Domain Growth in the Field Theoretic Version of the Potts Model

Man-hot Lau, Chandan Dasgupta, and Oriol T. Valls

School of Physics and Astronomy
University of Minnesota
Minneapolis, MN 55455

Abstract

We present a numerical study of the domain growth kinetics of the continuous, two-dimensional, q-state Potts model with a non-conserved order parameter. After the system is quenched from a high temperature disordered state to a finite temperature in the ordered region of the phase diagram, the development of ordered domains in time is analyzed by direct numerical solution of the associated Langevin equations. We find that the domain growth in the model, after a short initial transient time, is well-described by a characteristic length L(t) which increases with time as $L(t) \propto t^n$. The kinetic exponent n is determined to be 1/2 for both q = 4 and q = 15. The quasistatic structure factor $S(\vec{k},t)$ obeys dynamical scaling during the growth process, as expected. Our results are in agreement with those obtained from recent Monte Carlo studies of the discrete Potts model with stochastic dynamics, and indicate that the domain growth kinetics of the two-dimensional q-state Potts model is characterized by a Lifshitz-Allen-Cahn kinetic exponent n = 1/2, independent of q for a non-conserved order parameter.

I. INTRODUCTION

There has been recently a renewed surge of interest in the study of non-equilibrium phenomena through the use of field-theoretic methods with a Ginzburg-Landau free-energy and Langevin dynamics (see for example Refs. 1-7). These numerical studies have complemented the conventional Monte Carlo (MC) simulation approach involving discrete lattice-gas models and stochastic dynamics[8]. It has been found that Langevin equation methods have several advantages. From the numerical point of view, vectorization techniques often make them competitive[3] with lattice-gas dynamics. It appears also that transient effects are, at least in some cases[6], less severe in Langevin than in stochastic dynamics. An important advantage of Langevin methods is that they are more amenable to theoretical study. Indeed, the earlier development of the field of growth kinetics was dominated by field-theoretic models[9], and recent theoretical developments[10] along these lines have taken place. Therefore, a field theory approach to these problems is likely to be an important step in establishing an analytic theory of growth kinetics.

In this paper, we study the domain growth kinetics of the field theory version of the two-dimensional (2D) q-state Potts model. This field theoretic approach to the Potts model was first proposed by Zia and Wallace[11]. Using this approach, the static critical properties of this system near six dimensions were investigated by perturbative renormalization group calculations and the model was shown to exhibit a first order phase transition for general $q \geq 3$[11,12]. The non-equilibrium dynamics of this model, however, has not been investigated. We have studied the domain growth kinetics of this model in two dimensional space for $q = 4$ and $q = 15$.

While we do not know of any physical system corresponding to the case q = 15, the 2D q = 4 model can be realized in systems in which O_2 is adsorbed on the surface of nickel and N_2 is adsorbed on krypton-plated graphite[13]. There is also a recent experimental study[14] of the growth of domains in Cu_3Au alloys. The late stage coarsening of the system after a quench to a low temperature is consistent with a curvature driven growth, i.e., $L(t) \propto t^{1/2}$, where L(t) is the characteristic size of ordered domains at time t. The Cu_3Au system has a q = 4 fold degenerate ground state, but it is not a simple Potts system as discussed here because of the existence of more than one kind of domain walls[14].

The kinetics of domain growth in the conventional 2D q-state Potts model with stochastic dynamics when it is suddenly quenched to a finite temperature below the ordering temperature T_c has been the subject of many studies[15,19]. From these we know that the characteristic domain size increases algebraically with time as $L(t) \sim t^{n(q)}$, where n(q) is called the kinetic exponent. In the case of non-conserved order parameter, the q = 2 limit of the model (the Ising model) gives n(q) = 1/2. This is a well established result[16] from analytical[17] and computer[18] studies as well as experimental investigations. When q > 2, however, the value of n(q) has been a matter of some dispute. Initial studies[15] reported a n(q) value that depends on q, with n(q) decreasing from 0.5 for q = 2 to $\simeq$ 0.4 in the large-q limit. However, there are indications that the results obtained in these computer simulations may not be really "asymptotic" and thus do not describe the true long-time behavior of the model for q larger than two. Grest et al.[19] have recently obtained $n(q) \simeq 0.5$ with q > 30 in computer simulations of large samples over considerably longer times. This recent result is,

thus, in contradiction with earlier studies[15,20] which argued that the domain growth should be slower in the 2D Potts model with $q > 2$ due to the presence of vertices, pinning effects, and domain wall interactions.

It is a premise of our present study that through the use of Langevin dynamics, one can follow the evolution of the long-time growth of the system more easily than with conventional spin-flip dynamics. This premise is based on the expectation that the energy barriers one has to overcome while flipping spins in the conventional dynamics may not be present in Langevin dynamics. This is important in numerical studies of long time behavior within limited computer time. Whether our results for $n(q)$ in the asymptotic regime would apply to spin-flip dynamics depends on whether the two kinds of dynamics belong to the same "universality class". There is no rigorous proof that this is the case, although recent studies[3] have shown it to be true for $q = 2$ and nonconserving dynamics. To the extent that this may be a general result, some of the growth kinetics problems which are very difficult to resolve by conventional Monte Carlo studies can possibly be addressed by field theoretic methods. Thus, if the Langevin dynamics and the spin-flip dynamics belong to the same "universality class", our field theoretic study would help to remove some of the controversies still remaining on the question of the growth kinetics of the 2D q-state Potts model.

We summarize our results as the following: We quenched the field theoretic 2D q-state Potts model with non-conserved order parameter from a high temperature disordered state to a temperature far below the phase transition temperature. The evolution of the ordered domains in time was then monitored by solving the associated Langevin equation numerically. The

domain growth of the modeled system, after a short transient time, is well described by a characteristic length L(t), which varies in time as $L(t) \propto t^{1/2}$ for both q = 4 and q = 15. All dynamical quantities such as the susceptibility $\chi(t)$, the structure factor $S(\vec{k},t)$ ($\vec{k}$ is the wave vector), and the first moment of $S(\vec{k},t)$ exhibit time-dependences similar to those found in the Ising model[3]. Also $S(\vec{k},t)$ exhibits the expected dynamic scaling behavior. We have obtained the scaling function for this quantity, and found it to be very similar to that of the Ising model. Thus, our study shows that the field theoretic 2D q-state Potts model with non-conserving Langevin dynamics belongs to the 2D Ising model "universality class", characterized by a curvature-driven growth with a Lifshitz-Allen-Cahn growth exponent n = 1/2, independent of q.

The rest of this paper is organized as follows: in Section II we discuss the model and its associated Langevin equations. The results of our numerical calculations are presented in Section III and the conclusions are recapitulated in Section IV.

II. THE MODEL AND ITS LANGEVIN EQUATIONS

We consider the q-state Potts model in the field theoretic representation proposed by Zia and Wallace[11]. The free energy functional is written as:

$$F[\vec{\phi}] = \sum_i \left[\frac{1}{2} K' |\vec{\nabla}\vec{\phi}(i)|^2 - \frac{1}{2} r|\vec{\phi}(i)|^2 + \frac{1}{3!} g_3 Q_{\delta\mu\nu} \phi_\delta(i)\phi_\mu(i)\phi_\nu(i)\right.$$

$$\left. + \frac{1}{4!} (u S_{\delta\zeta\mu\nu} + f V_{\delta\zeta\mu\nu}) \phi_\delta(i) \phi_\zeta(i) \phi_\mu(i) \phi_\nu(i)\right] ,$$

$$\delta, \zeta, \mu, \nu = 1, 2, \ldots, q-1, \tag{2.1}$$

where $\vec{\phi}(i)$ is a real vector field with q-1 components, defined on a 2D square lattice site i, and repeated greek indices are summed over. The tensors Q, S, and V are of the form:

$$Q_{\delta\mu\nu} = \sum_{\alpha} e_{\delta}^{\alpha} e_{\mu}^{\alpha} e_{\nu}^{\alpha}, \tag{2.2a}$$

$$S_{\delta\zeta\mu\nu} = \frac{1}{3} (\delta_{\delta\zeta} \delta_{\mu\nu} + 2 \text{ permutations}), \tag{2.2b}$$

$$V_{\delta\zeta\mu\nu} = \sum_{\alpha} e_{\delta}^{\alpha} e_{\zeta}^{\alpha} e_{\mu}^{\alpha} e_{\nu}^{\alpha}, \tag{2.2c}$$

with $\{\vec{e}^{\alpha}\}$ being a set of q vectors in (q-1) - dimensional space which satisfy:

$$\sum_{\alpha} e_{\mu}^{\alpha} = 0; \qquad \sum_{\mu} e_{\mu}^{\alpha} e_{\mu}^{\beta} = \frac{q\delta^{\alpha\beta}-1}{(q-1)}; \qquad \sum_{\alpha} e_{\mu}^{\alpha} e_{\nu}^{\alpha} = \frac{q\delta_{\mu\nu}}{q-1}$$

$$\alpha, \beta = 1, 2, \ldots, q,$$

$$\mu, \nu = 1, 2, \ldots, q-1 \quad . \tag{2.3}$$

This set of $\vec{e}$'s can be visualized as a set of vectors defining the q vertices of a hypertetrahedron in (q-1)-dimensional space. For a proper choice of the values of the parameters, the global minima of the free energy functional occur for $\vec{\phi}$ along any one of these $\vec{e}$ directions, giving a q-fold degenerate ground state. The static properties of this model have been

studied by applying ϵ-expansion ($\epsilon = 6 - d$, where d is the spatial dimension) renormalization group methods[11,12]. Results from these studies show that the trilinear term in ϕ, $g_3 Q_{\delta\mu\nu} \phi_\delta \phi_\mu \phi_\nu$ leads to a first order phase transition for $q \geq 3$ (notice that $Q_{\delta\mu\nu}$ vanishes when $q = 2$, which corresponds to the Ising model). Such a first order phase transition due to a trilinear term is well known in the context of the mean-field theory of the q-state Potts model[21]. Therefore, the trilinear term in the free energy is important in the static and presumably, also in the dynamic behavior of the model. The quartic terms in $F[\vec{\phi}]$ give conventional symmetric couplings $(\vec{\phi}^2)^2$ for both $q = 2$ and $q = 3$ and cubic symmetric couplings for $q = 4$. For $q \geq 5$, however, a symmetric correspondence is hard to find, as pointed out by Zia and Wallace[11].

Since the overall scale of $|\vec{\phi}|^2$ is arbitrary, only four of the five parameters K', r, g_3, u, and f in $F[\vec{\phi}]$ are independent. For this reason, we can choose to write (2.1) in the following way:

$$F[\vec{\phi}] = \frac{K}{2} \{\sum_i [- \theta \, |\vec{\phi}(i)|^2 + |\vec{\nabla} \vec{\phi}(i)|^2 +$$

$$+ \frac{1}{3} g_3' Q_{\delta\mu\nu} \phi_\delta(i) \phi_\mu(i) \phi_\nu(i) +$$

$$+ \frac{1}{6} (1 + \theta) (S_{\delta\zeta\mu\nu} + f' V_{\delta\zeta\mu\nu}) \phi_\delta(i) \phi_\zeta(i) \phi_\mu(i) \phi_\nu(i)]\}. \qquad (2.4)$$

Here we have chosen the parameters in a similar fashion as in Ref 3: the choice involves a rescaling of every component of $\vec{\phi}$, i.e., $\phi_\mu^2(i)$ is scaled by a factor $u/2(|r| + K')$. The relations between the new and old parameters are then $\theta = |r| / K'$ and $K = 2K'^2 (1 + \theta)/u$. The coefficient of the

trilinear term is simply written down as g_3' since its actual value still remains to be chosen. The parameter f' is now equal to f/u.

The dynamics of the model is generated by a Langevin equation:

$$\partial\phi_\delta(i,t)/\partial t = -\Gamma\ \partial F[\vec{\phi}(i,t)]/\ \partial\phi_\delta(i,t) + \eta_\delta(i,t) \tag{2.5}$$

where Γ is a kinetic coefficient. We consider the case of a non-conserved order parameter, so that Γ is a constant which can be set equal to unity by an appropriate choice of the time unit, and $\eta_\delta(i,t)$ is a Gaussian noise term satisfying

$$\langle\eta_\mu(i,t)\ \eta_\nu(j,t')\rangle = 2\Gamma T\delta_{\mu\nu}\ \delta_{ij}\delta(t - t') \tag{2.6}$$

where T is the final quenching temperature. Combining (2.5) with (2.4), we have

$$\begin{aligned}\partial\phi_\delta(i,t)\ /\partial t = -\Gamma\ K\{ & -\theta\ \phi_\delta(i,t) - \nabla^2\phi_\delta(i,t) + \\ & + \frac{1}{2}\ g_3'\ Q_{\delta\mu\nu}\ \phi_\mu(i,t)\ \phi_\nu(i,t) + \\ & + \frac{1}{3}\ (1 + \theta)\ [\phi_\delta(i,t)\ |\vec{\phi}(i,t)|^2 + f'\ V_{\delta\zeta\mu\nu}\phi_\zeta(i,t)\ \phi_\mu(i,t)\ \phi_\nu(i,t)]\} \\ & + \eta_\delta(i,t)\end{aligned} \tag{2.7}$$

where ∇^2 is the usual discrete version of the Laplacian:

$$\nabla^2\phi_\delta(i) = \sum_{\vec{a}}\ (\phi_\delta(i + \vec{a}) - \phi_\delta(i)). \tag{2.8}$$

Here $\vec{a}$ is a nearest neighbor lattice vector, and we have chosen the unit of length to be the lattice constant a. Equation (2.7) could be used directly for the temperature quenching study. However, the presence of the tensors Q and V is inconvenient in the numerical calculation. To eliminate them, we introduce[11] the order parameter fields ψ^α, $\alpha = 1, 2, \ldots q$, obtained by projecting $\vec{\phi}$ onto the q unit vectors $\vec{e}^\alpha$:

$$\psi^\alpha = \vec{\phi} \cdot \vec{e}^\alpha = \sum_\mu \phi_\mu \, e^\alpha_\mu . \tag{2.9}$$

Equation (2.7) can then be written in the following form, with ψ^α instead of $\vec{\phi}$:

$$\begin{aligned} \partial\psi^\alpha(i,t)/\partial t = -\Gamma K\{ & -\theta\psi^\alpha(i,t) - \nabla^2\psi^\alpha(i,t) + \\ & + \frac{1}{2} g_3' \left[q \left(\psi^\alpha(i,t)\right)^2/(q-1) - \sum_\beta \left(\psi^\beta(i,t)\right)^2/(q-1)\right] + \\ & + (1+\theta)(q-1)\,\psi^\alpha(i,t) \sum_\beta \left(\psi^\beta(i,t)\right)^2/3q + \\ & + \frac{1}{3}(1+\theta) f' \left[q\left(\psi^\alpha(i,t)\right)^3/(q-1) - \sum_\beta \left(\psi^\beta(i,t)\right)^3/(q-1)\right]\} \\ & + \eta'_\alpha(i,t). \end{aligned} \tag{2.10}$$

We note that the $\psi^\alpha(i,t)$, $\alpha = 1, 2, \ldots, q$ are not all independent. From (2.3) and (2.9), it is clear that:

$$\sum_\alpha \psi^\alpha(i,t) = 0. \tag{2.11}$$

Also, the new noise term is given by $\eta'_\alpha = \sum_\mu e^\alpha_\mu \eta_\mu$, so that:

$$<\eta'_\alpha(i,t)\ \eta'_\beta(i,t)> = \sum_\mu \sum_\nu e^\alpha_\mu\ e^\beta_\nu <\eta_\mu(i,t)\ \eta_\nu(i,t)>$$

$$= 2\Gamma T\ (q\delta_{\alpha\beta} - 1)/(q-1). \qquad (2.12)$$

Equation (2.10) is the final expression of the Langevin equation used in the calculation. It preserves the constraints given in (2.11) provided that the initial values of $\psi^\alpha(i,t)$ satisfy these constraints.

The information on the dynamical behavior was obtained from the calculated correlation functions

$$C_{\alpha\alpha}\ (i,j,t) = <\psi^\alpha(i,t)\ \psi^\alpha(j,t)> \qquad (2.13)$$

where the average is over the noise and initial probability distribution of $\psi^\alpha(i,t=0)$. Here, the time t is measured from the time of quench. We note that all relevant information about two-point correlations in the system is contained in these "diagonal" correlation functions. From (2.3) and (2.9) it is clear that:

$$C_{\vec{\phi}\vec{\phi}}(i,j,t) = <\vec{\phi}(i,t) \cdot \vec{\phi}(j,t)>$$

$$= \frac{q-1}{q} \sum_\alpha C_{\alpha\alpha}(i,j,t). \qquad (2.14)$$

Also, by using (2.11) it is possible to express the off-diagonal correlation functions $C_{\alpha\beta}(i,j,t)$, $\alpha \neq \beta$, in terms of the diagonal ones. Since we expect

that the long time behavior of the model will be independent of the initial conditions chosen for ψ^{α} in our problem, we chose to work with $\{\psi^{\alpha}(i, t = 0)\} = 0$ in all the calculations. Of course, this choice does not correspond to any true physical state, but to a somewhat artificial "quasi-disordered" configuration. However, it has the advantage of simplicity.

In the present work, we set $f' = 1$ and $g_3' = -1$. We chose a negative value for g_3' because this choice produces the correct ordered state of the standard Potts model, i.e., the global minima of $F[\vec{\phi}]$ occur for $\vec{\phi}$ along the positive $\vec{e}$ direction[12,22]. We also chose $K = \theta = \Gamma = 1$ as our units of energy, length and time respectively. The choice for the temperature T, that is, the noise strength, is dictated in part by the requirement that the quench be into the thermodynamically ordered region. The phase diagram of this model has only been studied in detail for $q = 2^3$. We have chosen for our studies $T = 0.05$ (in units where $K = 1$) for $q = 15$ and $T = 0.2$ for $q = 4$. These values were chosen because very conservative estimates showed them to be well within the ordered region. As we shall see, we found that indeed the system is well-ordered at these quenching temperatures.

The discretized version of the Langevin equations were numerically integrated by using a simple Euler method. A detailed discussion of the integration method and the method used to generate the Gaussian noise fields is given in Ref. 3. These details will not be repeated here. Our numerical results are presented in the following section.

III. RESULTS

The physical quantities of interest are the correlation functions

$$C_{ij}(t) = \sum_{\alpha} C_{\alpha\alpha}(i,j,t) \tag{3.1}$$

where the sites i, j are on a square lattice of size N X N with periodic boundary conditions. Equivalently, we can consider the Fourier transform of $C_{ij}(t)$, that is, the quasistatic structure factor $S(\vec{k},t)$, where $\vec{k}$ is an appropriate discrete wavevector in the first Brillouin zone. We analyze our data on these correlation functions according to standard techniques[8], which include analysis of the structure factor $S(\vec{k},t)$ and its first moment. We will present these results in detail below. We also separately consider the susceptibility function $\chi(t)$ which is given in terms of the correlation functions as:

$$\chi(t) = \sum_{i,j} \sum_{\alpha} C_{\alpha\alpha}(i,j,t)/N^2 q = S(k = 0,t) \tag{3.2}$$

This quantity can be calculated numerically faster than $S(\vec{k} \neq 0,t)$.

Before proceeding further, we verified that our chosen quenching temperatures are below the transition temperature by using preliminary data on $\chi(t)$. This was done in the following way. For a given system size N, we quenched the system to a fixed final temperature T. After the quench, $\chi(t)$ would at first grow rapidly with time, and then saturate for large enough times. We then increased the size of the system to $N' > N$ and repeated this procedure. If χ saturated at a value independent of N for large N, then one would conclude that the system equilibrates without ordering, and one is therefore above T_c. If however, one finds that the saturated value of χ is proportional to N^2 (the number of sites in the system), then one can

conclude that the system has ordered for that choice of the quenching temperature, which is therefore clearly below T_c.

After this preliminary step, we studied in detail quenches to a temperature T well below T_c (see Sec. II). In the scaling regime, where the growth of local order in the system is controlled by a single characteristic length, $\chi(t)$ is expected to be of the form[23]:

$$\chi(t) = \text{Const. } L^d(t) = \text{const. } L^2(t) \tag{3.3}$$

where L(t) represents the characteristic length (or domain-size) at time t. We will verify below that our data is indeed taken in the scaling regime. Therefore, the exponent of the power law growth in t obtained from $\chi(t)$ can be readily translated into the kinetic exponent n that governs the growth of L. We found in our results that after a very short transient time measured from time t = 0 when the system is quenched, $\chi(t)$ increases linearly with t. The results for $\chi(t)$ (which do not include the results during the initial transient) for q = 4 and q = 15 are plotted in Fig. 1 and Fig. 2 respectively. These are for N^2 = 30 x 30 for q = 4 and N^2 = 60 x 60 and 90 x 90 for q = 15, and they are well fitted by straight lines. Preliminary results for N = 10, 15, 20, 30, 40, 60, and 90 on $\chi(t)$ and the structure factor $S(\vec{k},t)$ have been used to verify that these sizes are sufficient to avoid finite size effects (see also below). We thus conclude from (3.3) that $L(t) = \text{Const. } t^n$ with $n \simeq 1/2$. A least-squares fit of these data to a form:

$$\chi(t) \propto (t - t_0)^{2n} \tag{3.4}$$

gives $n = 0.52 \pm 0.01$, $t_0 = 0.56 \pm 0.04$ for $q = 15$; and $n = 0.51 \pm 0.02$, $t_0 = 2.20 \pm 0.03$ for $q = 4$. Results for $q = 4$ were obtained from averaging over 16 runs and those for $q = 15$ and $N^2 = 60 \times 60$ from 20 runs. We found that a linear relation between $\chi(t)$ and t was obtained for each individual run but with the constants of proportionality varying slightly from run to run due to statistical fluctuations (within about 20%). The absence of finite size effects can be seen (in addition to the checks referred to above) also in Fig. 2 where we show the results for $q = 15$ with a system size of $N^2 = 90 \times 90$. The data for $N = 90$ were averaged over only two runs, but nevertheless it shows a linear relation which is, well within statistical fluctuations, comparable to the data for $N^2 = 60 \times 60$.

The time scales considered here are quite large when compared to typical time scales in MC simulations, especially for large q values. From the data shown in Fig. 2, we estimate that the value $L(t) = 20$ units is attained at time $t \simeq 25$ units for $q = 15$. In MC simulations[15] with stochastic dynamics, similar values of L(t) are attained at $t_{MC} \geq 3 \times 10^3$ MC steps per spin, which indicates the existence of a factor of at least 100 between t and t_{MC}. Thus, our simulations extend to times equivalent to more than 6000 MC steps for $q = 15$ and $N = 90$.

We turn now to $S(\vec{k},t)$. Fig. 3 displays the circularly averaged structure factor data for $q = 15$ ($N^2 = 60 \times 60$). In this graph we observe the development of a Bragg peak at $k = 0$ as the system is ordering. We first wish to check if we are in the scaling regime where S(k,t) should be of the form[23]:

$$S(k,t) = M(t)\, G(k/k_w(t)) \tag{3.5}$$

where $M(t)$ ($= \chi(t)$), the normalization factor, ensures $G(0) = 1$, and k_w is the shrinking width of the Bragg peak of $S(k,t)$, determined from the condition $G(1) = 0.5$. We calculated k_w and G by carefully interpolating the data for $S(k,t)$ and the results are presented in Fig. 4. The full curve shown in Fig. 4 corresponds to $q = 4$. The results for $q = 15$ are, within the error bars shown, the same as those for $q = 4$. As we can see from Fig. 4, we have indeed confirmed the scaling behavior of $S(k,t)$ as predicted by (3.5). The scaling function $G(k/k_w)$ is a smooth curve as expected. For large k values, error bars are larger which we believe is due to poorer statistics. We plot also in Fig. 4 the results for the scaling function G for stochastic dynamics and two values of q, $q = 8$[15] and $q = 2$ (Ising)[24]. It appears from this Figure that there is a very large degree of universality in the scaling function, which seems to be very little sensitive not only to the value of q but also to whether the dynamics is of the spin-flip or Langevin type.

Having established that we are in the scaling regime, we can investigate again the growth law by studying any length associated with $S(\vec{k},t)$. The quantity $k_w^{-1}(t)$ can be used for this purpose. However, a precise analysis is somewhat impeded by the fact that, as an interpolated quantity, $k_w(t)$ is subject to additional uncertainties. We therefore used a different measure of the width of the Bragg peak, namely $k_1(t)$, the first moment of $S(\vec{k},t)$:

$$k_1(t) = \sum_{\vec{k}} k\, S(\vec{k},t) \;/\; \sum_{\vec{k}} S(\vec{k},t). \qquad (3.6)$$

Fig. 5 and Fig. 6 show the calculated $k_1^{-2}(t)$ vs. time t for q = 4 and q = 15 respectively. Both curves are fitted well by straight lines. Since $k_1^{-1}(t)$ is proportional to the characteristic length L(t) and $k_1^{-2}(t)$ is linear in time t, L(t) is then $\propto t^n$ with n = 1/2. We also performed a least-squares fit of the results for $k_1^{-1}(t)$ to the form:

$$k_1^{-1}(t) \propto (t - t_0')^n \tag{3.7}$$

giving n = 0.49 ± 0.01, t_0' = 1.96 ± 0.02 for q = 15(N^2 = 60 x 60) and n = 0.49 ± 0.02, t_0' = 2.40 ± 0.04 for q = 4 (N^2 = 30 x 30). The N^2 = 90 x 90 results for q = 15 show that there are no systematic finite-size effects over the time scales considered in our calculation. Thus, the results of the analysis of $S(\vec{k},t)$ agree with those obtained from $\chi(t)$, and we have established that n = 1/2 for both q = 4 and q = 15.

IV. CONCLUSIONS

In the study described above, we quenched the 2D field theoretic Potts model with a non-conserved order parameter from a high temperature disordered state to a temperature far below the phase transition temperature. We then numerically followed the time-evolution of the system under the Langevin dynamics. After solving the associated Langevin equations, we found that the characteristic length L(t), which describes the domain growth kinetics in this model, varies in time according to a power law behavior, $L(t) \propto t^n$. The kinetic exponent, n, was determined to be 1/2 for both q = 4 and q = 15.

The fact that the results obtained from our calculation are consistent with those obtained from the latest MC study of the discrete Potts model[19] is a strong indication that the field theoretic and the discrete version of the Potts model with non-conserving dynamics do belong to the same universality class for $q > 2$, as they were known to be in the Ising case. Although we did not investigate the model with still larger q values at the present time, we believe that a similar result would be obtained if such a study were conducted. We therefore conclude that the domain growth kinetics in the 2D q-state Potts model with non-conserved order parameter is characterized by a Lifshitz-Allen-Cahn growth exponent $n = 1/2$, independent of q. The apparent insensitivity of the scaling function to the value of q and to the kind of dynamics reinforces these conclusions.

We note also that although a computer study of this model for very large q is not practical, the $q \to \infty$ of the field theoretic model may be amenable to analytic treatments, along lines similar to that of Ref. 25.

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FIGURE CAPTIONS

Figure 1: Results for $\chi(t)$ at T (quenching temperature) = 0.2, in the units discussed in the text, for q = 4, N^2 = 30 x 30. The dependence of $\chi(t)$ on t is linear. The full line is a best least squares straight line fit.

Figure 2: Same as in Fig. 1, but at T = 0.05 for q = 15, N^2 = 60 x 60 and 90 x 90.

Figure 3: The circularly averaged structure factor S(k,t) at T = 0.05 for q = 15, N^2 = 60 x 60 calculated for t = 5, 10, 15, 20, 25 and 30.

Figure 4: The scaling function $G(k/k_w)$, Eq. (3.5), for q = 4 (full line). Error bars are estimated from the scatter of the individual data points in the scaling plot. The scaling function for q = 15 is, within the error bars, the same as that for q = 4. We also show for comparison the scaling functions for stochastic dynamics and two values of q, q = 8 (crosses, data taken from Kaski et al, Ref. 15) and q = 2 (squares , data taken from Ref. 24).

Figure 5: Results for $k_1^{-2}(t)$ (see Eq. (3.6)) at T = 0.2 for q = 4, N^2 = 30 x 30. $k_1^{-2}(t)$ is shown to be linear in t. The full line is the best least-squares fit to a straight line.

Figure 6: Same as in Fig. 5, but at T = 0.05 for q = 15, N^2 = 60 x 60 and 90 x 90.

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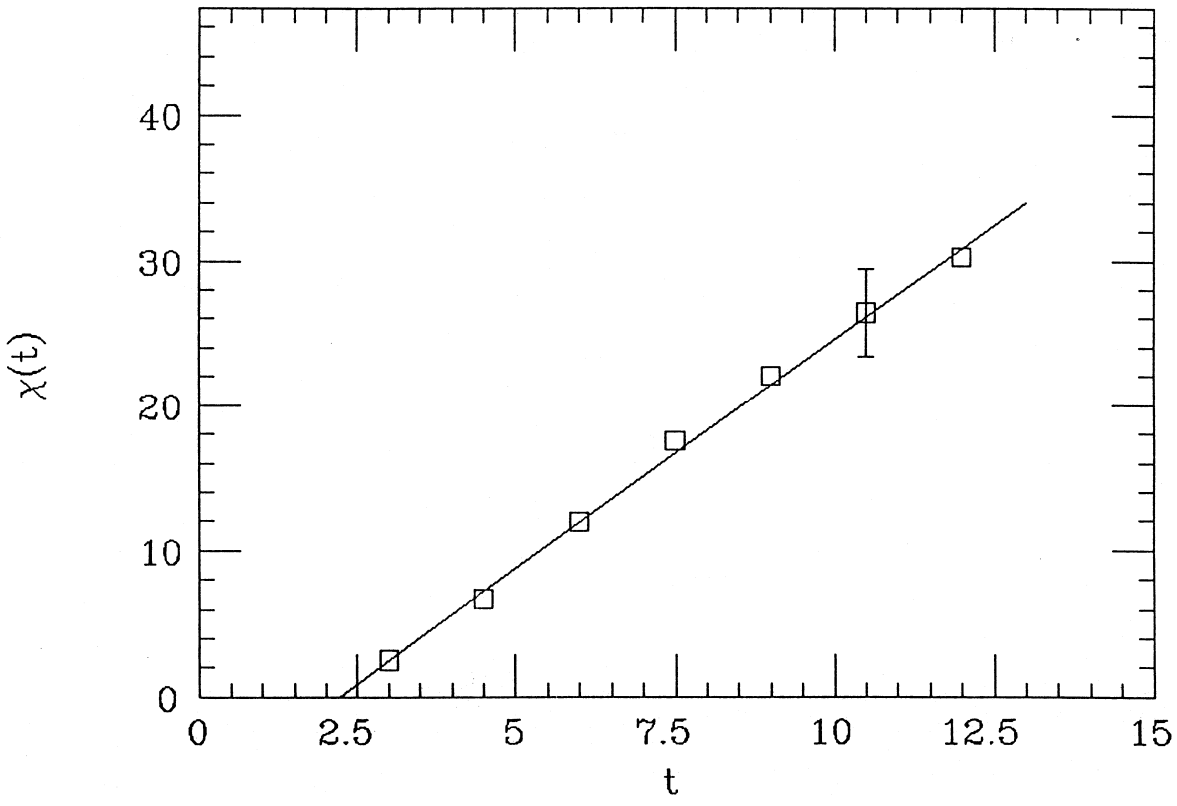
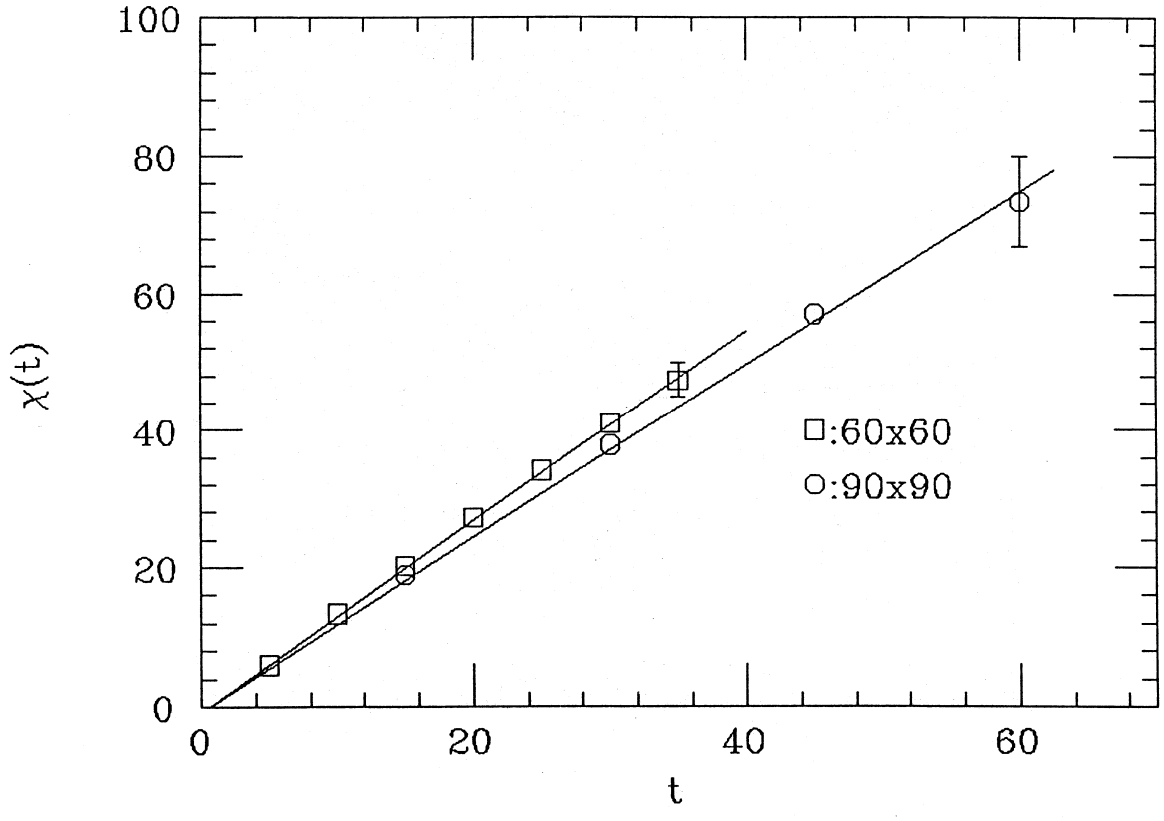


Fig 1



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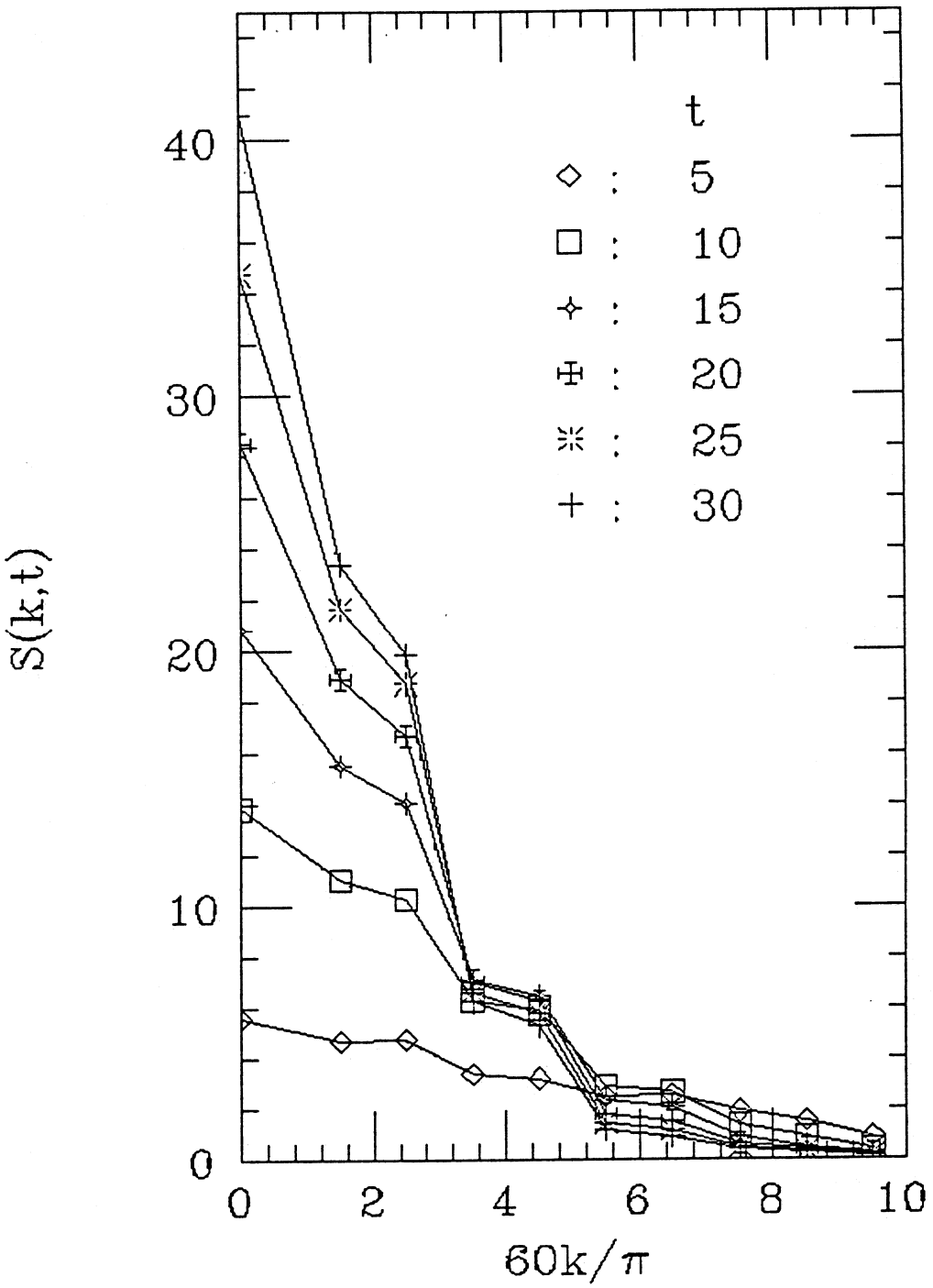

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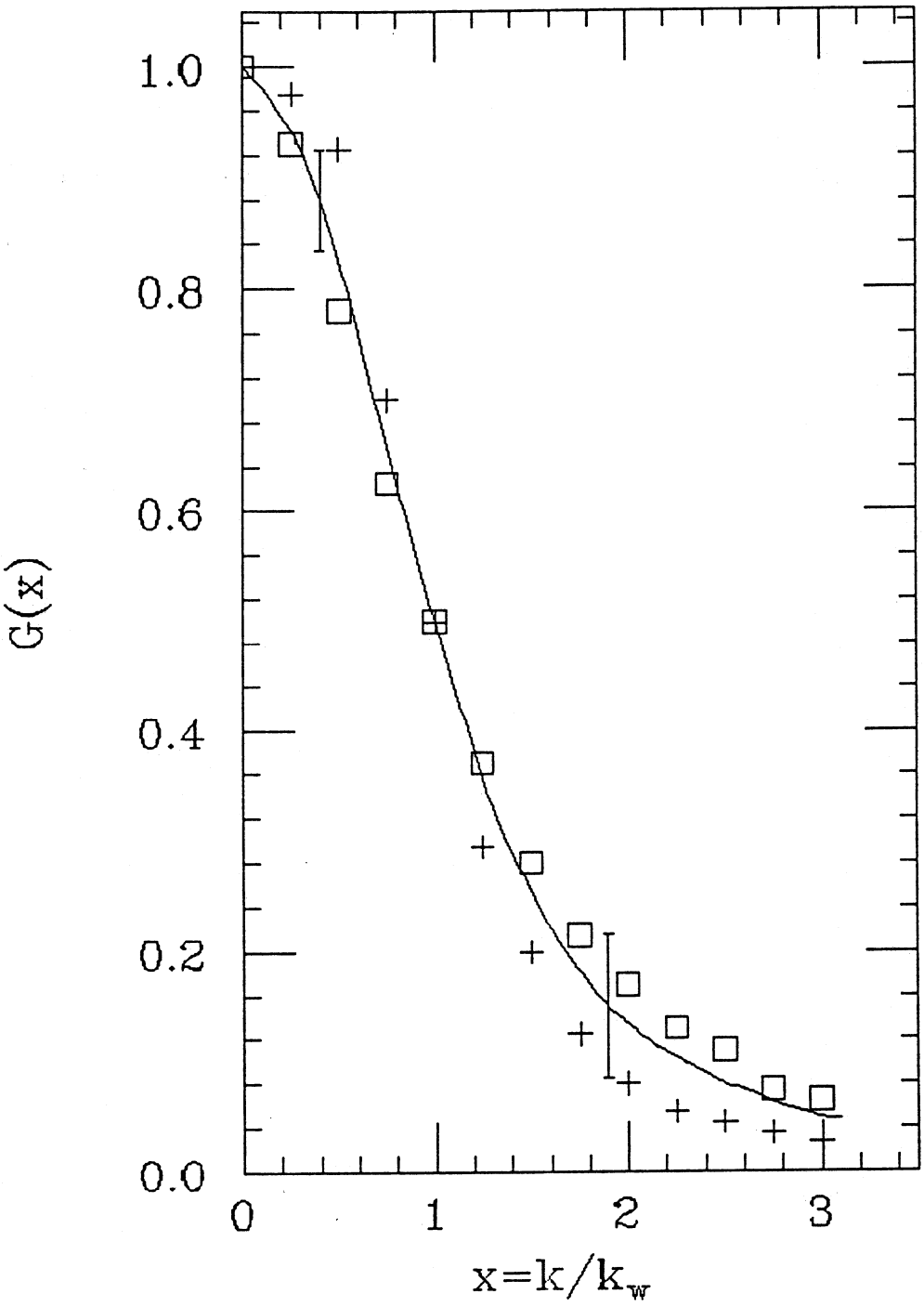

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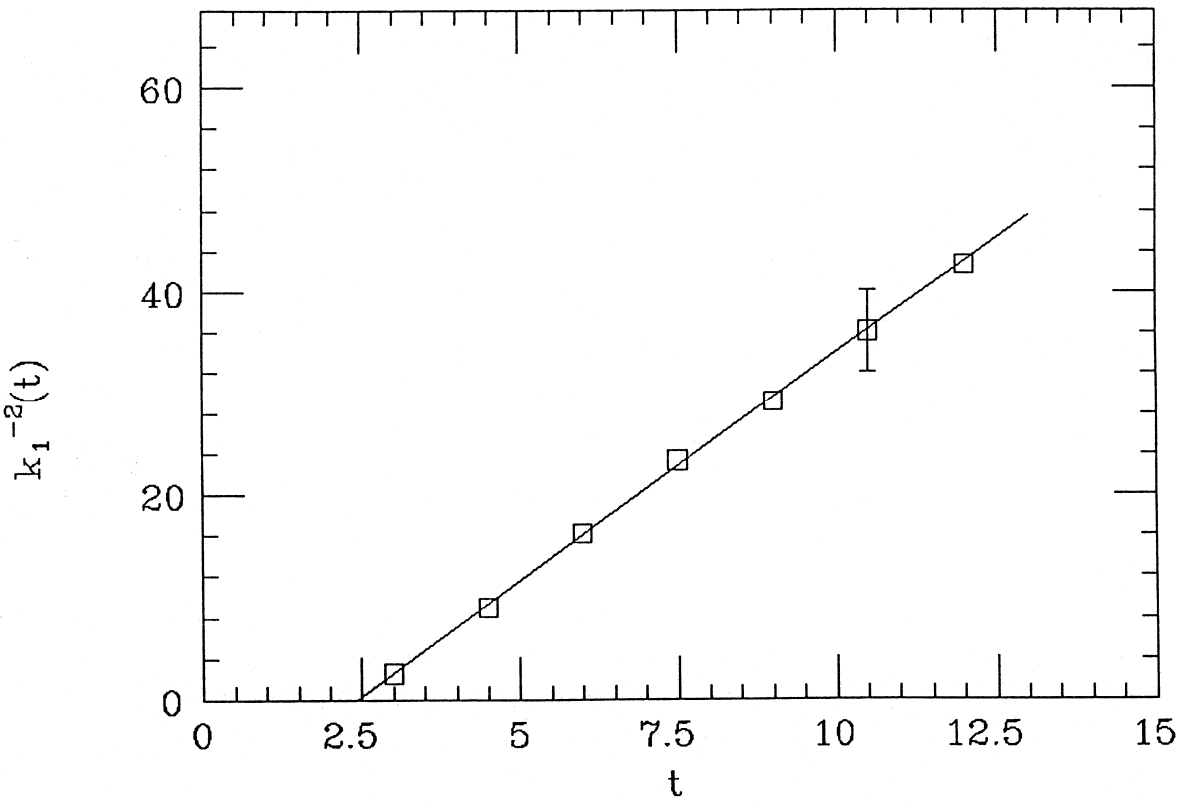

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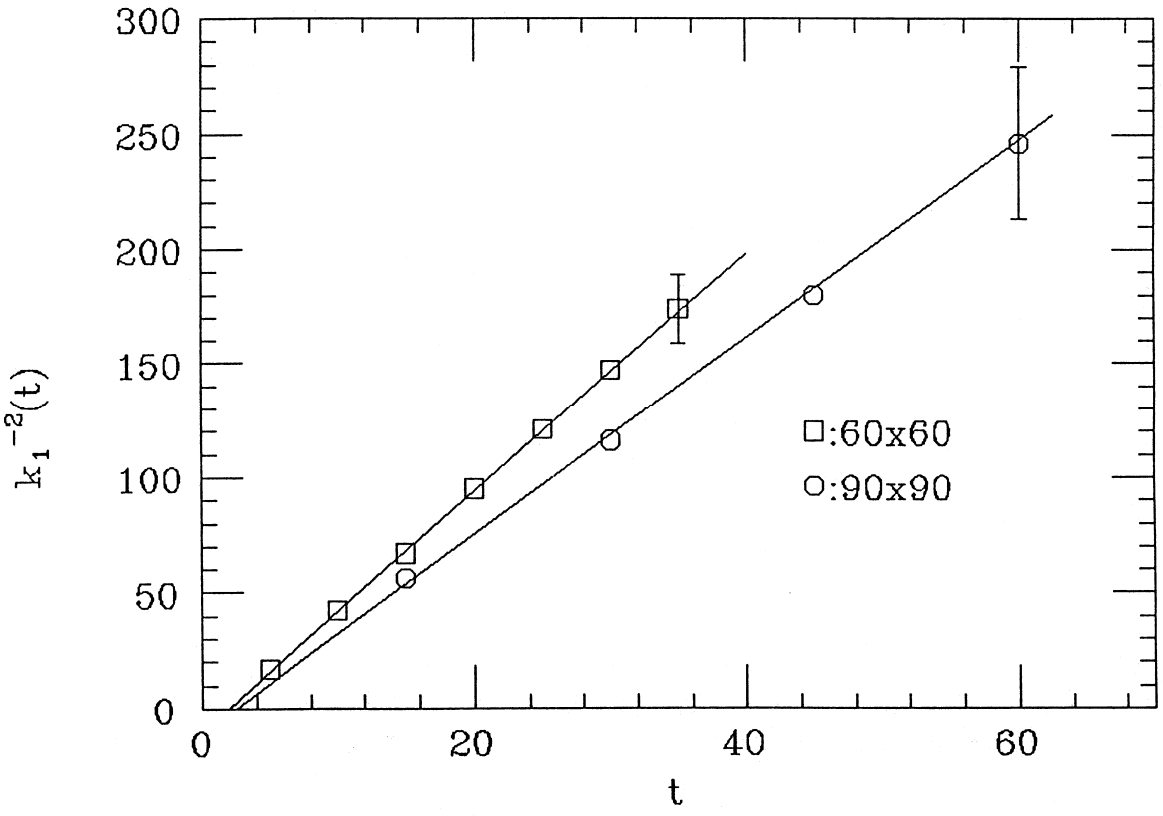

Fig. 6