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The moon illusion and the height of the sky

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Abstract. It is shown that the apparently flattened celestial vault can be represented by a geocentric sphere at a height of 3000–4000 km above the surface of the earth. It is also argued that our eye estimates the distance to this 'sky sphere' by its apparent opacity which is a function of the air mass. A hypothesis regarding the physiology of distance vision is proposed.

Key words : moon illusion—vision

1. Introduction

Rees (1986) has recently presented new observational data which confirms the commonly held hypothesis that the moon illusion, that is the apparently large size of the moon near the horizon, is caused by the flattened appearance of the celestial vault. He has fitted the observed relation between the apparent a' and true a elevations of the sun and the moon to a model of the celestial vault based on the cubic equation $y = 1 - (x/b)^3$ where $x \leqslant b$, $\theta = \tan^{-1}(y/x)$ is the zenith angle, and $r(\theta) = (x^2 + y^2)^{1/2}$ is the apparent distance to the sky vault. He found that b, the distance of the horizon measured in units of the vertical height, lies between 2 and 3, the best value being $b = 2.6$. However the sky appears to us as a sphere that follows the curvature of the earth. Although the earth looks flat the circular appearance of the horizon subcontiously seems to tell us that it is spherical and the sky is surrounding it at a fixed distance. We may, therefore, model the celestial vault as a geocentric sphere at a certain height h above the surface of the earth as shown in figure 1.

2. Sky as a geocentric sphere

If R is the radius of the earth, then from figure 1 it is clear that the distance of the horizon will be $bh = (2Rh + h^2)^{1/2}$ which gives

$$h = 2R/(b^2 - 1). \qquad ...(1)$$

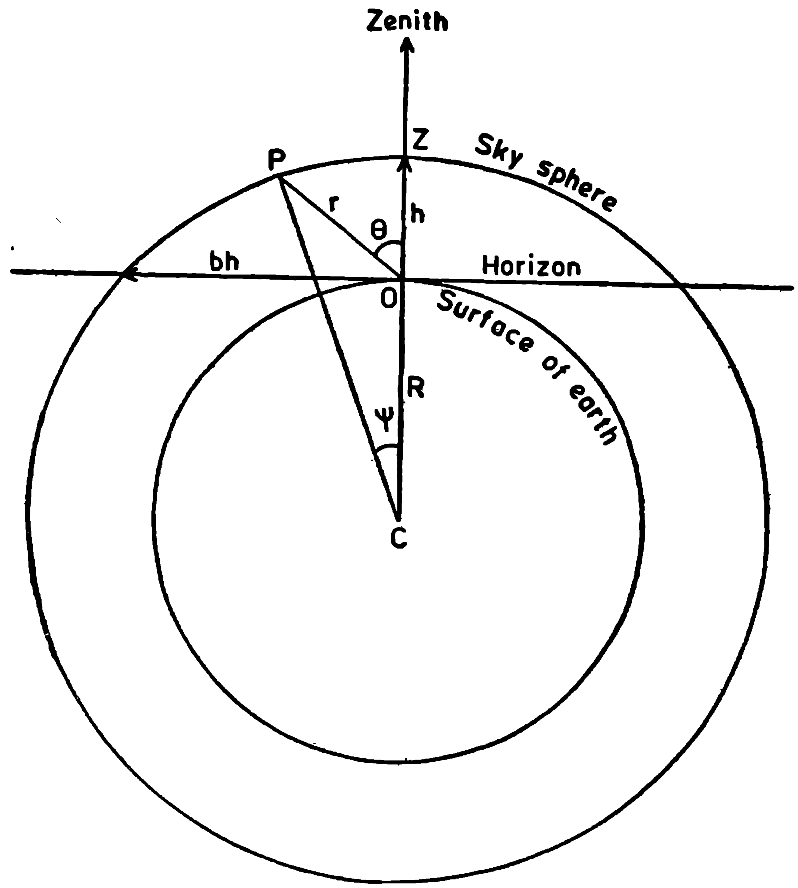

Figure 1. Representation of the celestial vault as a geocentric sphere.

Further, from the triangle COP we find that the distance $r(\theta)$ and the geocentric angle ψ are given by

$$\frac{r(\theta)}{h} = \frac{(b^2-1)}{2}\left[-\cos\theta + \sqrt{\cos^2\theta + 4b^2/(b^2-1)^2}\right] \qquad ...(2)$$

and

$$\psi = \sin^{-1}[r \sin\theta/(R+h)]. \qquad ...(3)$$

At the horizon ψ becomes

$$\psi_H = \sin^{-1}[bh/(R+h)]. \qquad ...(4)$$

Since the apparent zenith distance θ' is taken to be proportional to the corresponding length of the arc we get

$$\theta' = (\pi/2)(\psi/\psi_H). \qquad ...(5)$$

The relation between the apparent elevation $a' = 90 - \theta'$ and the true elevation $a = 90 - \theta$ obtained from this calculation is shown in figure 2 for $b = 1.0\ (0.5)\ 3.0$. Rees's observations are also plotted in the figure; they indicate that b lies between 1.5 and 3. It is likely that the actual value of b might depend on the atmospheric conditions. But taking b to be restricted to values between

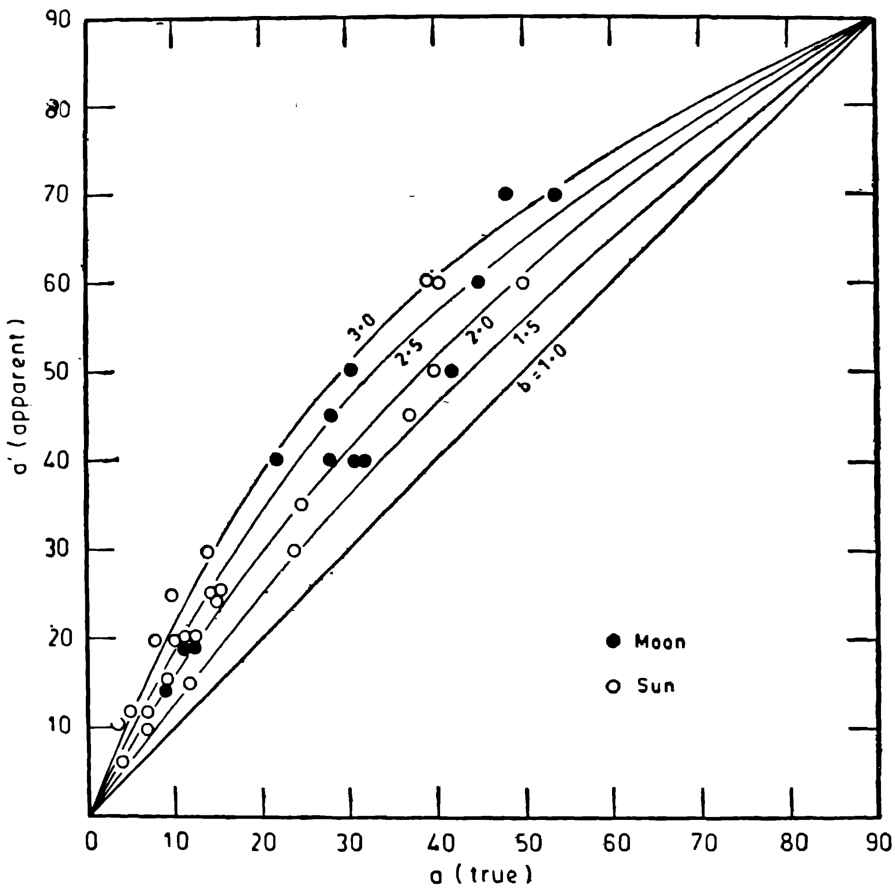

Figure 2. Relation between the apparent and true elevations for the sky vault of figure 1. Rees' (1986) observations are indicated by dots and circles.

2 and 2.5 we find that the 'height' of the sky above the earth's surface would be 3400 $\pm$ 1000 km according to equation (1).

When the sky is completely cloudy $h \approx 10$ km which makes $b \approx 35$; then the sky vault will appear much more flattened, which is indeed true.

3. Role of air mass

We may ask the question: How does the eye estimate the distance to the celestial vault ? My subjective impression is that the atmosphere appears to become thicker as we approach the horizon which is interpreted as an increase in the distance of the sky. In other words it is the airmass which plays the decisive role. In figure 3 we have plotted log (r/h) obtained from equation (2) as a function of θ for two values of $b = 2.0$ and 2.5 against log X, where X is the airmass given as a function of θ by Allen (1976). We see that for $X \approx 1$ both curves are close to the relation $r \propto \sqrt{X}$ indicated by the dashed line in figure 3. It reveals another important physiological effect. The luminosity l of an object at a given zenith distance is exp $(-XK)$ times its luminosity at zenith, where K is the absorption coefficient. As is well known, the response of the eye is logarithmic and the luminosity is perceived as a magnitude difference $\Delta m \propto X$. Now if we assume that the estimated distance is a luminosity distance, then $r \propto 1/\sqrt{l}$ which is translated by the eye as $(r/h) \propto \sqrt{\Delta m} \propto \sqrt{X}$. However as the extinction increases with X saturation sets in and the distance estimated from luminosity approaches a fixed value giving the flattened curves of figure 3.

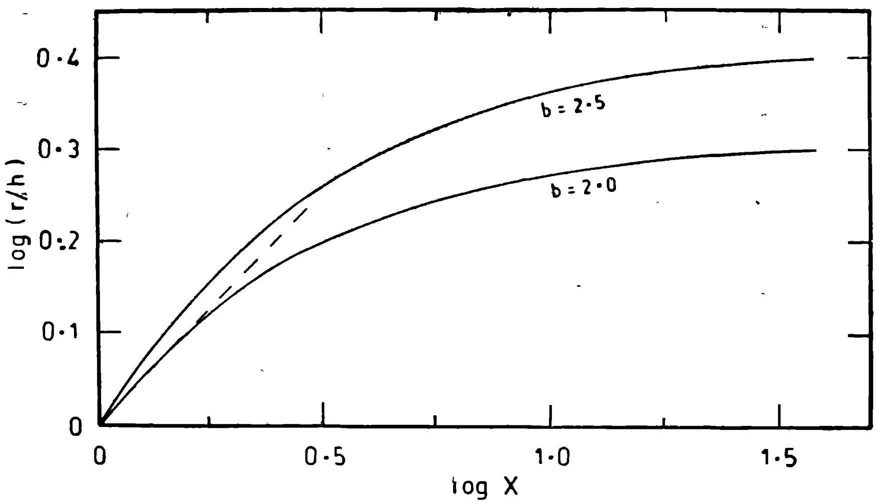

Figure 3. Relation between the distance of the sky vault and the airmass.

Minnaert (1954) has discussed several phenomena related to the moon illusion in his book on *Light and colour*. He has explained them on the basis of the theory of underestimation of distances by Von Sterneck formula

$$d' = \frac{cd}{c+d}, \quad \ldots(6)$$

where d is the true distance, d' the apparent distance and c a constant between 200 yards and 10 miles. It is mentioned that this theory applies to terrestrial objects for which distances are known to us by experience. But it cannot be applied to celestial objects. Further no light has been thrown on the origin of the underestimation. We would like to point out that saturation of luminosity distance by extinction due to airmass is akin to underestimation by a formula like equation (6). Further, here we have a physical explanation of the underestimation of distance.

4. Physiology of distance vision

If what we have noted above is valid we can make the following hypothesis regarding the physiology of distance vision which is remarkably close to the approach of an astronomer towards determining the distances of celestial objects.

For fairly close objects our binocular vision is effective in determining the relative distances of objects just like the trigonometric parallaxes of nearby stars.

For intermediate distances where the binocular vision begins to fail we estimate the distance by the relative angular sizes of familiar objects like trees, etc. There is an astronomical counterpart where we determine the distances of extended objects from their angular size. Finally, when the distance becomes very large and angular sizes of familiar objects cannot be estimated it is the brightness of the object which is used by the eye as criterion for judging the distance. In other words it is the luminosity distance which is the standard method used by the astronomers for obtaining the distances of very remote objects. Thus the distance to the sky vault is judged by us from the enhanced faintness of the objects. It is a common experience that objects appear farther away when seen through a fog. Similarly whenever we look at the stars from a mountain top they appear so bright and so close that we can easily imagine that we might be able to pluck them with our hands. The large size of the moon near the horizon is thus the result of its enhanced distance on the basis of the faintness of the objects near the horizon. The situation is quite akin to the increasing diameters of galactic clusters with distance found by Trumpler 50yr ago.

Our eye is thus a complex analogue computer which converts the appearances of objects into certain impressions according to a physiological printed program burnt into the brain. One can check the proposed hypothesis linking brightness to distance and angular size by considering the earth illusion on the moon.

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