CALCULATIONS ON THE CASCADE THEORY WITH COLLISION LOSS

By H. J. Bhabha, F.R.S., and S. K. Chakrabarty

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In a recent paper* we gave an exact solution of the equations of the cascade theory assuming the radiation and pair creation processes to be described by the Bethe-Heitler cross-sections for complete screening and taking the collision loss β to be independent of the energy. The solution is in the form of an infinite series, but unlike all the previous solutions, it is *not*, a simple series in powers of the collision loss β, since β enters essentially into the expression for each term. The solution satisfies the physically required boundary conditions at the surface of the layer exactly, whereas the previous treatments of collision loss given by Snyder (1938) and Serber (1938), and following them Corben (1941), are all faulty inasmuch as their solutions do not satisfy the correct boundary conditions at the surface. Moreover the method used by these authors for calculating the total number of particles is mathematically fallacious, so that the numerical results obtained by them must be regarded as uncertain.† Besides this their method is unsuitable for calculating the energy spectrum of particles in a shower near and below the critical energy.

The solution of our previous paper has the advantage that it is particularly suited for obtaining numerical results, since the series is so rapidly convergent that it is sufficient in general to calculate the first term only. Even the second term is small compared with the first both for particles above and below the critical energy, except at thicknesses so large that the cascade has been almost completely absorbed. Thus the first term alone gives to a very good approximation the whole energy spectrum of particles in a cascade from the lowest to the highest energies. In particular it shows that the energy spectrum at small thicknesses, where our method is particularly good, is quite different from what has been generally supposed and differs completely from the spectrum given by Arley (1938) which is based on inadequate physical assumptions.

In this paper we use the solution of the previous paper for calculating the total number of particles of all energies at any depth produced by a primary

* Communicated to the Royal Society and referred to in this paper as A.

† A detailed discussion of their solution is given in A.