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Variation of the galactic force law in the region of the Sun

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Abstract. From the radial velocity data for O–B5 stars within 3 kpc of the Sun the variation of the semi-axes Σ_1 and Σ_2 of the velocity ellipsoid in the solar region is estimated. It is found that both the semi-axes as well as their ratio Σ_2/Σ_1 decrease away from the galactic centre. The value of Σ_2/Σ_1 changes from $1 \cdot 17$ at $R = 9$ kpc to $0 \cdot 48$ at $R = 11$ kpc passing through a value of $0 \cdot 86$ at the solar distance of $R_0 =$ 10 kpc. These results are consistent with the usually assumed inverse square law of force in the outer regions of the galaxy.

Keywords. Galactic force law; velocity ellipsoid.

1. Introduction

The galactic force law and the mass distribution for the inner regions of the galaxy, with respect to the Sun, can be obtained from the rotation curve dervied from the observations of 21-cm line profiles. This is possible because the maximum positive velocity in any inner direction can be identified with that corresponding to the circular velocity at the point which is closest to the galactic centre in that line of sight. A similar procedure cannot be applied in the outer regions because there the line of sight passes monotonically farther and farther away from the galactic centre. It is normally assumed that the inverse square law of force prevails in the outer regions. This assumption has only a marginal effect on the overall structure of the galaxy on account of the small amount of matter occurring in the outer regions. However, an independent verification of the hypothesis is desirable for considering the dynamics of the galaxy in the solar neighbourhood.

It is known that the ratio Σ_2/Σ_1 of the semi-axes of the velocity ellipsoid in the galactic plane depends upon the force law in operation at the point in question (Trumpler and Weaver 1953). For example Σ_2/Σ_1 has a value of unity within a homogeneous ellipsoidal distribution of matter while $\Sigma_2/\Sigma_1 = 0 \cdot 5$ in an inverse square field of force. Thus the determination of this ratio is a useful tool for establishing the nature of the force law at a particular point in the galaxy. In this paper I have proposed and applied a new method of using the scatter of residual radial velocities for determining the semi-axes Σ_1 and Σ_2 and their ratio as functions of the distance from the galactic centre.

2. Procedure

Let P be a point in the galactic plane at a distance r and having the galactic longitude l as measured from the direction of the galactic centre (*see* figure 1). Let (R, θ) be the galactocentric coordinates of the same point. The semi-axes Σ_1 and

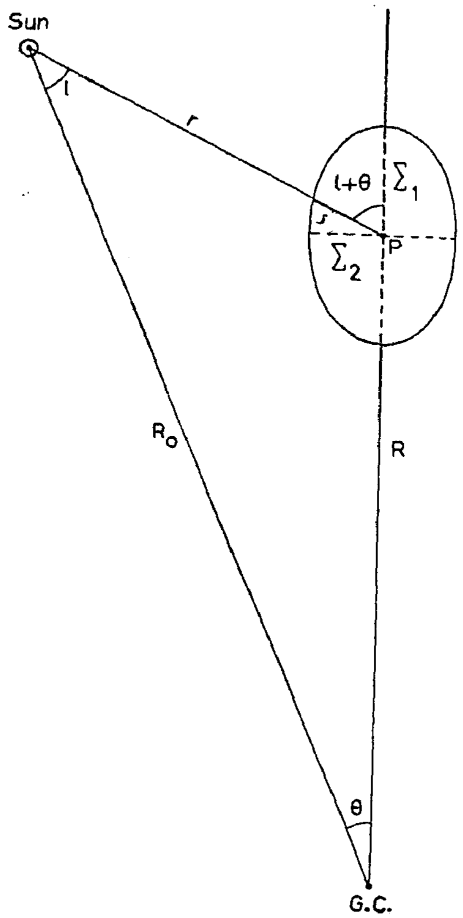

Figure 1. Relation between the velocity ellipsoid and the radial velocity dispersion at a point in the galactic plane.

Σ_2 of the velocity ellipsoid at P point in the radial and perpendicular directions respectively. The dispersion σ of the residual radial velocities of the stars in the neighbourhood of P is the radius vector of the velocity ellipsoid in the line of sight. It is related to Σ_1 and Σ_2 through the equation

$$\frac{\sigma^2 \cos^2 (l+\theta)}{(\Sigma_1)^2} + \frac{\sigma^2 \sin^2 (l+\theta)}{(\Sigma_2)^2} = 1 \tag{1}$$

Let

$$\left.\begin{aligned} \Sigma_1 &= \Sigma_1^{(0)} [1 + a(R - R_0)], \qquad a < 1 \\ \Sigma_2 &= \Sigma_2^{(0)} [1 + b(R - R_0)], \qquad b < 1 \end{aligned}\right\} \tag{2}$$

Then we have

$$\frac{\sigma^2 \cos^2 (l+\theta)}{(\Sigma_1^{(0)})^2} [1 - 2a(R - R_0)] + \frac{\sigma^2 \sin^2 (l+\theta)}{(\Sigma_2^{(0)})^2} \times [1 - 2b(R - R_0)] = 1$$

which can be written as

$$AX + BY + CZ + DT = E \tag{3}$$

where

$$\left.\begin{aligned} X &= (\Sigma_1^{(0)})^{-2}, & Y &= -2aX \\ Z &= (\Sigma_2^{(0)})^{-2}, & T &= -2bZ \\ A &= \cos^2 (l+\theta), & B &= (R - R_0) A \\ C &= \sin^2 (l+\theta), & D &= (R - R_0) C \\ E &= 1/\sigma^2 & & \end{aligned}\right\} \tag{4}$$

It may be noted that

$$R = [r^2 + R_0 (R_0 - 2r \cos l)]^{\frac{1}{2}} \tag{5}$$

and

$$R/\sin l = R_0/\sin (l + \theta)$$

so that

$$\left. \begin{aligned} \sin (l + \theta) &= R_0 \sin l/R \\ \cos (l + \theta) &= \pm \sqrt{1 - (R_0 \sin l/R)^2} \text{ for } R \lessgtr R_0 \end{aligned} \right\} \tag{6}$$

We have used $R_0 = 10$ kpc in our calculations.

Equation (3) has to be solved for X, Y, Z and T by the method of least squares. However, in order to avoid the effect of observational errors we would do so in two stages. For $| R - R_0 | \leqslant 0 \cdot 25$ kpc we can neglect B and D; then equation (3) reduces to

$$AX + CZ = E \tag{7}$$

This equation is solved first for X and Z by the method of least squares. Then writing

$$BY + DT = E - AX - CZ = F \tag{8}$$

for $| R - R_0 | > 0 \cdot 25$, we solve for Y and T by the method of least squares. Equations (4) then give us Σ_1 and Σ_2 as functions of R, the distance from the galactic centre.

3. Application

From the data on radial velocities given by Rubin *et al* (1962) and Bonneau (1967), Shatsova (1970) has determined the velocity dispersions of O and B stars within a few kiloparsecs of the Sun. She corrected the observations for the standard solar motion, Oort's terms of galactic rotation and the average residual velocity. We have used only the data for 1107 O–B5 stars contained in the 20 zones shown in figure 2. The stars in each zone were considered to give the radial velocity dispersion σ at the central point of the zone. The seven points represented by filled circles, which satisfy the condition $| R - R_0 | \leqslant 0 \cdot 25$ kpc, were used for solving equation (7) by the method of least squares. The remaining thirteen points were then used for solving equation (8) by the method of least squares. The following results were obtained:

$$X = 0 \cdot 00488 \pm 0 \cdot 00171 \text{ s.e. (km/sec)}^{-2}$$

$$Z = 0 \cdot 00655 \pm 0 \cdot 00128 \text{ s.e. (km/sec)}^{-2}$$

$$Y = 0 \cdot 00067 \pm 0 \cdot 00058 \text{ s.e. (km/sec)}^{-2}/\text{kpc}$$

$$T = 0 \cdot 00625 \pm 0 \cdot 00237 \text{ s.e. (km/sec)}^{-2}/\text{kpc}.$$

All quantities, except Y which is small, are fairly well determined. They give the following best values of the required quantities:

$$\Sigma_1^{(0)} = 14 \cdot 31 \text{ km/sec}, \qquad \Sigma_2^{(0)} = 12 \cdot 35 \text{ km/sec},$$

$$a = -0 \cdot 0685/\text{kpc}, \qquad b = -0 \cdot 4765/\text{kpc}.$$

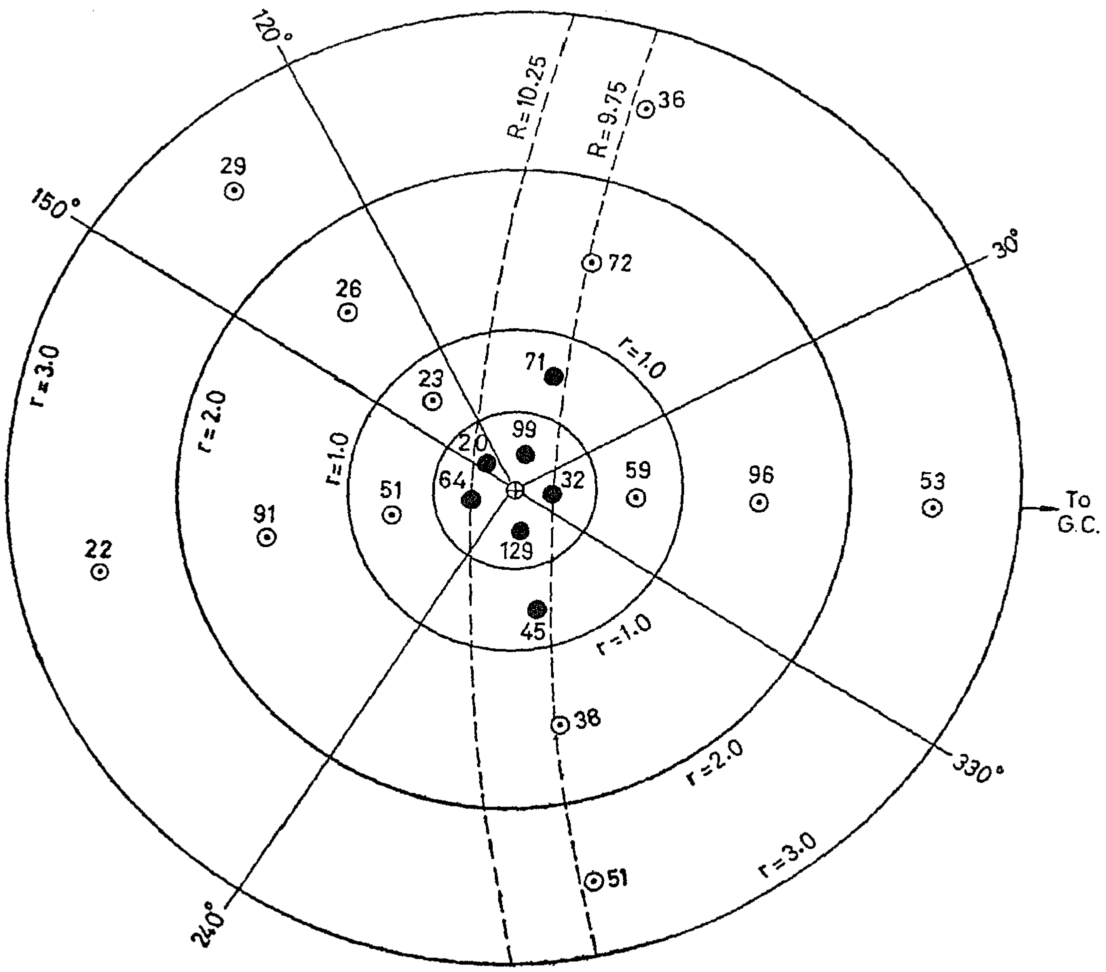

Figure 2. Zones in which the dispersion in the radial velocities of O–B5 stars has been determined by Shatsova (1970). The figure near each central point represents the number of stars within the zone.

Both a and b are small and negative, which indicates that the velocity ellipsoid shrinks as we go away from the galactic centre. Substitution in equations (2) gives the values of Σ_1 and Σ_2 and their ratio Σ_2/Σ_1 shown in table 1.

4. Conclusion

From table 1 we see that in the inner regions of the galaxy, $R < R_0 = 10$ kpc, $\Sigma_2/\Sigma_1 \approx 1$ which indicates that the force law there corresponds to a point within a homogeneous ellipsoid. On the other hand, at $R = 11$ kpc we have $\Sigma_2/\Sigma_1 \approx \frac{1}{2}$ which is in good agreement with the inverse square law of force in these outer regions. We have thus obtained an independent verification of the latter assumption.

Table 1. Variation of Σ_1 and Σ_2 with R.

R (kpc)	Σ_1 (km/sec)	Σ_2 (km/sec)	Σ_2/Σ_1
9	15·29	17·82	1·17
10	14·31	12·35	0·86
11	13.32	6·46	0·48

It might be argued that the above argument is somewhat circular because the inverse square law of force was already used in the calculation of Oort's terms of galactic rotation while deriving the velocity dispersion. However, it is gratifying that our result concerning the ratio of the axes of the velocity ellipsoid is consistent with the assumed inverse square law of force in the outer regions of the galaxy.

Finally we wish to draw attention to the generality of the present method of determining the force law. It can be profitably applied when the data about radial velocities and proper motions become available for stars at large distances from the Sun.

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