Variable property analysis – is there anything to it?

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Abstract. This paper discusses a few situations in combusting flows and attempts to demonstrate that including variable thermodynamic and transport properties in the analysis does more than simply improve the accuracy of predictions. The qualitative behaviour of the result itself is altered. Three examples are considered – single droplet combustion, forced convective turbulent boundary layer combustion, and free convective combustion.

The flame to droplet radius ratio is very well predicted by variable property theory and the improvement is a direct consequence of the property variation. In the case of turbulent boundary layer combustion of solid/liquid fuels it is shown that the fuel exerts a significant influence on regression rate as is found in the experiments. The constant property theory, however, shows relative independence of regression rate with regard to the nature of the fuel. The prediction of regression rate is improved substantially in the case of free convective combustion.

Keywords. Variable property analysis; single droplet combustion; forced convective turbulent boundary layer combustion; free convective combustion; regression rate.

1. Introduction

In many areas like fluid mechanics and aerothermochemistry the impression of a large number of workers is that the role of variable thermodynamic and transport properties is essentially cosmetic. Any reasonable choice of the constant property is adequate to describe the physics. The use of variable properties may improve the predictions. On the other hand, there is an impression amongst some that computer calculations including all effects is a sure cure for all bad comparisons between theory and experiments.

There is a middle path. It consists in making matters just as complex as is desirable to describe the behaviour appropriately. The same statement can be

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expressed differently: Matters can be made as simple as nature would accept. Three situations where it has been necessary to do this are described herein.

2. The droplet combustion problem

The classical problem which has received attention for over forty years is as follows: An atomised spherical droplet of fuel is burning in an atmosphere of oxidant (like air). It could burn in a normal gravitational environment as it does in many applications. Basic experimental studies on the combustion of droplets in zero-g environments have also been conducted. The measured parameters are the instantaneous drop diameter vs. time, flame diameter vs. time and initial diameter vs. time of burn-out. Typical measurements of combustion show that the flame to drop diameter ratio rarely exceeds 8–10 for many hydrocarbon liquids like heptane, octane and so on. The mass burn rate is expressed in terms of an evaporation constant which is the slope of (drop diameter)^2 versus time plot.

The theory consists of the solution of conservation equations of mass, species and energy. The momentum equation is generally not considered as it leads to negligible variations in pressure and velocity of flow. If thermodynamic and transport properties are taken to be constant, one obtains from a simple quasi-steady theory (SQST)

$$\dot{m} = 4\pi (k/c_p)_{av}\, r_s \ln(1+B), \tag{1}$$

$\dot{m}$ = mass burn rate (kg/s), $(k/c_p)_{av}$ = ratio of conductivity to specific heat of gas at average conditions (kg/m·s), r_s = drop radius (m),

$$d_f/d_s = \ln(1+B)/\ln(1+\beta/s), \tag{2}$$

$$B = [c_p(T_\infty - T_s) + H\beta/s]/Q,$$

T_∞, T_s = ambient and surface temperatures (K), β = oxidant mass fraction at infinity, s = stoichiometric ratio, H = heat of combustion per unit mass of fuel (MJ/kg) and flame temperature

$$T_f = [(c_p T_s - L + H)/s + c_p T_\infty]/[c_p(1 + \beta/s)]. \tag{3}$$

Values of $\dot{m}$ (or K, which is burning constant), d_f/d_s (flame diameter/droplet diameter) and T_f have been calculated for a few cases and compared with measurements under various conditions as in table 1. Of the three measurables, $\dot{m}$ (or K) is most successfully predicted by SQST. The success of SQST has been in isolating the final functional dependence of $\dot{m}$ on r_s or equivalently, the d^2-law which has been observed in the experiments. But the quantitative evaluation of $\dot{m}/r_s$ (or K) involves the judicious choice of thermodynamic and transport propeerties. One should conclude from the correlation of SQST predictions with experimental results obtained under natural convection (which is larger than under zero-g) that the prediction of $\dot{m}$ is an overestimation. In fact, if the value of (k/c_p) is evaluated at the mean temperature between the drop surface and the flame, the estimated burn rate will be significantly larger than is actually observed (Aldred & Williams 1966).

Similarly, the value of (d_f/d_s) assigned by SQST is also an example of gross overestimation (see table 1). Because of its logarithmic dependence on B, d_f/d_s has

Table 1. General comparison of experimental data with SQST (n-heptane).

Technique	Condition	d_0 (mm)	$k = 4\dot{m}/$ $\pi \rho_s d_s$ (mm^2/s)	d_f/d_s	T_f (K)	Reference
Stationary suspended drop	Unsteady natural convection	1·5	1·1	3·0	1800	Hedley *et al* (1971); Aldred & Williams (1966)
Porous burner	Steady natural convection	12·0 1·5	1·8 1·04	1·35 1·8	2000	Aldred & Williams (1966)
Drop in zero-g	Unsteady no convection	0·95	0·78	6–10		Isoda & Kumagai (1959)
Theory (SQST)	Quasi-steady no convection	Any size	1·1	29·0	2300	

negligible dependence on the chosen mean thermodynamic and transport properties, (2). Furthermore, d_f/d_s is considered to be independent of the diameter of the droplet inspite of the fact that the experimental observation has recorded that the initial movement of the flame is away from the droplet, followed by a gradual decrease in size as the drop continues to shrink steadily (Isoda & Kumagai 1959). These aspects deserve a careful reexamination.

Referring to table 1, the theoretically predicted values of T_f are again seen to be considerably larger than the experimental values (the free convection effect does not alter the flame temperature significantly). Although SQST predicts the adiabatic flame temperature, (3) predicts a strong dependence of T_f on c_p. While this ambiguity can be easily eliminated by considering the exact variation of c_p in the field, the inclusion of kinetics and unsteadiness becomes a necessary prerequisite for the realistic prediction of T_f.

Such discrepancies between the predictions of SQST and the experimental results have been attributed to various factors: finite kinetics, unsteadiness of the combustion process, variations of thermodynamic and transport properties, and natural convection. Of these, the effects of natural convection are not considered for the present due to the discrepancies which exist between SQST predictions and data from zero-g experiments. Lorell *et al* (1956) show through numerical calculation of the effect of single-step kinetics that for justifiable constants the effects of kinetics are small. It can therefore be concluded that the explanation for the above stated discrepancies must be in either the *nonsteady nature or variable properties*.

The analysis carried out by Raghunandan & Mukunda (1977) showed that the transient effects of drop-heating get completed in 20–25% of the burn time. Coupled transient heating and gas phase behaviour have confirmed that steady state results are recovered after this period of time. The variable property effects were investigated in some detail. The conservation equations were non-dimensionalised and transformed by

$$\eta = \exp \int (\dot{m} c_p)/(4\pi r^2 k) \, \mathrm{d}r. \tag{4}$$

It is worth noting that the original equations are written in r and later transformed into η. The solutions are set out in terms of η and then, the reverse transformation,

$$1/r = (4\pi)/\dot{m} \, J(\eta). \tag{5}$$

$$J(\eta) = \int_\eta^1 (k/c_p) \, (\mathrm{d}\eta/\eta),$$

was used to obtain (d_f/d_s) as

$$(d_f/d_s) = J(\eta_s)/J(\eta_f), \tag{6}$$

$$K_c = (\delta/\rho_c) \, J(\eta_s), \tag{7}$$

$$T_f = [\, T_\infty/I_2(1) + T_s/I_1(\eta_f) \, \exp P(\eta_s, \eta_f) +$$

$$+ (a_1 h_{1f} + a_2 h_{2f} + a_3 h_{3f})/(\eta_f a_1)\,] \div$$

$$\div \{1/I_2(1) + [1/I_1(\eta_f)] \, \exp P(\eta_s, \eta_f)\}, \tag{8}$$

where,

$$I_1(\eta) = \int_{\eta_s}^\eta \{(1/c_p) \, \exp[P(\eta_s, \eta)]\} \, \mathrm{d}\eta,$$

$$I_2(\eta) = \int_{\eta_f}^\eta \{(1/c_p) \, \exp[P(\eta_f, \eta)]\} \, \mathrm{d}\eta,$$

$$P(\eta_1, \eta_2) = -\int_{\eta_1}^{\eta_2} \sum_1^4 (c_{pi}/c_p) \, Le_i \, (\mathrm{d}Y_i/\mathrm{d}\eta) \, \mathrm{d}\eta.$$

The problem was solved for a single-step fast reaction with four species, fuel, oxidiser, product and inert. For n-heptane class of compounds, the products are CO_2 and H_2O. N_2 forms the inert. The species mass fractions in the product which is otherwise treated as a single species is assigned to the various product species in terms of stoichiometric mass proportions. The solution provides the distribution of mass fractions of various species and temperature. The transport properties are calculated with exact relations for these from Svehla (1962). The variation of properties is shown in figure 1. One can notice that (k/c_p) varies from 2×10^{-4} to 10×10^{-4} g/cm·s. Thus one would expect the results of variable properties not to be the same as those of constant properties.

The results of calculation and comparison are presented in table 2. In the case of constant property theory one has to choose a value of k and c_p. A value of $k = 1.2 \times 10^{-4}$ cal/cm·s·K is chosen to make predictions of K realistic. This corresponds to a temperature of 670–700 K for the mixture. This temperature is far below the mean of T_f and T_s. Table 2 also demonstrates that the improvement in the prediction of d_f/d_s is significant when variable c_p and k are employed. The value of $K = 0.72$ mm² agrees well with observations and the predicted flame temperature is the same as the adiabatic flame temperature. The values of d_f/d_s and T_f are still
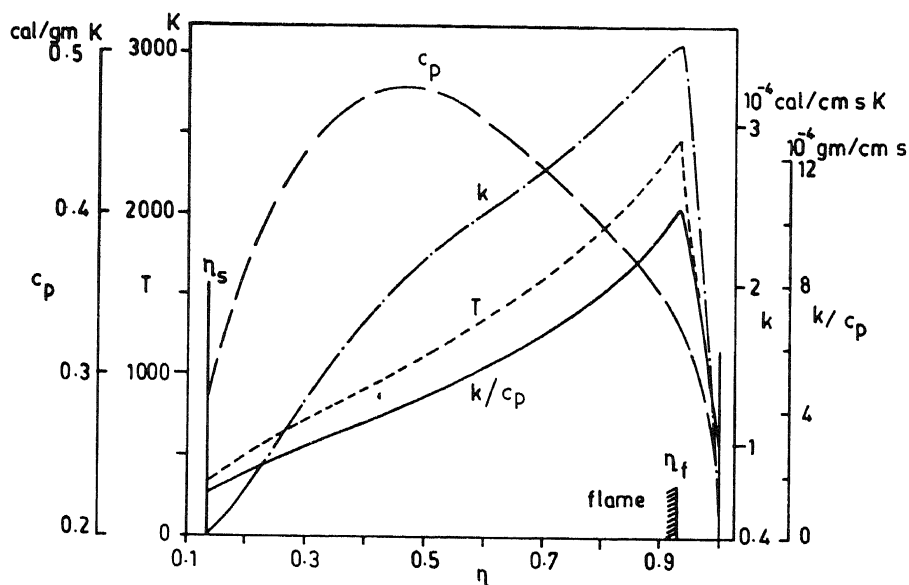
Figure 1. Variations of T, c_p, k & k/c_p with η-variable property analysis.

larger than observed (see table 1) and these are considered to be related to the treatment of finite kinetic calculations. These calculations of a meaningful nature are yet to be carried out. But the important feature is that the use of variable properties essentially entails computing integrals related to these in a rather straightforward manner and leads to improvement in result not related to any additional physical feature.

3. Forced convection turbulent boundary combustion

The physical flow field is simply a flat plate of solid fuel (could be liquid fuel as well) with gaseous oxidant flowing past. On ignition a flame will stabilize inside the boundary layer and fuel regression will take place. The question is to determine the dependence of the steady fuel regression rate on the thermochemical properties of the fuel-oxidant system. This geometry is of importance in hybrid rocket engine operation where oxidant flowing through the circular port of the fuel tube leads to combustion in the fuel-oxidiser interface region. A theory using turbulent boundary layer concepts was set out by Marxman and co-workers (Marxman &

Table 2. Comparison of results from various approximations

	k Constant c_p Constant	k Variable c_p Constant	k Constant c_p Variable	k Variable c_p Variable
$K(\mathrm{mm}^2/\mathrm{s})$	0·7193	1·11	0·543	0·723
d_f/d_s	28·3030	23·87	20·900	15·300
T_f (K)	2548·0000	2548·00	2506·000	2506·000

$k = 1·2 \times 10^{-4}$ is chosen; $c_p = 0·304$ is chosen.

Gilbert 1963; Marxman *et al* 1964; Marxman 1965; Marxman & Wooldridge 1968). Invoking Reynolds analogy and using the results of skin friction for zero injection they obtained an expression for regression rate of hybrid fuels as

$$\rho_p \dot{r} = 0.03 \, \mathrm{Re}_x^{-0.2} \, G(c_f/c_{f0}) \cdot B. \tag{9}$$

This result delineates the effects of fluid mechanics and chemistry. The effect of fluid mechanics is represented by the factor $\mathrm{Re}_x^{-0.2} G$ and thermochemistry by $B(c_f/c_{f0})$. The ratio c_f/c_{f0} represents the reduction in skin friction or heat transfer due to injection (called the blocking effect). The term was evaluated by Lees (1958) from simple film theory as

$$(c_f/c_{f0}) = [\ln(1+B)]/B. \tag{10}$$

Marxman & Gilbert (1963) used (10) to obtain c_f as

$$c_f = 0.0225 \, [\ln(1+B)]/B \, \mathrm{Re}_\delta^{-0.25}, \tag{11}$$

where Re_δ is the Reynolds number based on boundary layer thickness. Equation (11) is substituted in the momentum integral equation, when an expression for c_f/c_{f0} in a log-log plot leads approximately to

$$c_f/c_{f0} = 1.2B^{-0.77}. \tag{12}$$

To check this result Marxman (1965) compared it with the experimental results of Mickley & Davies (1957) and Pappas & Okuno (1960). The comparison between predictions of (12) and the results of the above workers was shown to be satisfactory.

Substitution of (12) into (9) leads to the final result

$$\rho_p \dot{r} = 0.036 \, G \, \mathrm{Re}_x^{-0.2} \, B^{0.23}. \tag{13}$$

It is seen from this equation that $\rho_p \dot{r}$ is relatively insensitive to B due to the low exponent (0.23) on B. This constitutes one of the principal results of the theory stressed by Marxman and co-workers. The above theory was compared with the experimental results of the Plexiglas-oxygen system obtained at ambient pressure in two-dimensional burners and the comparison was claimed to be satisfactory. The estimates of B were made for essentially diffusion-limited combustion in the above comparison. The aspect of diffusion-limitedness does not affect the basic nature of (13) but only alters the value of B which is influenced by the wall oxidiser concentration. Wooldridge *et al* (1969) in an elaborate study, not often referred to in the open literature, have presented extensive and useful data on the regression rate at various pressures obtained for Plexiglas (PMMA)*, polyurethane (PU) and PBAN* in a cylindrical rocket engine. Two of these are presented as fuel weight loss vs. burn time data as shown in the figure 2. The authors claim good comparison between the theory and experimental results at high pressures (about 215–265 psi, 1 psi $= 6.895 \times 10^3$ Pa). The fuel weight loss at low pressures (~ 45 psi) is substantially less than at 215 psi. It is to be recalled as noted earlier that good comparison between diffusion limited theory and near ambient pressure experiments were reported in Marxman & Gilbert (1963) and Marxman (1965). Thus one is led to the anomalous situation where the same theory for diffusion-limited conditions is said to have compared well with numerically widely different results.

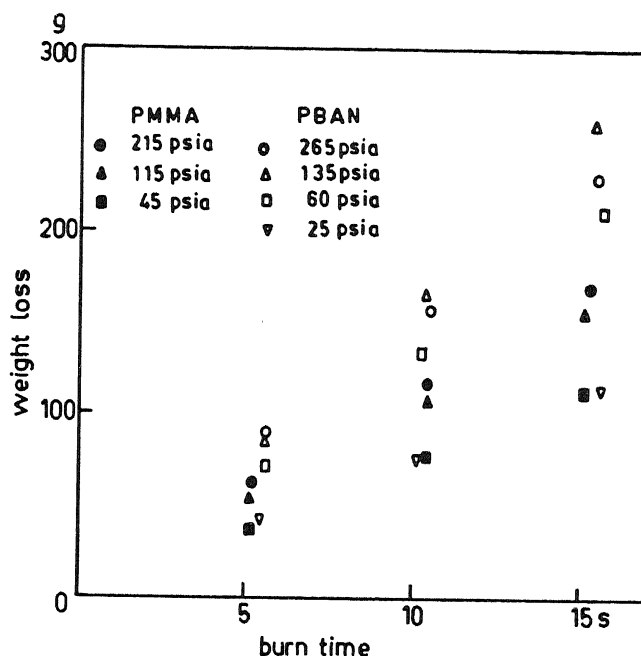PMMA–polymethylmethacrylonitrile; PBAN–polybutylacrylonitrile.

Figure 2. Results of Wooldridge *et al* (1969) on PBAN and PMMA-O_2 systems.

The situation has been carefully analysed by examining the work of Wooldridge *et al* (1969). It appears that the mass flux G used in the theoretical estimate goes up to 1·6 times the mean mass flux. The motivation for the use of increased flux comes from the fact that in the relation for $\rho_p r$ derived from boundary layer analysis, G is the free stream flux, and in an enclosed flow, by analogy the maximum flux rather than the average flux should be used. But it should be pointed out that the maximum flux in a fully developed pipe flow is only about 20% larger than the average flux. The factor used in the computer analysis of the data by Wooldridge *et al* (1969) increases to 60% in a distance of about 5 times the port diameter. This distance is based on the fact that the boundary layer growing on the wall will merge at about five times the port diameter. In actuality, the length for merger of boundary layers depends on the inlet Reynolds number. In view of the above, the claim of good comparison is not indeed justified. To further establish the laeuna of this theory it is worth examining the results of Wooldridge *et al* (1969) summarised in table 3. It can be observed that the predictions from Marxman's theory and the experimental results are widely different. Further, comparison of results for PBAN and PU with nearly identical B and at the same flux indicates a difference in mass regression rate of nearly 18%. Even if it is stated that the estimate of B needs modification to explain this variation in regression rate, the values of B for PBAN-O_2 and PU-O_2 would need to be different by a factor of two. This is too large a factor to be expected.

A further confirmation of the possible influence of B comes from the effect of pressure (figure 2). It is seen that the regression rate decreases by a factor of 0·7 for PMMA-O_2 and 0·5 for PBAN-O_2 when pressure is decreased from 215 to 45 psia and 265 to 25 psia respectively. Since pressure effects have to enter through

Table 3. Weight loss data of Wooldridge *et al* (1969) and theory

Fuel	B	G_{ox} Initial (g/cm²·s)	Observed fuel weight loss (g)	Fuel weight loss by Marxman's theory (g)	Molecular weight of gases near wall	Weight loss by present theory (g)
PMMA	9·3	7·03	160	130	90–100	180–190
PBAN	8·0	7·73	260	132	250–350	260–280
PU	8·0	7·73	220	132	150–250	220–250

Experiments with 1″ ID, 12″ long engine (Diffusion Limited) (1″ = 2·54 cm)

kinetics–largely of a gas phase nature in these nonhypergolic systems–their influence should be felt through B only. Again one can estimate the variation in B needed to account for the reduction of regression rate of these magnitudes. They turn out to be 4·7 and 20. These factors are too large to be realistic. Thus the examination of the work of Marxman and co-workers suggests that the law needs strong corrections particularly with the thermochemical factor.

Further evidence about it can be obtained by examining the relative regression rates of several fuels by two other groups of investigators – Blazowski *et al* (1975) and Julain *et al* (1978). The ordering of the fuels in terms of regression rate with oxygen is shown below.

Blazowski *et al*:   Delrin $\geqslant$ PMMA $\geqslant$ PP $\geqslant$ PE $\geqslant$ PSt

Julain *et al*:   PSt $\geqslant$ PMMA $\geqslant$ Delrin $\geqslant$ PP $\geqslant$ PE

(PP: polypropylene, PSt:polystyrene, PE = polyethylene).

The experiments of Blazowski are in the stagnation point apparatus and that of Julain *et al* (1978) in a two-dimensional flat plate burner. But the nature of the apparatus should not matter since the thermochemical influence is similar in both cases. The point to be noticed in the above ordering is that even qualitatively, fuels are classed differently by different workers. The mass regression rate ratio between extreme fuels is 1·6 and 2 for the two sets of workers respectively. It appears from these that even though fuels with the same name have been used by different workers, they would be chemically different because of antioxidants or additives which could affect regression.

Thus there is enough of a case made to state that the fuel does matter as far as regression rates are concerned and Marxman's theory is inadequate in describing this. After careful consideration of various aspects, Paul *et al* (1982) reasoned that the effect of wall injection on heat transfer (or skin friction) as evaluated in Marxman's theory does not include variation of density and other properties through the boundary layer. The theoretical considerations essentially set out to handle this led to

$$(c_f/c_{f0})^{1/2} = \int_0^1 \left[(\rho/\rho_\infty)\,(1/1 + B\zeta) \right]^{1/2} \, d\zeta. \tag{14}$$

Using the similarity between momentum and mass transfer, the expression for density ratio was put down as

$$\rho/\rho_\infty = \{1 - [B\rho_{\infty i}/(1+B)] \, (u/u_\infty) + (1 + B\rho_{\infty i})/(1+B)\}^{-1}, \qquad (15)$$

where $\rho_{\infty i}$ is the ratio of density of the free stream to that of injected species. Introducting (15) into (14) gives

$$(c_f/c_{f0})^{1/2} = 2\{(1+B)/B^2(1-\rho_{\infty i})\}^{1/2} \times$$
$$\times \ln\{(1+B)^{1/2}[1 + (1-\rho_{\infty i})^{1/2}]/[(1+B\rho_i)^{1/2} + (1-\rho_{\infty i})^{1/2}]\}$$
$$(16)$$

The predictions from this expression were checked with the experimental results of Romanenko & Kharchenko (1963) on the injection of CO_2 and Freon into air and found to be in excellent agreement (Paul *et al* 1982). The theory was then modified to include combustion and this leads to

$$\rho_p \dot{r} = 0 \cdot 056 \; G^{0 \cdot 8} \; (x/\mu)^{-0 \cdot 2} \; (\rho_f/\rho_\infty)^{0 \cdot 71} \; (\rho_w/\rho_\infty)^{0 \cdot 14} \times$$
$$\times B(1+B)^{-[0 \cdot 73 - 0 \cdot 002 \, (\rho_w/\rho_\infty)]}. \qquad (17)$$

This result shows that the dependence of $\dot{r}$ on B is stronger than is given by Marxman's expression. It is also stronger than is simply apparent in (17) because (ρ_w/ρ_∞) depends on B as well. A comparison of the predictions from the present theory and the experimental results are shown in table 3 and figure 3. As can be seen from both these, the success of the present theory is glaringly evident. Thus, it
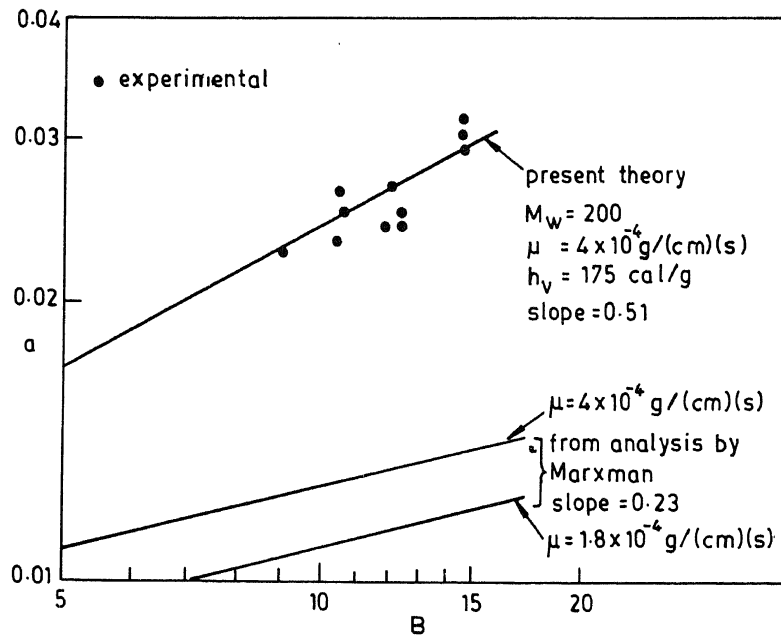

Figure 3. Results of a versus B from present experiments and theory as well as Marxman's theory.

is correct to summarize that the fuel exhibits a strong influence on the combustion rate and this feature is predicted by the theory which includes variable thermo-chemical properties.

4. Free convective combustion

A vertical flat fuel plate burning in a 1-g environment constitutes the free convective combustion situation. The evaluation of regression rate dependence on thermochemical properties is the central problem here. This subject has been treated by several workers. Kosdon *et al* (1969) solved the conservation equations for burning of vertical plates using similarity hypothesis. The predictions have been compared with experimental results on vertical cellulosic cylinders. While the flame stand-off is shown to behave qualitatively the same way ($y_f \sim x^{1/4}$) in both cases, quantitative comparison has called for adjustment of wall mixture properties suitably. The experimental stand-off behaves as

$$y_f \simeq 0 \cdot 0085 \, x^{1/4} \quad (x, y_f \text{ in m}),$$

and constant property. theory gives

$$y_f \simeq 0 \cdot 0135 \, x^{1/4} \quad (x, y_f \text{ in m}).$$

The correct prediction of mass burn rate calls for the use of 700 cal/g for the effective heat of vaporisation (L) and this is asserted by them to be reasonable for α-cellulose. Consistent adjustment of transport properties to yield better comparison of flame stand-off will in turn affect the mass burn rate and in fact it is not possible to meaningfully choose any set of parameters which permit simultaneous correct prediction of mass burn rate and flame stand-off within the frame work of constant property analysis.

Subsequently, Kim *et al* (1971) formulated the problem in a similar manner and carried through the solutions as a function of various parameters by treating Lewis number, $\rho\mu$ and ρk as constants in the field. They also developed an expression for mass burn rate using a profile technique. By far this development has been accepted in the combustion literature. They also obtained similarity solutions for various cases and compared their results with experiments. Table 4 summarizes the comparisons drawn from Kim *et al* (1971).

Table 4. Comparison of results of earlier theories and experiments

Fuel →	Benzene		Toluene		Methanol	
	Gr'	A	Gr'	A	Gr'	A
Experiment	$4 \cdot 8 \times 10^8$	167	$8 \cdot 5 \times 10^8$	150	10^8	100
Theory	$4 \cdot 8 \times 10^8$	244	$8 \cdot 5 \times 10^8$	271	10^8	120
Variation (%)	—	46	—	80		20

$$Gr' = g_0 \, \rho_w^2 x^3 / \mu_w^2; \quad A(x) = \left(1/\mu_w \right) \int_0^x \dot{m}'' \, dz.$$

As can be noticed, the predictions are different from theory by 45–80% in the case of benzene and toluene. In the case of methanol, the comparison is about satisfactory with a deviation of about 20%. In the case of PMMA-O_2 and α-cellulose-air, the theoretical predictions are coupled with experiments to determine the parameters B and L and to assess if these are reasonable. In the case of PMMA-O_2, B and L are obtained as 4·2 and 714·5 cal/g. In performing these calculations the authors (Kim *et al*) use a fuel surface temperature of 325°C and viscosity at wall equal to that of nitrogen at 395°C. This arbitrary choice of values is hard to justify. The parameters B and L from the literature (Krishnamurthy & Williams 1972) for Plexiglas are about 8 and 325 cal/g respectively. Thus the estimates of Kim *et al* (1971) are in substantial error. Stated differently, the predictions of burn rates for Plexiglas from its known properties would be in significant error [by a factor $\ln(1+8)/\ln(1+4\cdot18) = 1\cdot34$].

Kim *et al* (1971) argue that their markedly higher burn rate predictions for heavy fuels like benzene and toluene are related to low Lewis numbers in the fuel rich region. In the discussion of this point in the paper there are several statements which do not help resolution of the causes for the discrepancy. Thus, the two principal features of the experiments, namely the mass burn rate and flame stand-off, are inadequately explained by the current theories. Consequently, a theory based on free convection under diffusion limited conditions with thin flame approximation was evolved (Hegde *et al* 1986). A single-step reaction with possibility of four to five species (like in the earlier case of droplet combustion) was assumed. The governing partial differential equations were treated by similarly retaining the variation in thermodynamic and transport properties. The resulting differential equations are ordinary and of the ninth order. The boundary conditions are set out at the surface of the plate, at the thin flame location and at infinity. The three-point boundary value problem was solved by the quasilinearisation method. A comparison between the present theory, earlier theory and experiments is shown in table 5. It is noticed that the maximum error in the prediction of burn rate is about 15% and in most cases it is lower. The variable property theory predicts

Table 5. Comparison of results with experiments

Fuel	B	Constant properties		Variable properties		Experiment	% Error
		$y_f/x^{1/4}$ ($\times 10^3$)	$\dot{m}''_{av}$ ($\times 10^3$)	$y^f/x^{1/4}$ ($\times 10^3$)	$\dot{m}''_{av}$ ($\times 10^3$)	$\dot{m}''_{av}$ ($\times 10^3$)	Theory−experiment/experiment ($\times 100$)
Methanol	2·57	14·4	13·1	12·0	13·0	15·0	−13·3
Ethanol	2·73	16·9	15·3	12·1	13·0	15·0	−9·3
Acetone	4·61	18·7	20·2	12·8	15·6	15·9	−2·0
Benzene	5·58	21·4	24·4	13·9	16·7	16·5	1·2
Toluene	5·38	21·6	24·5	13·4	16·4	15·7	4·5
Xylene	5·10	21·7	23·5	13·6	15·4	14·3	7·7
Kerosene	3·15	21·2	20·1	11·6	14·1	16·5	−14·5
Cellulose	1·08	13·5	9·4	8·2	8·0	7·5	6·6
PMMA	1·69	15·6	12·1	10·2	11·2	10·4	7·9
PMMAl-O_2	7·79	20·1	29·0	10·3	23·1	24·0	3·8

Units for $y^f/x^{1/4}$, $m^{3/4}, m''_{av}$ are kg/m²s. Oxidant in all cases is air (except the last one).

much lower stand-off distance compared to the constant property theory. The flame stand-off distance obtained experimentally is $Y_f/x^{\frac{1}{4}} = 0.0085\ m^{\frac{3}{4}}$ and the variable property theory gives $Y_f/x^{\frac{1}{4}} = 0.0082\ m^{\frac{3}{4}}$. The improvement in prediction from the constant property theory can be seen to be dramatic. The good comparison of both burn rate and flame stand-off is due not to a single factor like Lewis number, but the complete treatment of variable properties.

5. Concluding remarks

This paper started with the assertion that it would demonstrate that accounting for variable properties does more than just improve the comparison between theory and experimental observations over the theories using constant properties, and would make the qualitative character of the predictions physically meaningful.

The example of forced convection turbulent boundary layer analysis demonstrates this. The conclusion arrived at by the simple constant property theory that the regression rate of the fuel is nearly independent of the thermochemical features of the fuel for a fixed oxidant is shown to be incorrect. Experiments show significant dependence on fuel characteristics and the variable property theory demonstrates this. Incidentally, a correlation for the regression rate as a function of various parameters has been established. The cases of droplet combustion as well as free convective plate combustion are those in which dramatic improvements in the quality of comparisons between theoretical and experimental results are seen. The flame to droplet radius ratio and flame stand-off distance are predicted quite accurately with no assumptions on free parameters. The burn rate also is better predicted by the variable property theory.

Dr B N Raghunandan, Dr P J Paul & Mr M Hegde have contributed to the understanding seen in this paper.

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