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# The Scattering of Positrons by Electrons with Exchange on Dirac's Theory of the Positron

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molecules have orientations similar to the dibenzyl orientation, and the other two can approximately be derived from them by a rotation of 180° about the $a$ axis, and a translation of $\frac{1}{2}c$. The resulting structure explains the pseudo-orthorhombic properties, the approximate halvings, and the principal X-ray intensities. It is contrary to a structure previously deduced from magnetic measurements by Krishnan, Guha, and Banerjee, who predicted a twisted and distorted molecule; but it is shown that the new structure is equally capable of explaining the magnetic data. Detailed measurements have not yet been made on tolane and azobenzene, but the preliminary data are sufficient to show that they are both closely similar to the stilbene structure.

# The Scattering of Positrons by Electrons with Exchange on Dirac's Theory of the Positron

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It has been shown by Mott† that exchange effects play a considerable part in the collision and consequent scattering of one electron by another. Mott's original calculation was non-relativistic, and there the exchange effect vanishes when the two electrons have their spins pointing in opposite directions. Møller‡ later developed relativistically invariant expressions for the collision of two charged particles with spin, and it may be seen directly from Møller's general formula for the collision cross-section that, in the collision of two identical particles, the effect of exchange does not in general vanish even when the two colliding particles initially have their spins pointing in opposite directions. It tends however to zero in this case as the relative velocity of the particles becomes small compared to $c$, the velocity of light, in agreement with the calculation of Mott.

The effect of exchange in the general relativistic case will still be considerable if one of the two electrons be initially (and therefore finally) in a state of negative energy. (If one of the electrons be initially in a negative energy state, then it follows from the conservation of energy

† 'Proc. Roy. Soc.,' A, vol. 126, p. 259 (1930).
‡ 'Ann. Physik,' vol. 14, p. 531 (1932).

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and momentum that one of them must be finally in a state of negative energy.) This at once leads to the conclusion that in the collision of an electron with a positron, the calculation of this process on the Dirac theory of the positron, where the positron is considered as an unoccupied state of negative energy, would lead to a result different from that which we should get if we did the calculation considering the positron as an independent positively charged particle in a state of positive energy whose behaviour is descibed by the Dirac equation.† The difference would be due to the effect of exchange between the electron we observe initially and the virtual electrons in states of negative energy. As we shall show, the effect of this exchange is considerable. It tends to zero, however, when the relative velocity of the electron and positron becomes small compared to $c$, as we should expect from what has been said in the last paragraph.

The process which we are considering is one which can be calculated on the theory of the positron in its primitive form. The effect on the collision of the polarization of the vacuum by a charge, and the effect of the infinite distribution of electrons in negative energy states in general would only affect the scattering *in a higher approximation* than the one considered here, and to calculate these we should have to treat the process on the basis of a more refined theory of the positron. Such theories have been put forward by Dirac,‡ and Heisenberg,§ where methods are given for treating the infinities of charge density and current which exist on the old theory of the positron. The existence of the exchange effect even in our approximation, however, is of particular interest, since it shows that considerable error may result from the fact that in the later theories mentioned above the effect of the exclusion principle has not been considered.

Since the exchange takes place between the electron we observe and one of the virtual electrons in states of negative energy which do not come directly under our observation, one might be inclined to suppose that the existence of this additional term in the scattering due to exchange is one of the incorrect predictions of the theory which does not correspond to any physical reality. We believe, however, that this is not so, since

† As will appear presently, it is important to realize that on the latter picture of the process electrons and positrons cannot be annihilated and created in pairs. Professor Wenzel has pointed out to me that, more conveniently for our purpose, we might *define* the independence of two kinds of particles as their inability to be annihilated and created in pairs.

‡ ' Proc. Camb. Phil. Soc.,' vol. 30, p. 150 (1934).

§ ' Z. Physik,' vol. 90, p. 209 (1934).

there is another way of looking at this exchange effect which is probably the more significant one, and which shows that additional terms must exist in the mutual scattering of two particles on any theory in which the particles can be annihilated and created in pairs.

The physical process we are considering is the following. Initially we have an electron in a state $a_+^0$ of positive energy and a positron in a state $b_+^0$, also of positive energy. After the scattering process the electron is to be found in a state $a'_+$ and the positron in a state $b'_+$. On the Dirac theory of the positron the process is considered in the following way. The two states of the positron $b_+^0$ and $b'_+$ correspond to two unoccupied states of negative energy which we call $a'_-$ and $a_-^0$ respectively. We then have, initially, an electron, which we shall denote by the suffix 1, in the state $a_+^0$, another electron, which we shall denote by the suffix 2, in the state of negative energy $a_-^0$, and an unoccupied state of negative energy $a'_-$ representing the positron. After the scattering, the electron 1 goes over to the final state $a'_+$, and the electron 2 jumps into the unoccupied state $a'_-$, leaving the state $a_-^0$ unoccupied, which then appears as the scattered positron. This is the normal scattering process. The effect of exchange arises in this, that we should get to the same physically observable final state if the electron 1 jumped into the unoccupied state $a'_-$, and the electron 2 jumped to the final state $a'_+$. But we may clearly consider this process as one in which the original electron and positron have annihilated one another with the simultaneous creation of a new pair. It appears then that we should expect extra terms in the mutual scattering of any two particles which can be annihilated and created in pairs. For example, this extra scattering exists on a recent theory put forward by Pauli and Weisskopf† in which particles without spin and of opposite charge can be created and annihilated in pairs. Here the particles do not even satisfy the exclusion principle, as in the Dirac theory, but obey the Einstein-Bose statistics instead.

We shall carry out the calculation of the collision cross-section in § 1, and in § 2 we shall discuss the results.

## 1—Calculation of the Scattering

We consider the scattering process in a system L in which one of the particles, say the positron, is initially at rest, and the electron moves along the $z$-axis with an energy E and momentum $p$. We wish to know the differential effective cross-section $d$Q for the scattering of the

† 'Helv. Phys. Acta,' vol. 7, p. 709 (1934).

electron through an angle between $\theta$ and $\theta + d\theta$ about its initial direction of motion. We now consider the same elementary process as seen from a system L* in which the electron and positron move in opposite directions along the $z$-axis with equal velocity. The energy of either particle in this system is E*, which is connected with E by the relativistic formula

$$\left.\begin{aligned} \gamma^* \equiv \frac{E^*}{mc^2} &= \sqrt{\frac{\gamma + 1}{2}} \\ \text{where} \qquad \gamma &= \frac{E}{mc^2} \end{aligned}\right\}. \tag{1}$$

In the system L* the electron moves initially with a momentum $p^*$ along the $z$-axis, and the positron with a momentum $p^*$ in the opposite direction. After the scattering the electron moves in a direction making an angle between $\theta^*$ and $\theta^* + d\theta^*$ with the initial direction of motion. $\theta^*$ is connected with $\theta$ by the formula

$$\tan\theta = \frac{1}{\gamma^*}\tan\frac{\theta^*}{2}. \tag{2}$$

The differential effective cross-section $dQ$ is most simply calculated if we notice that it is an area perpendicular to the direction of relative motion of the systems L and L*, and consequently is an invariant for a Lorentz transformation from one system to the other. We therefore calculate the differential effective cross-section $dQ^*$ for the scattering of the electron through an angle between $\theta^*$ and $\theta^* + d\theta^*$. This will then just be equal to $dQ$, the cross-section for the scattering of the electron in the system L through an angle between $\theta$ and $\theta + d\theta$. The angle $\theta$ is connected with $\theta^*$ by the relation (2).

We write the Dirac equation for a free particle in the system L* in the form†

$$\{E^* + c(\boldsymbol{\alpha}, \mathbf{p}^*) + \alpha_4 mc^2\}\psi^* = 0,$$

where $m$ is the mass of the electron, and the $\alpha$'s are the four anticommuting matrices given by Dirac. The solutions are of the form

$$a(E^*, \mathbf{p}^*, s)\, e^{i\{(\mathbf{p}^*, \mathbf{x}^*) - E^* t^*\}/\hbar},$$

where the $a$'s are matrices‡ of one column and four rows satisfying the equation

$$\{E^* + c(\boldsymbol{\alpha}, \mathbf{p}^*) + \alpha_4 mc^2\}\, a(E^*, \mathbf{p}^*, s) = 0. \tag{3}$$

† Letters in Clarendon type denote vectors.

‡ To be consistent with our notation, we should write these as $a^*$. The asterisk will be omitted here for convenience, as no confusion arises thereby.

For each value of the energy and momentum there are two independent orthogonal solutions of (3), which we shall denote by $a\,(\mathrm{E}^*, \mathbf{p}^*, s_1)$ and $a\,(\mathrm{E}^*, \mathbf{p}^*, s_2)$ respectively, or in short just by $a\,(s_1)$ and $a\,(s_2)$. We normalize our solutions so that

$$(a\,(\mathrm{E}^*, \mathbf{p}^*, s)\,.\,a\,(\mathrm{E}^*, \mathbf{p}^*, s)\,) \equiv \sum_\sigma \bar{a}_\sigma\,(\mathrm{E}^*, \mathbf{p}^*, s)\,a_\sigma\,(\mathrm{E}^*, \mathbf{p}^*, s) = 1, \quad (4)$$

where a bar over a symbol denotes the conjugate complex. The $a$'s then satisfy the orthogonality and normalization relations

$$\left.\begin{array}{ll}(\bar{a}\,(\mathrm{E}^*, \mathbf{p}^*, s_q)\,.\,a\,(\mathrm{E}^*, \mathbf{p}^*, s_r)\,) = \delta_{qr} & \\ (\bar{a}\,(\mathrm{E},^* \mathbf{p}^*, s_q)\,.\,a\,(-\mathrm{E}^*, \mathbf{p}^*, s_r)\,) = 0 & q, r = 1, 2\end{array}\right\}. \quad (4)$$

This normalization represents a density of one particle per unit volume.

With this normalization we have initially in the system L* one electron per unit volume in a state of energy E* and momentum $p^*$ along the $z$-axis, which we shall denote by $(\mathrm{E}^*, 0, 0, p^*)$. We have also one unoccupied state $(-\mathrm{E}^*, 0, 0, p^*)$ of negative energy representing one positron per unit volume. This unoccupied state represents one of the *final* states of one of the electrons. It then follows at once from the conservation of momentum that if $(\mathrm{E}'^*, \mathbf{p}'^*)$ be the final state of one of the electrons, where $(\mathbf{p}'^*) \equiv (p'^* \sin\theta^* \cos\phi^*,\ \mathrm{p}'^* \sin\theta^* \sin\phi^*,\ p'^* \cos\theta^*)$, the other initial state must be $(-\mathrm{E}'^*, \mathbf{p}'^*)$. It will follow presently from the conservation of energy that $\mathrm{E}'^* = \mathrm{E}^*$. We have therefore to calculate the total number of transitions per unit volume per unit time of a system of two electrons from their initial states $(\mathrm{E}^*, 0, 0, p^*)$, $(-\mathrm{E}'^*, \mathbf{p}'^*)$ to their final states $(\mathrm{E}'^*, \mathbf{p}'^*)$, $(-\mathrm{E}, 0, 0, p^*)$, and divide this number by $\mathrm{J}_z^*$ to get the differential effective cross-section $d\mathrm{Q}^*$. Here $\mathrm{J}_z^*$ represents the number of electrons in the system L* which cross a unit area round each positron perpendicular to the direction of their relative motion per second. It is given by

$$\mathrm{J}_z^* = 2v^* = \frac{2c^2p^*}{\mathrm{E}^*}, \quad (5)$$

where $v^*$ is the initial velocity of either particle in the system L*. We then find that $d\mathrm{Q}^*$ is given according to Møller† by

$$d\mathrm{Q}^* = \frac{8\pi e^4}{\mathrm{J}_z^*}\,(\tfrac{1}{4}\mathrm{S})\,\delta\,(\mathrm{E}_1'^* + \mathrm{E}_2'^* - \mathrm{E}_1{}^{0*} - \mathrm{E}_2{}^{0*})\,p_1'^{*2}\,dp_1'^*\sin\theta^*\,d\theta^*. \quad (6)$$

† 'Ann. Physik,' vol. 14, p. 531 (1932). The formula (6) differs from formula (70′) of Møller's paper by certain constant factors. This is due to our $a$'s being normalized differently from Møller's.

The suffixes 1 and 2 refer to the two electrons, one of which is initially in a state of negative energy, and the affixes $^0$ and $'$ refer to their initial and final states respectively. S is defined in (8) below. In our problem one of the initial and one of the final states (the vacant state representing the positron) are fixed, and we have $E_1^{0*} = - E_2'^* = E^*$. Further, since we have conservation of momentum in the collision, the momentum $\mathbf{p}_2^{0*}$ of the electron initially in the state of negative energy is connected with the final momentum $\mathbf{p}_1'^*$ of the electron in the state of positive energy by the relation $\mathbf{p}_2^{0*} = \mathbf{p}_1'^*$. We therefore have $E_2^{0*} = - E_1'^*$ and the $\delta$ function in (6) reduces to $\delta\,(2E_1'^* - 2E^*)$. The $p_1'^*$ integration in (6) may then be carried out, remembering that

$$dp_1'^* = \frac{E_1'^*\, d\,(2E_1'^*)}{2c^2 p_1'^*}.$$

Using (5), we then get

$$dQ^* = \frac{2\pi e^4}{c^4} E^{*2}\,(\tfrac{1}{4}S)\sin\theta^*\, d\theta^*. \tag{7}$$

S is given by

$$S = \Sigma\left|\frac{(\bar{a}_2' \,.\, a_2^0)\,(\bar{a}_1' \,.\, a_1^0) - (\bar{a}_2'\alpha a_2^0 \,.\, \bar{a}_1'\alpha a_1^0)}{|\mathbf{p}_1^{0*} - \mathbf{p}_1'^*|^2 - \left(\dfrac{E_1^{0*} - E_1'^*}{c}\right)^2} - \frac{(\bar{a}_1' \,.\, a_2^0)\,(\bar{a}_2' \,.\, a_1^0) - (\bar{a}_1'\alpha a_2^0 \,.\, \bar{a}_2'\alpha a_1^0)}{|\mathbf{p}_1^{0*} - \mathbf{p}_2'^*|^2 - \left(\dfrac{E_1^{0*} - E_2'^*}{c}\right)^2}\right|^2. \tag{8}$$

$a_1^0$ here stands for $a\,(E_1^{0*}, \mathbf{p}_1^{0*}, s)$, with a similar meaning for the other $a$'s. The summation extends over the two independent states of spin of each initial and final state. The first term in (8) represents the direct scattering, the second term being the exchange term.

The summation over the spins can be carried out in the usual way.† We take a specimen term from (8), say

$$\overline{(a_2'\alpha_\mu a_2^0)\,(\bar{a}_1'\alpha_\mu a_1^0)}\,(\bar{a}_2'\alpha_\nu a_2^0)\,(\bar{a}_1'\alpha_\nu a_1^0),$$

in which we shall carry out the summation over the two spin directions $a'_1\,(s_1)$, $a'_1\,(s_2)$. We get

$$\sum_{s\,=\,s_1,\,s_2} (\bar{a}_2^0\alpha_\mu a_2')\,(\bar{a}_2'\alpha_\nu a_2^0)\,(\bar{a}_1^0\alpha_\mu a_1'\,(s))\,(\bar{a}_1'\,(s)\,\alpha_\nu a_1^0)$$
$$= (\bar{a}_2^0\alpha_\mu a_2')\,(\bar{a}_2'\alpha_\nu a_2^0)\,(\bar{a}_1^0\alpha_\mu \rho_1'\alpha_\nu a_1^0),$$

where $\rho_1'$ is a matrix defined by

$$(\rho_1')_{\sigma\tau} = a_1'\,(s_1)_\sigma \,.\, \bar{a}_1'\,(s_1)_\tau + a_1'\,(s_2)_\sigma \,.\, a_1'\,(s_2)_\tau. \tag{9}$$

† Casimir, 'Helv. Phys. Acta,' vol. 6, p. 287 (1933).

From (4) we see that it satisfies the equations

$$\left.\begin{array}{l}\rho_1' a\,(E_1'^*, \mathbf{p}_1'^*, s) \equiv \rho_1' a_1'(s) = a_1'(s) \quad \text{for} \quad s = s_1, s_2 \\ \rho_1' a\,(-E_1'^*, \mathbf{p}_1'^*, s) = 0,\end{array}\right\}, \tag{10}$$

The equations (10) define $\rho'_1$ completely. It then follows from (3) that if we take

$$\rho_1' = \frac{1}{2E_1'^*}\{E_1'^* - c\,(\boldsymbol{\alpha}, \mathbf{p}_1'^*) - \alpha_4 mc^2\}, \tag{11}$$

the equations (10) are satisfied. Similarly summing over $a_1^0(s_1)$, $a_1^0(s_2)$ gives

$$(\bar{a}_2^0 \alpha_\mu a_2')\,(\bar{a}_2' \alpha_\nu a_2^0)\ \text{spur}\,(\alpha_\mu \rho_1' \alpha_\nu \rho_1^0),$$

where $\rho_1^0$, defined in a similar way to $\rho'_1$ in (9) is given by an expression like (11). The summation over the two directions of spin of the other states can be carried out in a similar manner.

The denominator of the first term in (8) is equal to $(2p^* \sin \frac{1}{2}\theta^*)$, since $|p_1^{0*}| = |p_1'^*| = p^*$, and $E_1^{0*} = E_1'^* = E^*$. The denominator of the second term gives $-(2E^*/c)^2$, since $\mathbf{p}_1^{0*} = \mathbf{p}_2'^*$, and

$$E_1^{0*} = -E_2'^* = E^*.$$

We finally get for S

$$\begin{aligned} S = \Bigg[ & \frac{1}{16p^{*4}\sin^4\frac{1}{2}\theta^*}\left\{\sum_{\mu,\nu=0}^{3} c_\mu c_\nu\ \text{spur}\,(\alpha_\mu \rho_1' \alpha_\nu \rho_1^0)\ \text{spur}\,(\alpha_\mu \rho_2' \alpha_\nu \rho_2^0)\right\} \\ & + \frac{c^4}{16E^{*4}}\left\{\sum_{\mu,\nu=0}^{3} c_\mu c_\nu\ \text{spur}\,(\alpha_\mu \rho_2' \alpha_\nu \rho_1^0)\ \text{spur}\,(\alpha_\mu \rho_1' \alpha_\nu \rho_2^0)\right\} \\ & - \frac{c^2}{16E^{*2}p^{*2}\sin^2\frac{1}{2}\theta^*}\left\{\sum_{\mu,\nu=0}^{3} c_\mu c_\nu\ \text{spur}\,(\alpha_\mu \rho_2' \alpha_\nu \rho_2^0 \alpha_\mu \rho_1' \alpha_\nu \rho_1^0) \right. \\ & \left. \qquad + \text{conj. complex}\right\}\Bigg], \end{aligned} \tag{12}$$

where the $c_\mu$'s are defined by

$$c_\mu = 1, \text{ for } \mu = 1, 2, 3; \text{ and } c_0 = -1. \tag{13}$$

$\alpha_0$ denotes the unit matrix. The $\rho$'s are given by

$$\left.\begin{aligned} \rho_1^0 &= \frac{1}{2E^*}\{E^* - c\alpha_z p^* - \alpha_4 mc^2\} \\ \rho_1' &= \frac{1}{2E^*}\{E^* - c\,(\boldsymbol{\alpha}, \mathbf{p}'^*) - \alpha_4 mc^2\} \\ \rho_2^0 &= \frac{1}{2E^*}\{E^* + c\,(\boldsymbol{\alpha}, \mathbf{p}'^*) + \alpha_4 mc^2\} \\ \rho_2' &= \frac{1}{2E^*}\{E^* + c\alpha_z p^* + \alpha_4 mc^2\} \end{aligned}\right\}, \tag{14}$$

where we have inserted in expressions like (11) the values of $E_1'^*$, $\mathbf{p}_1'^*$, etc., in terms of $E^*$, $p^*$, $\mathbf{p}'^*$. The spurs in (14) are easily evaluated if we remember that the spurs of all the Dirac matrices and their products are zero, excepting that of the unit matrix.

We get finally for the differential effective cross-section $dQ^*$ for the scattering of the electron through an angle between $\theta^*$ and $\theta^* + d\theta^*$ in the system $L^*$ the expression

$$\begin{aligned} dQ^* = \frac{\pi}{8}\frac{e^4}{m^2c^4\gamma^{*2}}\Big[&\frac{1}{(\gamma^{*2}-1)^2\sin^4\frac{1}{2}\theta^*}\{1+4(\gamma^{*2}-1)\cos^2\tfrac{1}{2}\theta^* \\ &\qquad + 2(\gamma^{*2}-1)^2(1+\cos^4\tfrac{1}{2}\theta^*)\} \\ &+\frac{1}{\gamma^{*4}}\{3+4(\gamma^{*2}-1)+(\gamma^{*2}-1)^2(1+\cos^2\theta^*)\} \\ &-\frac{1}{\gamma^{*2}(\gamma^{*2}-1)\sin^2\frac{1}{2}\theta^*}\{3+4(\gamma^{*2}-1)(1+\cos\theta^*) \\ &\qquad +(\gamma^{*2}-1)^2(1+\cos\theta^*)^2\}\Big].\sin\theta^*\,d\theta^*. \end{aligned} \tag{15}$$

This is just $dQ$. We may, if we choose, express it in terms of $\theta$ and $\gamma$ by using the relations (1) and (2). This would only lead to very complicated expressions, and it is more convenient to leave it in its present form. $dQ$ is the differential effective cross-section for the scattering of the electron through an angle between $\theta$ and $\theta + d\theta$ in the system in which the positron is initially at rest. But (15) is clearly quite symmetrical between the positron and electron, so that $dQ$ also gives the effective cross-section for the scattering of the positron through an angle between $\theta$ and $\theta + d\theta$ in the system in which the electron is initially at rest. We shall henceforth use L to denote any system in which either the electron or the positron is initially at rest.

For many purposes it is more convenient to express the scattering in terms of the number of particles initially at rest which after the collision receive a certain fraction $\varepsilon$ of the kinetic energy of the colliding particle. Let $E'_R$ denote in the system L the energy after the collision of the particle which was initially at rest. (It may be either an electron or a positron.) Then $E'_R$ is connected with $\theta$ by the usual relativistic formula†

$$E'_R = \tfrac{1}{2}mc^2\{\gamma + 1 - (\gamma - 1)\cos\theta^*\}. \tag{16}$$

If $\varepsilon$ be the ratio of the kinetic energy of this particle after the collision to

† Møller, 'Ann. Physik,' vol. 14, p. 531 (1932), formula (70).

the initial kinetic energy of the colliding particle in the system L, then by (1) and (16)

$$\left.\begin{aligned} \varepsilon \equiv \frac{E'_R - mc^2}{E - mc^2} = \tfrac{1}{2}(1 - \cos\theta^*) = \sin^2 \tfrac{1}{2}\theta^* \\ \text{and} \qquad d\varepsilon = \tfrac{1}{2}\sin\theta^*\, d\theta^* \end{aligned}\right\}. \tag{17}$$

Using (1) and (17), $dQ$ given by (15) can at once be expressed in terms of $\varepsilon$ and $\gamma$. We write it in the form

$$dQ = 2\pi \frac{e^4}{m^2c^4} \cdot \frac{\gamma}{(\gamma - 1)^2 \varepsilon^2} F(\gamma, \varepsilon)\, d\varepsilon, \tag{18}$$

where

$$\begin{aligned} F(\gamma, \varepsilon) = \frac{1}{\gamma(\gamma+1)} \Big[ & \{1 + 2(\gamma - 1)(1 - \varepsilon) + (\gamma - 1)^2 (1 - \varepsilon + \tfrac{1}{2}\varepsilon^2)\} \\ & + \frac{(\gamma - 1)^2 \varepsilon^2}{(\gamma + 1)^2} \{3 + 2(\gamma - 1) + (\gamma - 1)^2 (\tfrac{1}{2} - \varepsilon + \varepsilon^2)\} \\ & - \frac{(\gamma - 1)\varepsilon}{\gamma + 1} \{3 + 4(\gamma - 1)(1 - \varepsilon) + (\gamma - 1)^2 (1 - \varepsilon)^2\} \Big]. \end{aligned} \tag{19}$$

In the limit of very high energies, $E^* \gg mc^2$, we may neglect terms of the order unity compared to $\gamma^{*2}$ and using (1) write $dQ$ in the form

$$\begin{aligned} dQ = \frac{\pi}{4} \frac{e^4}{m^2c^4\gamma} \Big[ & \frac{2(1 + \cos^4 \frac{1}{2}\theta^*)}{\sin^4 \frac{1}{2}\theta^*} + (1 + \cos^2\theta^*) \\ & - \frac{(1 + \cos\theta^*)^2}{\sin^2 \frac{1}{2}\theta^*} \Big] \sin\theta^*\, d\theta^*. \end{aligned} \tag{15A}$$

## 2—Discussion and Results

The first term in square brackets in (15) or (19) is the ordinary scattering term. We should have got just this term if we had considered the positron as an independent positively charged particle in a state of positive energy. The other two terms represent the effects of exchange. From what has already been said in the introduction, we may look upon the second term as the one due to the annihilation of the initial pair and the simultaneous creation of a new one. The third term then represents the interference between the direct scattering and the latter process.

The three expressions in curly brackets in (15) or (19) are of the same order of magnitude for all values of the energy. The relative order of magnitude of the three terms in square brackets in (15) or (19) is then determined by the expressions outside the curly brackets. These are proportional to $1/p^{*4}\sin^4\frac{1}{2}\theta^*$, $c^4/E^{*4}$, and $c^2/E^{*2}p^{*2}\sin^2\frac{1}{2}\theta^*$ respectively. The ratio of the third term to the first term in square brackets is then of the order

$$\frac{(\gamma-1)\,\varepsilon}{\gamma+1}=\frac{v^{*2}}{c^2}\,\varepsilon=\frac{v^{*2}}{c^2}\sin^2\tfrac{1}{2}\theta^*, \tag{20A}$$

and the ratio of the second term to the first is of the order

$$\frac{(\gamma-1)^2\,\varepsilon^2}{(\gamma+1)^2}=\frac{v^{*4}}{c^4}\,\varepsilon^2=\frac{v^{*4}}{c^4}\sin^4\tfrac{1}{2}\theta^*. \tag{20B}$$

The extra terms in the scattering are then smaller than the normal terms in the ratio $v^{*2}\sin^2\frac{1}{2}\theta^*/c^2$. Their effect on the scattering therefore tends to zero in the limit of low velocities, $v^*\ll c$, as was stated in the introduction.

Even in the limit of large velocities, $v^*\sim c$, the first term will be very much larger than the other two if $\theta^*$ or $\varepsilon$ be small enough. This is due to the appearance of the factor $\sin^4\frac{1}{2}\theta^*$ in the denominator of the first term in (15) only. It follows at once from (20) that for those scattering processes in which the initially stationary particle receives a large fraction of the kinetic energy of the colliding particle ($\varepsilon\sim 1$) the extra scattering is considerable. For those processes where the stationary particle receives but a small fraction of the kinetic energy of the colliding particle ($\varepsilon\ll 1$), the extra scattering is less than the normal scattering by a factor of the order $\varepsilon$.

In fig. 1 we have plotted $F(\gamma,\varepsilon)$ as a function of $\varepsilon$ for different values of $\gamma$. We have also plotted $F_0(\gamma,\varepsilon)$ for the same values of $\gamma$, where $F_0(\gamma,\varepsilon)$ is given by

$$F_0(\gamma,\varepsilon)=\frac{1}{\gamma(\gamma+1)}\,[1+2(\gamma-1)(1-\varepsilon)+(\gamma-1)^2(1-\varepsilon+\tfrac{1}{2}\varepsilon^2)]. \tag{21}$$

$F_0(\gamma,\varepsilon)$ is just the first term of $F(\gamma,\varepsilon)$ and gives what we have called the " normal " scattering, *i.e.*, the scattering in the absence of exchange effects. In other words, it gives the scattering we should get if the positron were an independent particle which with an electron could not be annihilated and created in pairs.

There is just one more point we must consider before we apply our theory to the passage of positrons through matter. We have considered the electron which was initially at rest as free, whereas all the electrons are bound in atoms. We should, however, expect the effect of binding to be negligible in all those cases where the electron and the positron after the collision both have energies large compared to the binding energy of the electron in the atom. But these are just the collisions for which the extra scattering is important. For those cases where the electron after the

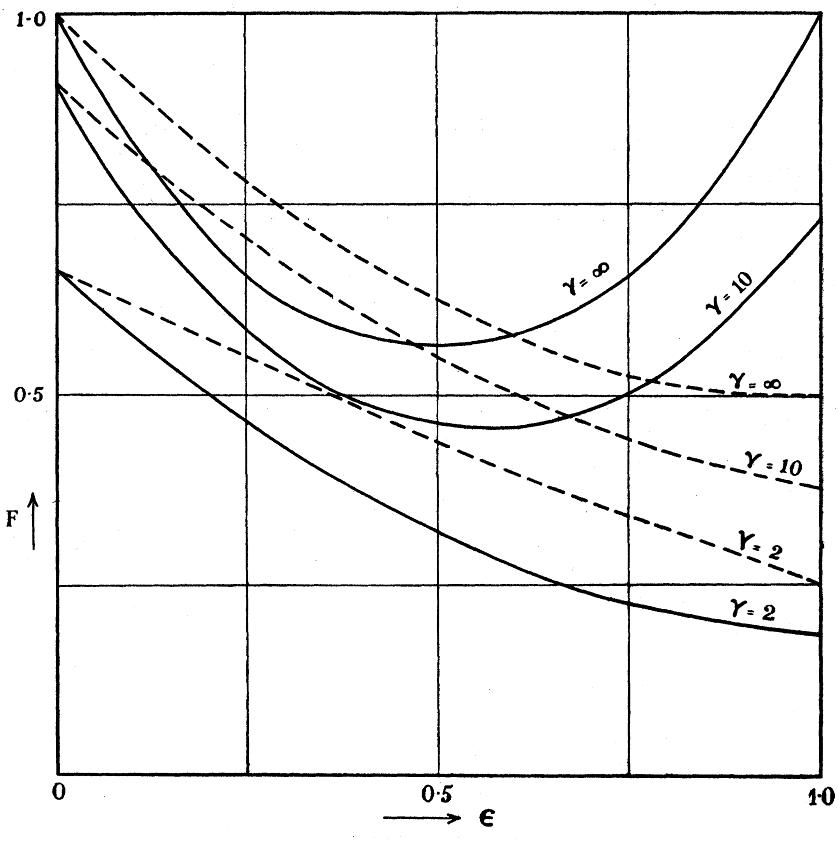


FIG. 1—The continuous lines represent F ($\gamma$, $\varepsilon$) as a function of $\varepsilon$, the broken lines represent $F_0$ ($\gamma$, $\varepsilon$).

collision has energy comparable with its binding energy, the effect of exchange is in any case small, and we may use the cross-sections calculated by Møller, Bethe, and Bloch. Lastly we have cases where the energy of the positron after the collision is comparable with the binding energy of the electron in the atom. Here the extra scattering is considerable, and it is also not legitimate to treat the particles as free. The total number of such collisions is, however, small.

We may then sum up roughly by saying that when positrons pass through matter, the number of slow secondary and ionization electrons produced will be the same as on the usual theory, whereas the number of fast secondary electrons ejected will be considerably changed. The number of fast secondaries is considerably increased for high energies of the positron. It should be possible to observe the difference experimentally.

## Summary

The ordinary collision of an electron with a positron, and the consequent scattering is considered on the Dirac theory of the positron. It is shown that exchange may take place between the electron we initially observe, and one of the virtual electrons in states of negative energy, and that this exchange very considerably modifies the scattering. It is further shown that an alternative way of looking at this exchange is to consider the process as one in which the initial electron and positron have been annihilated, giving rise simultaneously to a new pair. The ordinary scattering must then be modified on any theory in which the electron and positron can be created and annihilated in pairs.

The practical result of the extra scattering is that when positrons pass through matter, the number of slow secondary and ionization electrons produced is not changed, but the number of fast secondary electrons is considerably increased for high initial energies of the positron.