

SW Lacertae—A quadruple system

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Abstract. Orbital period study using photoelectric times of minima of SW Lac reveals that the period change is caused by the orbital motion of a triple system around a fourth body. The orbital periods of the third and the fourth body are 19.67 yr and 70.25 yr respectively, and their masses are close to $1 M_{\odot}$ each.

Key words : SW Lac—multiple star systems

1. Introduction

SW Lac is a W UMa-type eclipsing binary. The masses of the primary and secondary are $1.05 M_{\odot}$ and $0.89 M_{\odot}$ respectively (Giannone & Giannuzzi 1974). Their spectral type has been identified as G3p + G3p (Koch *et al.* 1963). Many detailed investigations have been made on the nature of its orbital period. Schilt (1923) analysed the available data including his own photographic minima. He expected that the observed change in the period would be easily explained on the basis of the motion of the eclipsing binary around a third body. Chou (1963) reserved his comment on the cause of the period change in SW Lac on account of meagre data available then. Lang & Vettesnik (1966) found that there was no ‘monotonous’ change in the period, only discontinuous changes. They explained this feature by means of ‘sudden changes in the distribution of matter in the system’, or by ‘loss of matter out of the system’. In addition Faulkner (1978), Brownlee (1957), Kreiner & Frasinka (1977), Bookmyer (1965) and Semeniuk (1971) have made period studies of this system.

In order to understand clearly the actual nature of the variation of the orbital period of this system, we preferred only photoelectric times of minima to other types to avoid unnecessary scattering of points in the (O-C) diagram. The present analysis of SW Lac based on the available data in the literature neither supports mass exchange hypothesis nor does it confirm Woodward’s (1956) suspicion that the changes of the period might be due to the so-called rotation of the apsidal line. But we modify Schilt’s (1923) idea of a triple system into that of a quadruple system. In other words

we believe that the observed changes in the period of SW Lac can be explained convincingly on the basis of light time effect of a triple system around a fourth body. Also, the masses of the third and fourth components turn out to be reasonable.

2. Period analysis

A plot of the time residuals of all the available photoelectric minima, listed in Table 1, using the light elements of Kreiner & Frasinka (1977) is shown in Figure 1. Retaining the initial epoch of the above authors we used a different period which is 0.89 s longer than the one given by them. Figure 2 corresponds to this modified ephemeris :

$$\text{Heliocentric minimum } I = JD \ 2442697.404 + 0^d.320724716E \quad \dots (1)$$

In Figure 2 it may be seen that the points follow a curve which is a combination of two sine curves of different frequencies. Physically this means that the eclipsing pair is moving around a third component while this triplet itself is in motion around a fourth body. Therefore SW Lac can be rightly called a quadruple system. We assumed a third body period of 19.67 yr (22400 binary cycles) and the fourth body period of 70.25 yr (80000 binary cycles); the latter is of course less certain than the former. The orbits of these bodies have been taken to be circular so that (O-C) can be represented by equation of the type

$$(O-C) = \frac{a \sin i}{c} \sin \frac{360}{P_n} (E + E_0) \quad \dots (2)$$

where P_n stands for the long term period.

First, a sine curve shown in solid line in Figure 2 was fitted by the method of least squares using points between -14000 and -23000 cycles, for the third body orbital period $P_1 = 19.67$ yr. It is represented along with standard errors, by

$$(O-C)_I = 0.0426 - 0.0202 \sin \frac{360}{22400} [E + 1749.3]. \quad \dots (3)$$

$$\pm 7 \quad \pm 11 \quad \pm 1.2$$

The deviations of all the points from this sine curve C_1 were found and plotted in Figure 3(a). Secondly, another sine curve, represented by

$$(O-C)_{II} = -0.0398 - 0.0409 \sin \frac{360}{80000} [E - 2414.4] \quad \dots (4)$$

$$\pm 8 \quad \pm 45 \quad \pm 3.3$$

was fitted through the points in Figure 3(a) for the fourth body orbital period $P_2 = 70.25$ yr. Therefore the theoretical representation of the points in the (O-C) diagram turns out to be the algebraic sum of equations (3) and (4) :

$$(O-C)_{III} = 0.0028 - 0.0202 \sin \frac{360}{22400} [E + 1749.3]$$

$$\pm 11 \quad \pm 11 \quad \pm 1.6$$

$$- 0.0409 \sin \frac{360}{80000} [E - 2414.4] \quad \dots (5)$$

$$\pm 45 \quad \pm 3.3$$

which is shown by a dashed line in Figure 2. The deviations of points $\Delta(O-C) = (O-C) - (O-C)_{III}$, calculated from equation (5) are given in column 5 of Table 1 and

Table 1. Photoelectric times of minima of SW Lac

Sl. No.	<i>J</i>	<i>D</i> ⊙	<i>E</i>	(O-C)*	Δ(O-C)	Reference
1	243	3923.	5446 II	—27357	+0.04630	—0.00176
2		3928.	51748	—27341	+0.04794	—0.00010
3		3931.	40381	—27332	+0.04775	—0.00028
4		3993.	30416	—27139	+0.04823	+0.00048
5		4271.	37095	—26272	+0.04669	+0.00081
6		4600.	58995 II	—25246	+0.04177	—0.00087
7		4600.	59008 II	—25246	+0.04190	—0.00074
8		4637.	7950 II	—25130	+0.04275	+0.00053
9		4658.	8024	—25064	+0.04268	+0.00071
10		4660.	7272	—25058	+0.04313	+0.00118
11		4663.	7730 II	—25049	+0.04205	+0.00013
12		4664.	5766	—25046	+0.04384	+0.00193
13		4664.	7367 II	—25046	+0.04358	+0.00167
14		4665.	6976 II	—25043	+0.04230	+0.00040
15		4665.	8575	—25042	+0.04184	—0.00005
16		4666.	8216	—25039	+0.04376	+0.00188
17		4667.	7833	—25036	+0.04329	+0.00142
18		4668.	7451	—25033	+0.04292	+0.00106
19		4680.	6115	—24996	+0.04250	+0.00078
20		4681.	5737	—24993	+0.04253	+0.00082
21		4681.	7348 II	—24993	+0.04327	+0.00156
22		5036.	6130	—23886	+0.03957	+0.00227
23		5037.	5748	—23883	+0.03919	+0.00190
24		5037.	7350 II	—23883	+0.03903	+0.00174
25		5040.	6180 II	—23874	+0.03551	—0.00174
26		5040.	6182 II	—23874	+0.03571	—0.00154
27		5055.	5357	—23827	+0.03951	+0.00245
28		5055.	6940 II	—23827	+0.03745	+0.00039
29		5055.	6945 II	—23827	+0.03795	—0.00089
30		5057.	6190	—23821	+0.03810	+0.00107
31		5374.	4900 II	—22832	+0.03307	+0.00009
32		5379.	4638	—22817	+0.03564	+0.00272
33		5390.	3683	—22783	+0.03550	+0.00272
34		5390.	5266 II	—22783	+0.03344	+0.00066
35		6037.	5829	—20765	+0.02763	+0.00162
36		6045.	4384 II	—20741	+0.02537	—0.00058
37		6045.	43843 II	—20741	+0.02540	—0.00055
38		6045.	6005	—20740	+0.02711	+0.00117
39		6045.	60053	—20740	+0.02714	+0.00120
40		6046.	4020 II	—20738	+0.02680	+0.00086
41		6046.	56336	—20737	+0.02780	+0.00186
42		6046.	5634	—20737	+0.02784	+0.00190
43		6048.	4870	—20731	+0.02709	+0.00117
44		6048.	6470 II	—20731	+0.02673	+0.00081
45		6049.	4490	—20728	+0.02691	+0.00100
46		6049.	4493	—20728	+0.02721	+0.00130
47		6049.	6091 II	—20728	+0.02665	+0.00074
48		6049.	60913 II	—20728	+0.02668	+0.00077
49		6050.	41146	—20725	+0.02720	+0.00129
50		6050.	4115	—20725	+0.02724	+0.00133
51		6050.	57086 II	—20725	+0.02624	+0.00033

Continued

Table 1—Continued

S1 No.	<i>J</i>	<i>D</i> ⊙	<i>E</i>	(O-C)*	Δ(O-C)	Reference		
52	243	6050.	5709	II	—20725	+0.02628	+0.00037	6
53		6461.	4147	II	—19444	+0.02172	—0.00198	7
54		6462.	3761	II	—19441	+0.02094	—0.00275	7
55		6463.	3391	II	—19438	+0.02177	—0.00192	7
56		6480.	3361	II	—19385	+0.02036	—0.00328	7
57		6843.	3967	II	—18253	+0.02058	—0.00293	7
58		6844.	3593	II	—18250	+0.02101	—0.00250	7
59		6844.	3590	II	—18250	+0.02071	—0.00280	7
60		6847.	4066		—18240	+0.02142	—0.00210	7
61		6848.	3695		—18237	+0.02215	—0.00137	7
62		7173.	7460	II	—17223	+0.02342	—0.00145	2
63		7191.	7065	II	—17167	+0.02334	—0.00164	2
64		7192.	8301		—17163	+0.02440	—0.00059	8
65		7194.	7552		—17157	+0.02515	—0.00015	8
66		7194.	9139	II	—17157	+0.02349	—0.00151	8
67		7201.	6485	II	—17136	+0.02287	—0.00217	8
68		7202.	7726		—17132	+0.02443	—0.00062	8
69		7220.	5720	II	—17077	+0.02361	—0.00156	2
70		7220.	7335		—17076	+0.02475	—0.00042	2
71		7225.	5445		—17061	+0.02488	—0.00032	2
72		7225.	7039	II	—17061	+0.02392	—0.00128	2
73		7233.	5629		—17036	+0.02516	—0.00010	2
74		7258.	5795		—16958	+0.02523	—0.00020	2
75		7262.	5873	II	—16946	+0.02398	—0.00148	2
76		7556.	6958	II	—16029	+0.02791	—0.00013	2
77		7572.	5719		—15979	+0.02814	—0.00006	2
78		7572.	7328	II	—15979	+0.02868	+0.00048	2
79		7573.	6952	II	—15976	+0.02890	+0.00069	2
80		7577.	7038		—15963	+0.02844	+0.00018	2
81		7578.	6660		—15960	+0.02847	+0.00020	2
82		7611.	7012		—15857	+0.02902	+0.00040	2
83		7614.	5879		—15848	+0.02920	+0.00055	2
84		7615.	5502		—15845	+0.02932	+0.00066	2
85		7619.	5582	II	—15833	+0.02827	—0.00043	2
86		7845.	5115		—15128	+0.03100	—0.00035	4
87		7846.	4740		—15125	+0.03133	—0.00003	4
88		7846.	4760		—15125	+0.03333	+0.00197	4
89		7869.	4062	II	—15054	+0.03171	+0.00006	4
90		7871.	4904		—15047	+0.03120	—0.00048	4
91		7875.	3397		—15035	+0.03180	+0.00007	22
92		7875.	4994	II	—15035	+0.03114	—0.00059	4
93		7879.	5097		—15022	+0.03238	+0.00060	4
94		7876.	3014		—15032	+0.03133	—0.00041	22
95		7876.	4683	II	—15031	+0.03786	+0.00612	22
96		7877.	4234	II	—15028	+0.03079	—0.00097	22
97		7878.	3846	II	—15025	+0.02981	—0.00196	22
98		7879.	3476	II	—15022	+0.03064	—0.00114	22
99		7879.	6662	II	—15021	+0.02851	—0.00328	22
100		7881.	2744	II	—15016	+0.03309	+0.00128	22
101		7881.	4339		—15016	+0.03228	+0.00042	22
102		7883.	3590		—15010	+0.03298	+0.00115	22

Continued

Table 1—Continued

Sl. No.	<i>J</i>	<i>D</i> ⊙	<i>E</i>	(O-C)*	Δ(O-C)	Reference		
103	243	7885.	4419	II	—15003	+0.03117	—0.00069	22
104		7886.	4045	II	—15001	+0.03160	—0.00027	4
105		7887.	3671	II	—14998	+0.03203	+0.00015	4
106		7887.	5276		—14997	+0.03217	+0.00029	4
107		7890.	4144		—14988	+0.03244	+0.00052	4
108		7897.	4698		—14966	+0.03190	—0.00011	4
109		7903.	4034	II	—14948	+0.03209	0	4
110		7916.	7153		—14906	+0.03392	+0.00166	9
111		7919.	7603		—14897	+0.03203	—0.00027	9
112		7926.	6559		—14875	+0.03205	—0.00034	9
113		7926.	8139		—14875	+0.02969	—0.00270	9
114		7929.	7008		—14866	+0.03007	—0.00236	9
115		7929.	8627		—14865	+0.03160	—0.00083	9
116		7940.	6019	II	—14832	+0.02653	—0.00604	9
117		7941.	4091		—14829	+0.03191	—0.00067	4
118		7959.	3705		—14773	+0.03273	—0.00009	4
119		7961.	2945		—14767	+0.03280	—0.00004	4
120		8235.	5180		—13912	+0.03625	—0.00037	10
121		8670.	4214		—12556	+0.03693	—0.00584	4
122		8670.	4252		—12556	+0.04073	—0.00204	4
123		8708.	2710		—12438	+0.04102	—0.00227	10
124		8709.	3930	II	—12435	+0.04048	—0.00282	10
125		9039.	42074	II	—11406	+0.04249	—0.00496	11
126		9040.	38393	II	—11403	+0.04351	—0.00395	11
127		9041.	34579	II	—11400	+0.04319	—0.00428	11
128		9041.	50748		—11399	+0.04452	—0.00295	11
129		9059.	4680		—11343	+0.04442	—0.00326	10
130		9393.	18563	II	—10303	+0.04802	—0.00282	11
131		9393.	34719		—10302	+0.04921	—0.00163	11
132		9443.	3810		—10146	+0.04997	—0.00124	10
133	244	0035.	4224**		—8300	+0.03354	...	12
134		0110.	4923		—8066	+0.05386	+0.00129	13
135		0128.	4540		—8010	+0.05497	+0.00246	12
136		0202.	2197		—7780	+0.05399	+0.00179	12
137		0373.	4866		—7246	+0.05389	+0.00277	12
138		0419.	3502		—7103	+0.05386	+0.00311	12
139		0441.	4806		—7034	+0.05425	+0.00370	13
140		0467.	4591		—6953	+0.05405	+0.00374	13
141		0497.	2869		—6860	+0.05445	+0.00442	12
142		0515.	2460		—6804	+0.05297	+0.00312	12
143		0542.	3457	II	—6720	+0.05143	+0.00187	14
144		0836.	4479	II	—5803	+0.04907	+0.00335	15
145		0842.	3820		—5784	+0.04976	+0.00413	15
146		0843.	5030	II	—5781	+0.04822	+0.00261	15
147		0848.	4758		—5765	+0.04979	+0.00426	15
148		1167.	4277	II	—4770	+0.04095	+0.00115	23
149		1172.	4009		—4755	+0.04292	+0.00321	23
150		1192.	7656	II	—4692	+0.04161	+0.00231	16
151		1192.	9264		—4691	+0.04204	+0.00275	16
152		1249.	3737		—4515	+0.04179	+0.00368	23
153		1583.	4060	II	—3473	+0.03929	+0.00886	24

Continued

Table 1—Continued

Sl No.	<i>J D</i> ⊕	<i>E</i>	(O-C)*	Δ(O-C)	Reference
154	244 1598.	3161	—3427	+0.03570	+0.00563
155	1598.	3154	—3427	+0.03500	+0.00493
156	1683.	3055	—3162	+0.03305	+0.00511
157	1900.	4293	—2485	+0.02622	+0.00393
158	2361.	7776 II	—1046	+0.01201	+0.00219
159	2369.	7962 II	—1021	+0.01249	+0.00288
160	2630.	3750	—209	+0.00247	-0.00025
161	2723.	8271 II**	+82	-0.03668	...
162	2724.	7881 II**	+85	-0.03786	...
163	2738.	7390**	+129	-0.03848	...
164	2697.	4039	0	-0.00010	-0.00110
165	2768.	2821	+221	-0.00206	-0.00127
166	3013.	7899 II	+986	-0.00903	-0.00240
167	3049.	8707	+1099	-0.00976	-0.00231
168	3398.	4899	+2186	-0.01833	-0.00384
169	3411.	4804 II	+2226	-0.01718	-0.00246
170	3411.	6343	+2227	-0.02364	-0.00891
171	3459.	7476	+2377	-0.01905	-0.00348
172	3460.	5490 II	+2379	-0.01946	-0.00388
173	3487.	6504	+2464	-0.01930	-0.00326
174	3488.	6128	+2467	-0.01907	-0.00302
175	3802.	5987	+3446	-0.02267	-0.00210

Note : *Obtained using $JD \odot I = 2442697.404 + 0d.320724716 E$.

**not included in the diagram as well as in analysis.

References: 1. Kwee (1958), 2. Bookmyer (1965), 3. Brownlee (1957), 4. Lang & Vettesnik (1966), 5. Hinderer (1960), 6. Broglia (1962), 7. Widorn (1962), 8. Muthsam & Rakos (1974), 9. Chou (1963), 10. Pohl (1967), 11. Rucinski (1968), 12. Kizilirmak & Pohl (1970), 13. Semeniuk (1971), 14. Muthsam (1972), 15. Pohl & Kizilirmak (1971), 16. Meyer (1972), 17. Baldinelli (1973), 18. Kizilirmak & Pohl (1976), 19. Baldinelli & Ghedini (1976), 20. Kizilirmak *et al.* (1978), 21. Faulkner (1979), 22. Kalchaev *et al.* (1968), 23. Pohl & Kizilirmak (1972), 24. Pohl & Kizilirmak (1974), 25. Skillman (1977), 26. Faulkner & Bookmyer (1980).

shown in Figure 3(b). Even a casual observation of Figure 3(b) will reveal that there is a very small fluctuation of points. This fluctuation is rather small when compared with other W UMa type binaries, say, 441 Bootis B.

3. Masses of third and fourth components

The mass functions for the triple and quadruple systems can be written respectively as

$$f(m)_3 = \frac{(a_{12} \sin i_1)^3}{P_1^2} = \frac{m_{12} \sin^3 i_1}{\alpha_1 (1 + \alpha_1)^2} \quad \dots (6)$$

and

$$f(m)_4 = \frac{(a_{123} \sin i_2)^3}{P_2^2} = \frac{m_{123} \sin^3 i_2}{\alpha_2 (1 + \alpha_2)^2} \quad \dots (7)$$

where $\alpha_1 = \frac{m_{12}}{m_3}$, $\alpha_2 = \frac{m_{123}}{m_4}$,

- m_{12} = total mass of the eclipsing binary,
 m_3, m_4 = individual masses of third and fourth components of the system,
 m_{123} = total mass of the triple system,
 i_1 = angle of inclination of the third body orbit,
 i_2 = angle of inclination of the fourth body orbit,
 a_{12} = semi major axis of the third body orbit,
 a_{123} = semi major axis of the fourth body orbit.

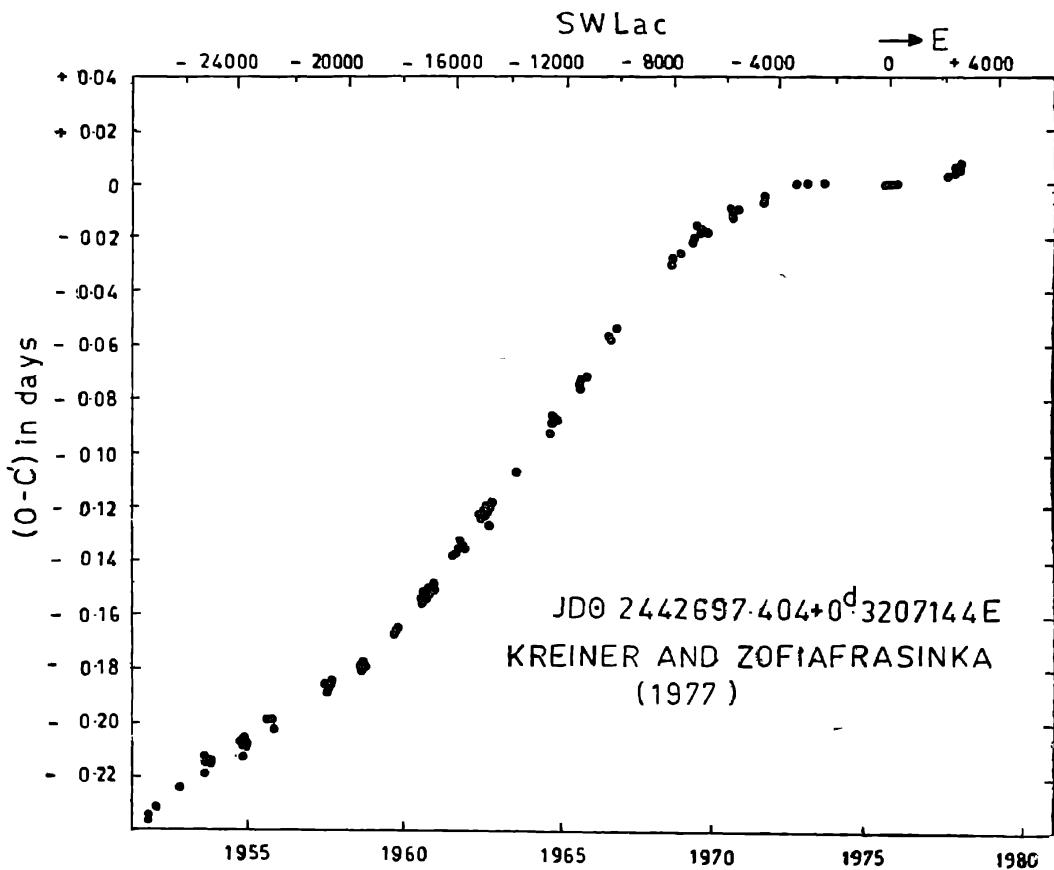


Figure 1. Time residuals (O-C') versus the number of cycles elapsed E , based on the light elements given by Kreiner and Frasinka (1977).

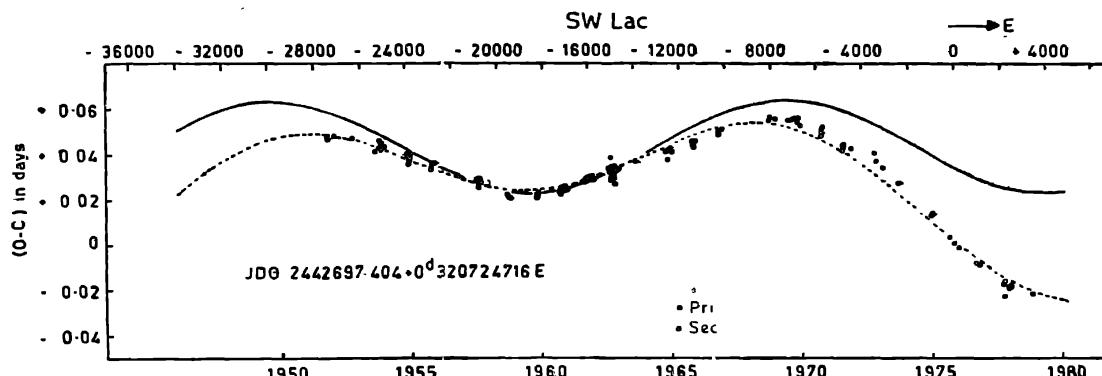


Figure 2. (O-C) diagram based on the ephemeris. Heliocentric primary minimum = JD 2442697.404 + 0^d.320724716 E . Solid curve represents equation (3) while dashed curve indicates equation (5).

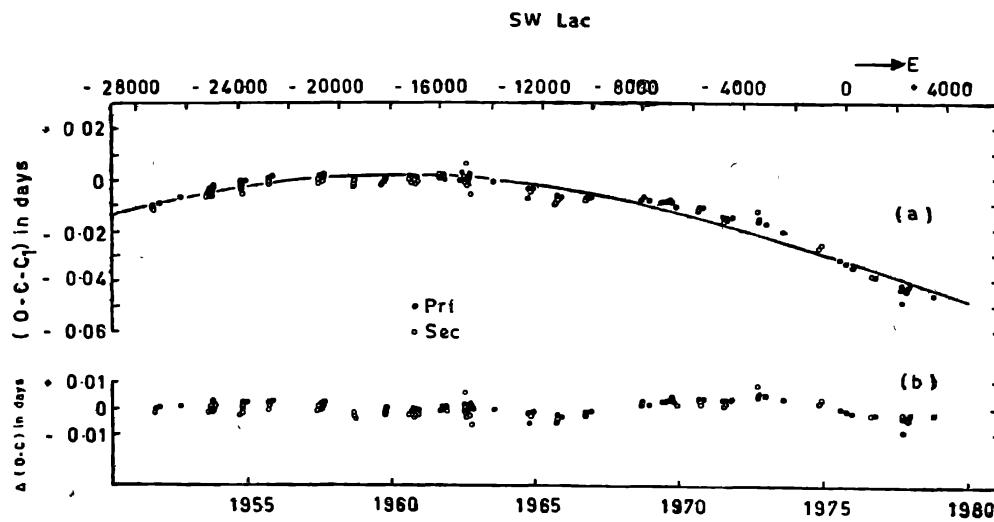


Figure 3. (a) Time residuals after the effect due to third body has been eliminated. Solid curve represents equation (4). (b) Time residuals after the effects due to third and fourth bodies have been eliminated.

The last two quantities multiplied by $(\sin i)/c$ are given by the amplitude coefficients of their respective Fourier terms. From equations (3) and (4) we obtain $a_{12} \sin i_1 = 3.51$ a.u. and $a_{123} \sin i_2 = 7.11$ a.u. Then equations (6) and (7) give

$$f(m)_3 = \frac{(3.51)^3}{(19.67)^2} = 0.112 M_\odot$$

$$f(m)_4 = \frac{(7.11)^3}{(70.25)^2} = 0.073 M_\odot$$

If we take the total mass of the eclipsing system to be $1.94 M_\odot$ (Giannone & Giannuzzi 1974) the third mass m_3 turns out to be $0.99 M_\odot$ for $i = 90^\circ$ and $1.19 M_\odot$ for $i = 60^\circ$. In case we assume that the third mass is $1 M_\odot$, the mass of the fourth body comes out to be $1.05 M_\odot$ for $i_2 = 90^\circ$, and $m_4 = 1.26 M_\odot$ for $i_2 = 60^\circ$. It may be observed that the masses of the third and fourth components are almost below the Chandrasekhar mass limit, for moderate values of the angle of inclination $90^\circ \geq i_1 (i_2) \geq 60^\circ$. Hence they are likely to be white dwarfs with low luminosity which makes them undetectable by other means. A more reliable estimates for these quantities can be made if the entire cycle of the triple system around the fourth body is covered with accurate observations, and astrometric studies made to determine i_1 and i_2 .

4. Conclusion

A convincing interpretation for the observed period change in SW Lac is that of the light-time effect of a triple system around a fourth body. The third and fourth bodies in SW Lac are of equal mass of about $1 M_\odot$. The short-term period variation in this system is not so conspicuous as in other W UMa systems.

References

- Baldinelli, L. (1973) *Inf. Bull. Var. Stars* No. 838.
- Baldinelli, L. & Ghedini, S. (1976) *Inf. Bull. Var. Stars* No. 1134.
- Bookmyer, B. B. (1965) *Astr. J.* **70**, 415.
- Broglia, P. (1962) *Contr. Oss. astr. di Milano-Merate*, 190.
- Brownlee, R. R. (1957) *Ap. J.* **125**, 372.
- Chou, K. C. (1963) *Astr. J.* **68**, 342.
- Faulkner, D. R. (1978) *Inf. Bull. Var. Stars* No. 1503.
- Faulkner, D. R. (1979) *Inf. Bull. Var. Stars* No. 1685.
- Faulkner, D. R. & Bookmyer, B. B. (1980) *Publ. astr. Soc. Pacific* **92**, 92.
- Giannone, P. & Giannuzzi, M. A. (1974) *Ap. Sp. Sci.* **26**, 289.
- Hinderer, F. (1960) *J. des Obs.* **43**, 161.
- Kalchaev, K., Laytschev, I. N., & Trutse, Yu. L. (1968) *Variable Stars* **16**, 440.
- Kizilirmak, A., Ebersberger, J. & Pohl, E. (1978) *Inf. Bull. Var. Stars* No. 1449.
- Kizilirmak, A. & Pohl, E. (1970) *Inf. Bull. Var. Stars* No. 456.
- Kizilirmak, A. & Pohl, E. (1976) *Inf. Bull. Var. Stars* No. 1163.
- Koch, R. H., Sobieski, S. & Wood, F. B. (1963) *Publ. Univ. Pa. Astr. Series* **9**.
- Kreiner, J. M. & Frasinka, Z. (1977) *Inf. Bull. Var. Stars* No. 1285.
- Kwee, K. K. (1958) *Bull. astr. Inst. Neth.* **14**, 131.
- Lang, K. & Vetešnik, M. (1966) *Bull. astr. Inst. Czech.* **17**, 21.
- Meyer, A. (1972) *Inf. Bull. Var. Stars* No. 668.
- Muthsam, H. (1972) *Inf. Bull. Var. Stars* No. 631.
- Muthsam, H. & Rakos, K. D. (1974) *Astr. Ap. Suppl.* **13**, 127.
- Pohl, E. & (1967) *Inf. Bull. Var. Stars* No. 198.
- Pohl, E. & Kizilirmak, A. (1971) *Inf. Bull. Var. Stars* No. 530.
- Pohl, E. & Kizilirmak, A. (1972) *Inf. Bull. Var. Stars* No. 647.
- Pohl, E. & Kizilirmak, A. (1974) *Inf. Bull. Var. Stars* No. 937.
- Rucinski, S. M. (1968) *Acta Astr.* **18**, 49.
- Schilt, F. (1923) *Bull. astr. Inst. Neth.* **2**, 1.
- Semeniuk, I. (1971) *Acta Astr.* **21**, 49.
- Skillman, D. R. (1977) *J. Am. Assoc. Var. Stars Obs.* **7**, 23.
- Widorn, Th. (1962) *Mitt. Univ. Sternwarte Wien Bon.* **11**, 1.
- Woodward, E. J. (1951) *Astr. J.* **56**, 771.