THE MEAN SQUARE DEVIATION OF THE NUMBER OF ELECTRONS AND QUANTA IN THE CASCADE THEORY

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1. Introduction

The cascade theory enables one to calculate the mean number of particles to be expected in a shower started by a primary electron or photon of a given energy after a given thickness of material. When one comes to the interpretation of rare cosmic ray events it is also necessary to know something about the fluctuations from this mean in order that one should be able to decide whether an observed shower can be interpreted as a fluctuation, or excluded from this interpretation as being highly improbable. A measure of the fluctuation is provided by the mean square deviation of the number of shower particles in a given energy interval, and the purpose of this paper is to give an explicit formula for this quantity.

Efforts to calculate the fluctuations in cascade showers have been made by several authors, among whom may be named Furry (1937), Euler (1938), Nordsieck, Lamb and Uhlenbeck (1940), and Scott and Uhlenbeck (1942). A lengthy discussion of this problem is to be found in the book of Arley (1943). All the above-mentioned authors except Scott and Uhlenbeck have replaced the actual cosmic-ray problem by a model which corresponds to it to a lesser or greater extent. In some cases one has dealt with a model in which there is only one type of particle having a certain probability of splitting into two per unit distance of travel. In other cases two types of particles, corresponding to the electrons and photons are considered, but the dependence of the radiation and pair-creation processes on the energy of the particles is completely neglected. The only treatment of the problem based on the actual quantum mechanical cross-sections for radiation loss and pair creation has been given by Scott and Uhlenbeck.* The present paper goes further than theirs only in strictly carrying through the calculations to the end. In this way we have calculated and retained certain

* We were unfortunately not aware of the paper by Scott & Uhlenbeck till after our calculations had been completed.