure* 1 can be wrong by as much as 60% at extremely high Reynolds numbers; this new analysis is not yet either confirmed or refuted.

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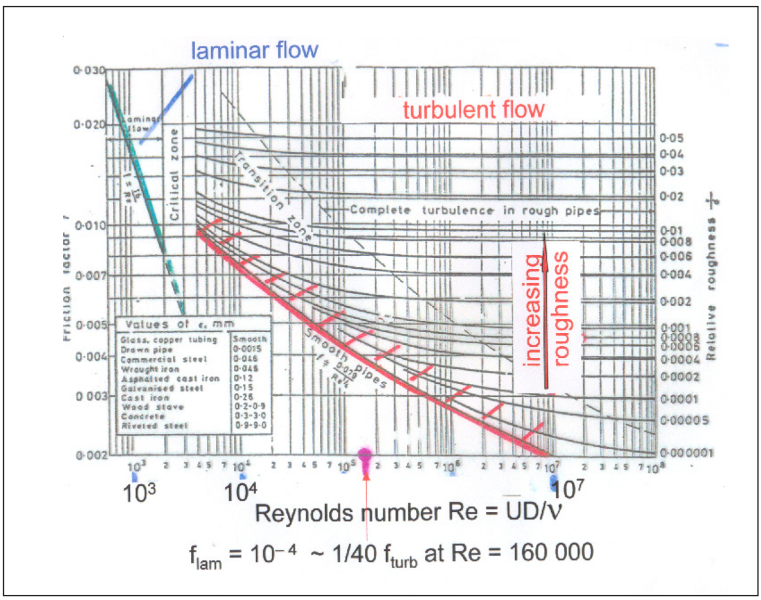

Figure 1. Chart giving pressure-drop data in rough pipes, of the kind that engineers use to design piping systems. The only data on this chart that can be derived from first principles (like the Navier–Stokes equations) is the green laminar line on top left. All the rest have to appeal to experimental data in some way, supplemented by some very clever scaling arguments.

Richard Feynman called turbulent flow "the greatest puzzle of classical physics".

Deterministic Chaos

This has been the enduring mystery of fluid flows: its governing laws are known, nevertheless even everyday phenomena, seen by our eyes all the time, cannot always be explained solely from those laws. Richard Feynman called turbulent flow "the greatest puzzle of classical physics". But the adjective 'classical' there should be carefully interpreted, for 'classical' is often equated with 'understood', i.e. intellectually dead. This of course is far from being the case with regard to turbulent flows. I think it would be more accurate to say that Newtonian mechanics has turned out to be full of deep, unresolved, sometimes even unsuspected mysteries (in spite of having been dubbed as 'classical' with the advent of relativity and quantum mechanics). One such phenomenon, unsuspected till some forty years ago, is deterministic chaos. This forces together two concepts – of necessity and chance, of law and accident, or (in Upanishadic terms) of *niyati* and *yadrccha* – concepts that had earlier been thought of as two competing, mutually incompatible views of the nature of the universe. The discovery that paradigmatically deterministic Newtonian systems can behave in ways that appear random has had such a profound effect on our thinking that Sir James Lighthill, occupant of the same prestigious Lucasian Chair that Newton had held some 300 years earlier in Cambridge, felt compelled to say in 1986:

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> We [i.e. the community of scientists pursuing 'classical' mechanics] collectively wish to apologize for having misled the general educated public by spreading ideas about the determinism of systems satisfying Newton's laws of motion that, after 1960, were to be proved incorrect.

And chaos has now been detected in such exemplars of the alleged 'clockwork' of the universe as the planetary system, the pendulum and the elastic string. Einstein famously said that he did not believe in a God who played dice, but would he have believed in a Newtonian God who played deterministic nonlinear games whose outcomes would be effectively indistinguishable from the results of playing dice?

Incidentally Newtonian chaos is not unrelated to the turbulence of fluid flow – chaos is generic turbulence, so to speak, that may be encountered in non-fluid dynamical systems as well. Indeed, a key advance in the emergence of the concept of chaos was the study of a highly idealized form of convective weather – going one or two nonlinear steps beyond the saree-border pattern of *Figure* 3, Part 1. That study, undertaken in the early 1960s by the American meteorologist E N Lorenz, showed how convection can become erratic, explained why weather is unpredictable beyond a certain time horizon, showed the relation of these properties to those of some simple nonlinear maps, and visually displayed for the first time the mathematical object that later came to be called a strange attractor. This theory of chaos has certainly solved the philosophical problem of how turbulence can emerge out of the Navier–Stokes equations. But it has unfortunately not otherwise been of great help in understanding or predicting turbulent flows for the simple reason that the number of degrees of freedom in a flow diverges as the Reynolds number increases.

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The Fundamental Problem

I believe it was John von Neumann who came closest to seeing the true nature of the fundamental problem of fluid dynamics. He first of all realized that:

> From the point of view of theoretical physics turbulence is the first clear-cut instance calling for a new form of statistical mechanics.

He then went on to say:

> The impact of an adequate theory of turbulence on certain very important parts of pure mathematics may be even greater [than on fluid dynamics].

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– J von Neumann

How can the pressure on a body whose cross-section is a platonically perfect circle vary with Reynolds number in such a non-simple way?

Von Neumann thought that there was some hope to "break the deadlock by extensive but well-planned computational efforts"; and this was one of the reasons that he got so deeply involved in the development of computer technology. But computing solutions of the Navier–Stokes equations is more like doing experiments (– only they are numerical instead of physical), and will not automatically provide understanding. In any case, even on the most powerful computers available in the world today, we cannot reach Reynolds numbers higher than of order 10^4. To see that we still have a long way to go, look at *Figure* 2, which is a collection of data compiled by Anatol Roshko on the pressure at the back (more precisely rear stagnation point) of the same kind of cylinder that sheds Kármán vortices at lower Reynolds numbers. First of all note that the data (collected from many different sources) show surprisingly small scatter. Now how can the pressure on a body whose cross-section is a platonically perfect circle vary with Reynolds number in such a non-simple way? (Could that crazy variation be a set of signatures of the many transitions that keep occurring as the flow folds into complex layers?) Who would dare to guess (based on that data) what happens to the pressure in the limit as the Reynolds number Re tends to infinity? Or, to put it in equivalent terms, how is it that even at $\mathrm{Re}^{-1} = 10^{-7}$ we clearly cannot be sure we are

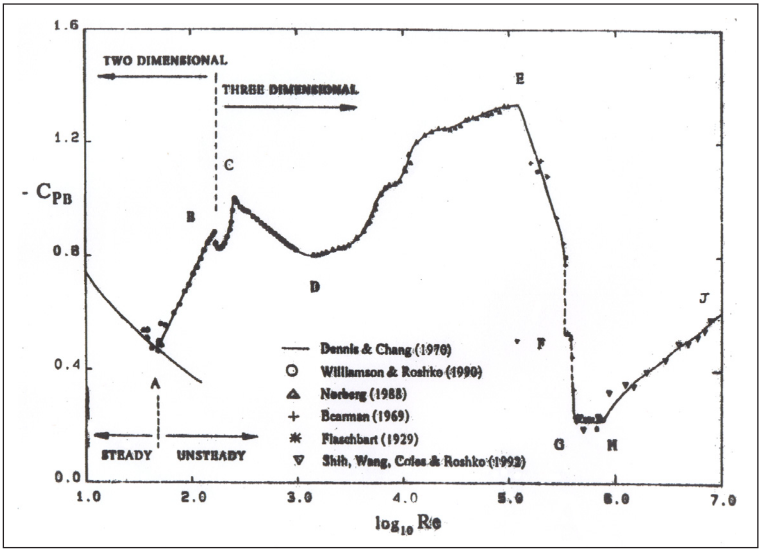

Figure 2. The remarkable variation of the pressure at the rear stagnation point ('back') of a circular cylinder, as the flow Reynolds number varies over six orders of magnitude. The kinky nature of the variation probably indicates a complex sequence of flow transitions, and does not give unambiguous answer about the nature, or even the existence, of a limiting state as Reynolds number tends to infinity – suggesting a value of 10^7 may not be large enough for this parameter.

close to the limit $\mathrm{Re}^{-1} \to 0$? (There must be a discontinuity at $\mathrm{R}^{-1} = 0$, showing the limit is singular.) And, to cap it all, even on the most powerful computers in the world today, we are not even half-way past the abscissa in the diagram. So you can see how far we are from final solutions.

That the problem is basically mathematical is at last being more widely recognized, for two problems on the Navier–Stokes equations are among the seven million-dollar prize problems posed by the Clay Foundation (*Box* 2) – along with the Riemann hypothesis and the Poincaré conjecture. That I feel is the right company for turbulence, and shows why fluid dynamics continues to be such an enduring challenge.

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What Can be Done

While waiting for mathematical paradise, there is a great deal that fluid dynamicists can do and have done about these

Box 2. The Clay Institute's Million-dollar Prize Problems

The Clay Mathematics Institute (Cambridge, MA) is a private foundation financed by the businessman Landon T Clay. The Institute has offered a million-dollar prize for solutions of each of 7 'Millennium Prize' problems. These include such celebrated and long-standing open questions as the Riemann hypothesis and the Poincaré conjecture. One of the seven problems is concerned with 'Navier–Stokes existence and smoothness'.

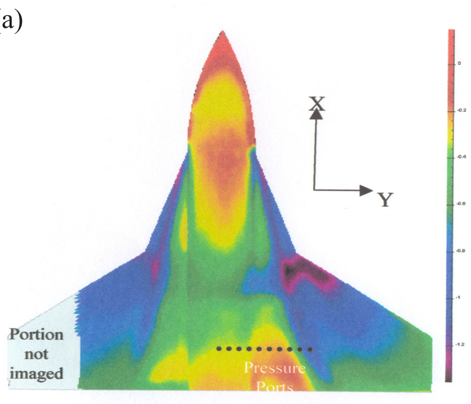
The basic question underlying the Navier–Stokes problem is to show whether smooth, physically reasonable solutions of the three-dimensional Navier–Stokes equations exist or not. (The two-dimensional problem was solved by the Soviet mathematician Ladyzhenskaya (1922-2004) in the 1960s.)

As solution of the problem, proof is demanded of one of four statements. The flavour of these statements is conveyed by the following question. If the initial velocity field is sufficiently smooth everywhere, and the forcing function $\boldsymbol{F}(\boldsymbol{x}, t)$ is also similarly sufficiently smooth everywhere and at all times, does the Navier–Stokes solution for the velocity field remain smooth with finite energy, or can it blow up? (For a statement of the problem with more precise mathematical qualifications, see the Clay Mathematical Institute website: http://en.wikipedia.org/wiki/ Millennium_Prize_Problems.) Even more simply stated, do smooth initial conditions always yield smooth solutions for all times or not? For the Euler equations (valid for an inviscid fluid, $\nu = 0$), the evidence seems to suggest that blow-up is possible. Answers to such questions can help us to understand the way that viscous fluid flows are drawn out into thin sheets or filaments of vorticity, as in *Figure* c in *Box* 4.

Box 3. What We Can Measure

Here are some typical examples of what one can measure by recent techniques. The current trend in fluid flow measurements is towards development of tools that measure whole pressure, velocity and temperature fields, rather than making point measurements.

(a)



(b)

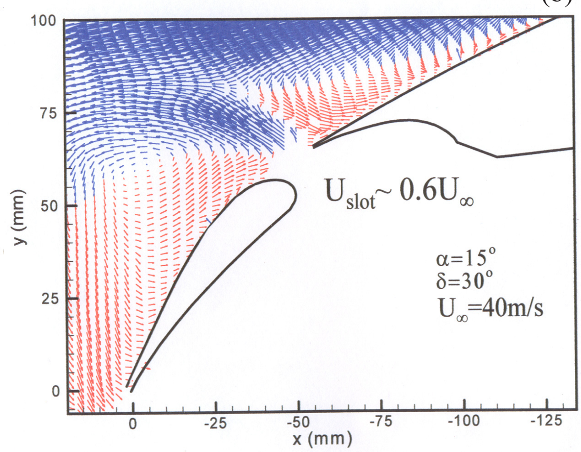


Figure a (left). A recently developed method of measuring pressure distribution on models in wind tunnels is the use of pressure-sensitive luminescent coatings ('paints'). This picture shows pressure measurements made on an aircraft model using a pressure-sensitive paint developed at NAL.

Figure b (right). Whole-field velocity measurements using particle image velocimetry near the trailing edge of a flapped aircraft wing.

problems. Engineers of course cannot always wait for understanding (as the great electrical engineer Oliver Heaviside pointed out, we do not stop eating just because we do not understand digestion). But engineers would love a good theory – it would cut development costs dramatically (I can hear some of you saying ugh!). In the absence of a theory, one engineering way is to treat each flow on its merits, carry out extensive tests (see *Box* 3) and make handbooks or catalogues with diagrams like *Figure* 1 (using physical or computational experiments) – or their more modern computerised equivalents. But I think a great deal more can be done.

While waiting for mathematical paradise, there is a great deal that fluid dynamicists can do and have done.

Some Basic Ideas

First of all, I believe there are two keys to *appreciating* fluid flows

Box 4. What We can Compute

Here are some typical examples of the kind of computations that can be made today. Each of them is thought to be an exact numerical solution of the governing equations and initial-boundary conditions – that is, without the aid of any additional empirical or semi-empirical assumptions, hypotheses, models etc.

a

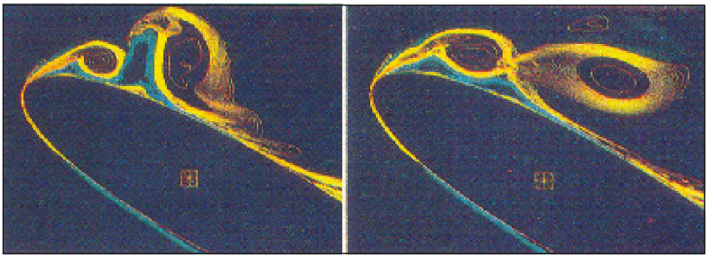


Figure a. Solution of Navier-Stokes equations for an aerofoil at high angle of attach (~29 deg.). Instantaneous contours of vorticy. Reynolds number = 45 000 based on chord.

b

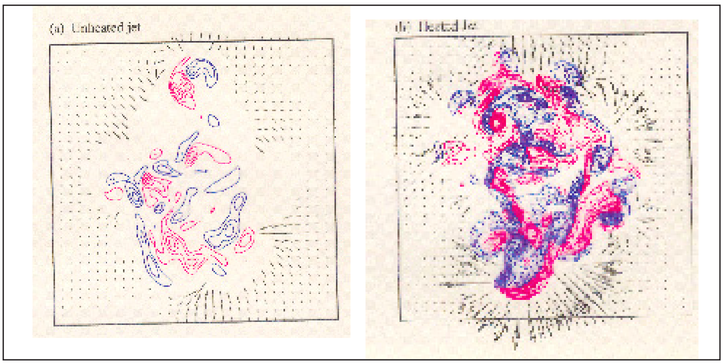
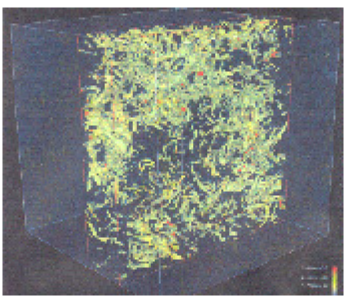


Figure b. Diametral cross-section of a turbulent jet, like that shown in Figure 9, Part 1. Solutions of the full Navier–Stokes equations, Re = 1600 based on nozzle velocity and diameter. Colour coding indicates values of the vorticity component along the direction of jet flow, i.e. perpendicular to the plane of the paper. Arrows show velocity vectors in the plane of the paper just outside the core of the jet. (Left) Conventional jet. (Right) Jet with volumetric heating, simulating the release of latent heat of condensation of water vapour in a cloud. Notice the dramatic changes in both velocity and vorticity fields, explaining in part why flight through clouds in an aircraft can be so bumpy.

c



d

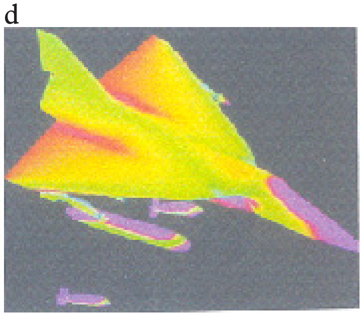


Figure c (left). A solution of the Navier–Stokes equations for decaying turbulence stirred up in a box of size 2048^3. Reynolds number = 270 based on r.m.s. fluctuating velocity and the Taylor microscale. Picture shows contours of constant vorticity. The solution (with others like it) takes several weeks on one of the most powerful computers in the world, the Advanced Simulation and Computing Q machine at Los Alamos.

Figure d (right). A solution of the Euler equations, colour-coded to indicate pressure distribution on the Light Combat Aircraft.

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– i.e. to getting an intuitive feel for their structure (which is still far short of *predicting* them from first principles). The two keys, I propose, are

INSTABILITY, NONLINEARITY

As we have already seen, fluid flows tend naturally to instability under most conditions barring the mildest (very low Reynolds or Rayleigh numbers, for example). The nature of the instability depends on the particular flow type (jet, boundary layer, convection, rotation etc.), and can vary widely from flow to flow. That is, instability is general, but its character is flow-specific. This tendency to crumple into instability, enabling small disturbances to grow, can be the first step leading to chaos and turbulence. It is no wonder therefore that instability has been seen as a central issue by some of the biggest names in physics and fluid dynamics: Sommerfeld, Rayleigh, Heisenberg, Chandrasekhar – and most relentlessly and successfully by Prandtl and his pupils and by G I Taylor. But the final stages of transition, ending up in breakdown, are essentially nonlinear, and not yet understood.

That leads to nonlinearity, which has several effects. The first, strangely, is to fix order. By this I mean that a linear instability mode which has started growing can have its amplitude limited by nonlinear saturation, producing the *stable* order of which we saw several illustrations earlier (e.g. convective rolls, Kármán vortices), without necessarily changing the *mode* much.

The second effect of nonlinearity is to fold the flow into layers, as first analysed by Prandtl in the simple case of flow past a flat plate. But as thin layers fold into even thinner ones (see e.g. *Figure* 3), there is a cascade in scales. And there can be layers within layers! Of course the folding and squashing can result not only in sheets, but scrolls, filaments, shocks and various other types of singularities.

Thirdly nonlinearity can generate chaos – as we have already described.

And, by a combination of all these mechanisms nonlinearity can produce strange mixtures of chaos and order over a wide range of scales – the *vicitra-vibhava* of the Vāsiṣṭha.

To these two pillars of intelligent thinking about fluid flows, we should perhaps add a third,

SCALING AND MATCHING

which is a way of reasoning about length and time *scales* in the flow. This is important for several reasons. First of all fluid flows have a wide range of scales (because of the layer-making property discussed above). Secondly scaling arguments help us to separate the more nearly universal features of a flow from those that are less so. Thirdly a successful scaling argument helps us to condense vast quantities of experimental data into manageable *patterns*. For example, if the scales characterizing wake flows are known, then all wakes become *instances* of a universal wake, i.e. all of them collapse into universal functions and numbers in appropriately scaled variables. (Of course these functions and numbers cannot still be *predicted*, and usually have to be found from physical or numerical experiments.)

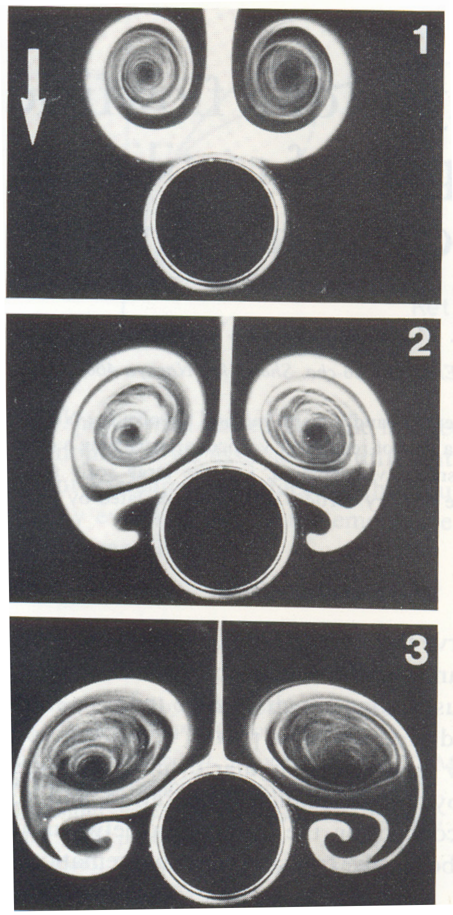

Figure 3. A vortex pair descends on a circular cylinder, and folds itself into layers. Smoke flow visualization at 40 ms intervals, basic cylinder Reynolds number 1500.

One of the most celebrated of such scaling arguments is due to Kolmogorov (although the fundamental core of the argument had been used earlier in turbulent channel flow by Clark Millikan – at least that is the way I see it). Kolmogorov analysed the spectrum of turbulence, which he considered as determined by an energy cascade from large scales in the motion to small scales (rather as the crushing of coal leaves us with pieces of various sizes – from a few big lumps to a lot of fine dust). He proposed that the 'small eddies' (high wave numbers) are universal and depend only on the viscosity and the energy dissipation, and the large eddies are flow-specific and inviscid. Crucially, he further postulated that there is a range of intermediate scales over which

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both scaling arguments are valid, i.e. they are matchable. This led to the prediction that, over that range of intermediate scales (often called the inertial subrange), the spectrum should be proportional to $k^{-5/3}$ (where k is the wave number). This should be true in *any* flow – whether it is a jet stream in the atmosphere, tidal flow in the oceans, or the boundary layer on an aircraft wing. *Figure* 4 shows how successful the argument is.

I have vastly oversimplified the reasoning here, and must hasten to caution you that universality may not be as common as it is sometimes thought to be. Kolmogorov himself felt compelled to revise his argument nearly twenty years after he first put it forward. Nevertheless, the organizing power of a successful scaling argument is enormous, its chief attraction being that it is minimalist in the hypotheses it makes (unlike in the specialized industry that churns out basically *ad hoc* turbulence models, for example). But then we cannot prove when (or even whether) such scaling arguments follow from the Navier–Stokes equations, and, even if the arguments are valid, the corresponding universal numbers and functions have of course to be determined from experiment of one kind or other.

Box 5. The Spectrum

When the flow velocity in turbulent flow fluctuates in an apparently random way (as seen for example in Figure 7, Part 1), one useful method of description is in terms of its (frequency) spectrum, or more precisely the power spectral density. In the case of the turbulent velocity field ($\boldsymbol{u}'$) this is commonly done in terms of the wave-number, which is a 'spatial' frequency measured say in cycles per unit length, related to the more familiar temporal frequency (cycles per second) through a translational velocity. If k is the magnitude of the wave number, one can express the mean kinetic energy in the fluctuating motion per unit mass of fluid as ½ $(\boldsymbol{u}')^2$, and write it as the integral

$$\tfrac{1}{2}\,\boldsymbol{u}'^2 = \int_0^\infty \mathrm{E}(k)\, \mathrm{d}k.$$

The spectral density $\mathrm{E}(k)$ can be inferred by measurement of the mean square value of outputs from appropriate filters through which components of the velocity signal $\boldsymbol{u}'$ are made to pass. Today it is more convenient to evaluate the spectrum by digitizing measured $\boldsymbol{u}'$ components and doing a fast Fourier transform on them in a computer.

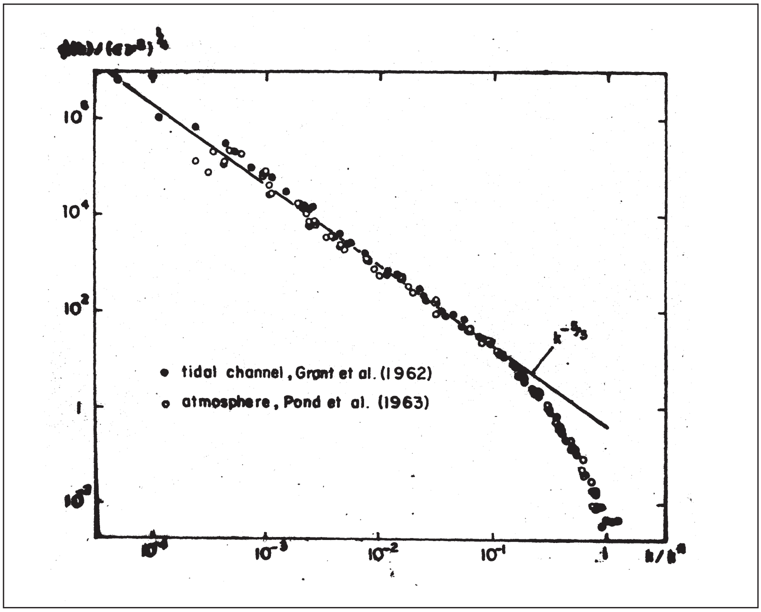

Figure 4. The high-wave number end of turbulent spectra, scaled according to Kolmogorov. The agreement between atmospheric and oceanographic measurements, and the presence of an extended $k^{-5/3}$ region, demonstrate the effectiveness of scaling and matching arguments.

Such scaling arguments have been used very widely – some times with success, at other times in controversy. They can be seen as simple applications of a kind of group theory, the centrepiece of the argument being a postulate on what the relevant group is.

There is, of course, THEORY – a lot of it in fact, although the central problem of the turbulent solutions of the Navier–Stokes equations in the limit of high Reynolds numbers remains unsolved. So the theories we possess have mostly to do either with linear problems such as inviscid flows without vorticity, small disturbance flows or low amplitude waves, highly viscous flows (i.e. at low Re) etc., or with limiting flow situations where the nonlinearity can be simplified because it is localized or approximated in some way.

What engineers often do today is use any tool or method that will help: theory of course, whenever it is available, testing, computing, simulation, scaling arguments and, increasingly, mathematical modelling. This modelling has to be distinguished from making direct appeal solely to first principles (e.g. Navier–Stokes). Instead, essentially new equations are devised, taking

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inspiration from but abandoning equivalence to the Navier–Stokes equations. Ingenuity lies in inventing equations that give the engineer the reduced information he wants, like the pipe data of *Figure* 1. Such models can be at many different levels – from codification into charts or tables like *Figure* 1, through ordinary differential equations for each flow type (boundary layers, jets etc.), all the way to systems of nonlinear partial differential equations (for mean quantities) expected to be useful for wide classes of flows – inspired by but not deducible from the Navier–Stokes equations. Although none of them is spectacularly successful, many of these models are useful – sufficiently so that there is a minor industry across the world generating, testing and applying such models to a vast variety of fluid flow problems – from making better aircraft wings to estimating how pollutants disperse to forecasting weather or ocean state.

So I hope you may see in some small way why some of us love exploring fluid flows: they can be beautiful, crazy, fearsome, important; a challenge whether you want to stare at nature's free displays, or visualize flows in the laboratory, or measure them with great precision, or control them so that they do your bidding; a happy hunting ground if you want to match your skills – of any kind – to try and unravel their secrets and to add to the great deal that is known; but, in essence, still beyond any mathematics or computers invented by man; still out there, so to speak, taunting us to see if we can *understand*, as we claim to know the basic laws.

If you also feel taunted by those flows, welcome to the fluid dynamicists' club!

Acknowledgements

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Picture credits

Part 1

Figure 1b: T F Hornbein: *Everest: The West Ridge*, Sierra Club, New York, 1966.
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Part 2.

Figure 2: A Roshko: Perspectives on Bluff Body Aerodynamics, *J. of Wind Engineering and Industrial Aerodynamics*, Vol. 49: 79-100, 1993. Courtesy: A Roshko
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Box 3. Figure a: Channaraju and P R Viswanath: PSP measurements on an aircraft model in NAL 1.2m blowdown wind tunnel, *NAL PD EA* 0313, 2003. Courtesy: P R Viswanath
Box 3. Figure b: L Venkatakrishnan, S Sundarama, P R Viswanath: Observations of hysteresis and broad flow features on a swept wing at high life, *NAL PD EA* 0503, 2005. Courtesy: P R Viswanath
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Box 4. Figure d: ADA/HAL Brochure.

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