

This article was downloaded by:

On: 20 January 2011

Access details: *Access Details: Free Access*

Publisher *Taylor & Francis*

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



## Chemical Engineering Communications

Publication details, including instructions for authors and subscription information:

<http://www.informaworld.com/smpp/title~content=t713454788>

### MASS TRANSFER AUGMENTATION DUE TO WALL SLIP IN HAEMODIALYSERS

A. Dutta<sup>a</sup>; R. A. Mashelkar<sup>a</sup>

<sup>a</sup> Polymer Engineering Group Chemical Engineering Division, National Chemical Laboratory, Pune, India

**To cite this Article** Dutta, A. and Mashelkar, R. A.(1982) 'MASS TRANSFER AUGMENTATION DUE TO WALL SLIP IN HAEMODIALYSERS', *Chemical Engineering Communications*, 16: 1, 361 – 375

**To link to this Article:** DOI: 10.1080/00986448208911108

**URL:** <http://dx.doi.org/10.1080/00986448208911108>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.informaworld.com/terms-and-conditions-of-access.pdf>

This article may be used for research, teaching and private study purposes. Any substantial or systematic reproduction, re-distribution, re-selling, loan or sub-licensing, systematic supply or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

## MASS TRANSFER AUGMENTATION DUE TO WALL SLIP IN HAEMODIALYSERS†

A. DUTTA and R.A. MASHELKAR

*Polymer Engineering Group  
Chemical Engineering Division  
National Chemical Laboratory  
Pune-411 008, India*

*(Received October 2, 1981; in final form March 18, 1982)*

The inadequacy of the current theories to accurately predict the haemodialyser performance is shown to be a result of the effective slip at the membrane surface. This slip phenomenon, manifested by the Fahraeus-Lindqvist effect is peculiar to (heterogeneous) blood flow. Literature data have been analyzed to substantiate the slip hypothesis proposed in this work. Both the theoretical analysis and the experimental observations indicate that slip enhances haemodialyser performance. Some design considerations have been provided, which will account for the beneficial effect of wall slip in practical haemodialyser operations.

### INTRODUCTION

Regular haemodialysis treatment has proved to be a viable alternative for a large number of people suffering from chronic renal diseases (Leonard and Dedrick, 1968). Haemodialysers permit mass transfer between dialysate and blood streams separated by a semipermeable membrane. Low mass transport rates imply larger transfer areas for efficient solute removal and larger equipment sizes and hence, the costs. In commonly used coil and flat plate dialysers, large areas are generally provided by allowing the blood to flow through very fine tubes or channels. The development of hollow fibre artificial kidneys, which combine the advantages of both coil and flat-plate dialysers, is a major breakthrough. A typical unit consists of several thousand hollow fibres in a cylindrical shell. The blood flows inside the fibres, whereas dialysate is pumped through the shell side. Typically, these fibres are 0.1 mm in diameter (internal), 0.01 mm in thickness and 20 cm in length.

Detailed analyses, which form a basis for successful design of haemodialysers, have been conducted by a number of workers. For example, Grimsrud and Babb (1966), Davis and Parkinson (1970), Colton *et al.* (1971a), Cooney *et al.* (1974), Ramachandran and Mashelkar (1980). In all of these analyses, appropriate convective diffusion equations are solved using the no slip assumption, which implies that the velocity is zero at the membrane surface. Interestingly, when compared with experimental results, these theoretical calculations generally underpredict the haemodialyser performance. This is somewhat surprising, specially in view of the fact that the theory based on solution of convective diffusion equations appears to work perfectly well for homogeneous fluids. In the present work, we shall be focussing attention on this somewhat intriguing aspect and presenting its interpretation.

---

† NCL Communication No. 2868.

*Previous Experimental Observations*

In order to demonstrate the difference, which currently exists between the theory and practice, it is useful to cite some typical data in the literature. Figure 1 shows the normalised bulk concentration as a function of normalised dialyzer length. Two sets of data have been presented.

Grimsrud and Babb (1966) have provided experimental results on diffusion of sodium chloride in water through permeable membranes using a parallel plate dialyzer with channel size varying from 0.005 to 0.155 cm, whereas Davis and Parkinson (1970) have reported data on urea removal from blood in tubular dialyzer of 0.315 cm radius. It can be seen from Figure 1 that the agreement between the theory and experiments is quite good for sodium chloride solution. The same, however, is not true for urea removal.

It may be remarked here that whereas there is no ambiguity in the literature as regards diffusivity of sodium chloride through water, there has been some ambiguity concerning diffusivity of urea in blood. Reliable data have been presented by Colton *et al.* (1971b) which show that there is a minor influence of shear rate on diffusivity of urea in blood such that when the shear rate is changed from 30 to 50  $\text{sec}^{-1}$ , the diffusivity changes from  $0.9 \times 10^{-5}$  to  $1.1 \times 10^{-5}$   $\text{cm}^2/\text{sec}$ . However, the detailed studies of Goldsmith and Karino (1979) show clearly that it is most unlikely that in the range of RBC concentration in which the data have been obtained, there will be any appreciable shear effect on diffusivity. Further, Mashelkar and Dutta (1982) have shown that the apparent shear rate influence obtained by Colton *et al.* (1971b) could be at least partially due to the plasma layer separation at the wall. We have used a diffusivity value of  $1 \times 10^{-5}$   $\text{cm}^2/\text{sec}$  for the sake of comparison.

At this juncture, the obvious question is why the theory which performs adequately for a homogeneous fluid (NaCl solution) yields anomalous results when applied to heterogeneous liquids like blood. A possible explanation may pertain to the non-Newtonian nature of blood, whereas the theories usually assume Newtonian behaviour. This, however, appears to be unlikely as it has generally been determined that the non-Newtonian characteristics are not of great importance under conditions usually encountered in blood flow. The non-Newtonian nature of blood is characterized by the existence of a yield stress. Kooijman (1972), for typical haemodialyser operations, has shown that this yield stress is only a very small fraction (0.1–0.7%) of the wall shear stress, and hence its effect on the dialyser performance is practically negligible. An alternative explanation, therefore, must exist for the aforementioned discrepancy. We propose that this anomalous behaviour is most likely to be due to effective slippage of the blood at the membrane surface.

Mashelkar and Dutta (1982), for a large variety of flow situations, have shown that even minor wall slip can play a very prominent role in enhancing mass transfer rates. A number of experimental evidences of mass transfer augmentation due to effective wall slip are provided for laminar flow of heterogeneous fluids, such as, polymer solutions and suspensions. It, therefore, seems quite plausible that large slip effects, if present during (heterogeneous) blood flow, will undoubtedly play a major part in determining the overall haemodialyser performance. Fortunately, there is considerable amount of direct experimental observations that confirm the presence of wall slip in blood flow, particularly through slender passages.

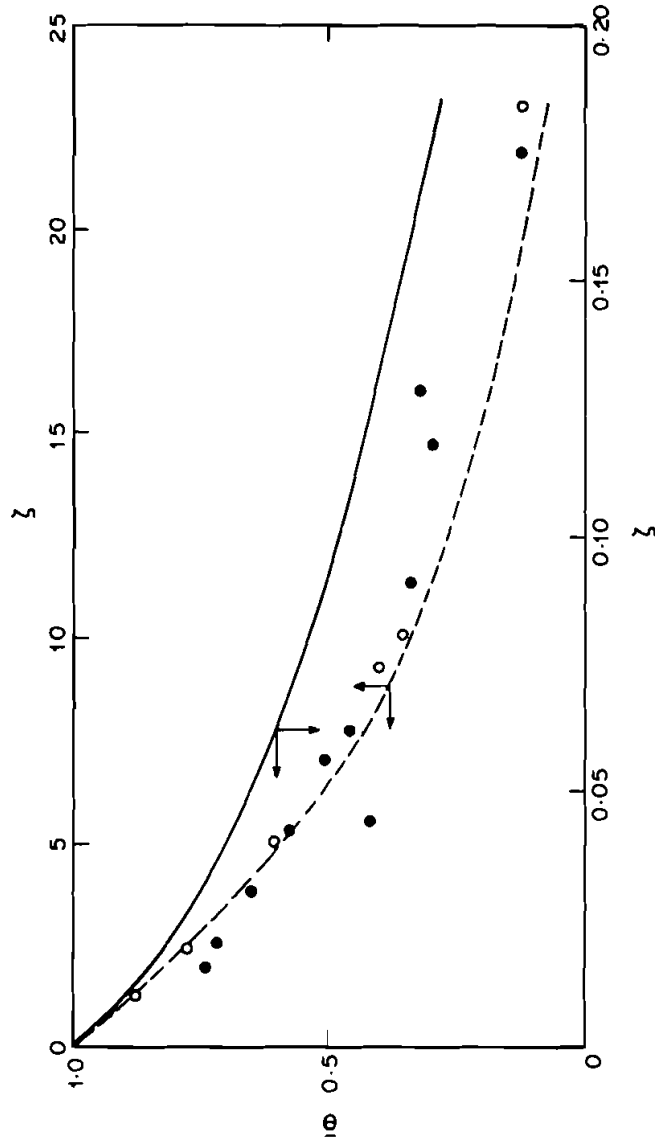


FIGURE 1 Comparison of theoretical and experimental results. Tubular geometry: — calculated, (●) experimental (Davis and Parkinson, 1970),  $Sh_w = 7.6$ , blood flow. Plane geometry: ---- calculated, (○) experimental (Grimsrud and Babb, 1966),  $Sh_w = 0.96$ , sodium chloride solution.

It is common knowledge that when blood flows through long and narrow ( $<1$  mm) capillaries, the cells tend to migrate away from the walls resulting in a cell-depleted layer adjacent to the walls (Bugliarello and Sevilla, 1970). This layer is not well defined and its thickness generally depends on the haematocrit concentration, flow rate, and capillary dimensions. Typically, this cell-depleted layer is very thin (2–6 microns) and hence, occupies a small portion of the flow area. Nevertheless, this layer is significant in that it serves as a lubricating layer for the concentrated core region, generating an apparent slip velocity,  $v_s$ , at the wall.

This phenomenon of cell migration accounts for the well known Fahraeus–Lindqvist effect, where the apparent viscosity of blood decreases rapidly with decreasing capillary dimensions. Conversely, wall slip explains the occurrence of anomalously low pressure drops for blood flow as compared to the predicted values calculated from the measured viscosity and flow rate. Apart from the indirect evidence of wall slip in blood flow, direct experimental confirmation is also available from the velocity profile measurements of Kried and Goldstein (1974), Blackshear *et al.* (1971), Duffaux *et al.* (1980), among many others. As an illustration, a typical velocity profile from the work of Blackshear *et al.* is shown in Figure 2. The data, obtained by means of a 3D flowmeter, are for simulated blood (dilute RBC in 50% ghost suspension) flowing in a 0.1 mm channel. Clearly, the blood velocity at the wall is effectively non-zero and is as high as 25 to 60% of the maximum velocity. Presence of such large slip velocities is also evident from the Laser–Doppler measurements of blood flow in a 0.37 mm channel as reported by Duffaux *et al.* (1980).

Surprisingly, inspite of considerable amount of experimental evidence supporting the phenomenon of effective wall slip in blood flow through slender capillaries, no attempt has been made to investigate its effect on the mass transfer processes occurring in biomedical devices, such as haemodialysers and blood oxygenators. In order to fill this important gap, we shall present a theoretical analysis of haemodialyser operation taking into account the presence of slip at the membrane surface.

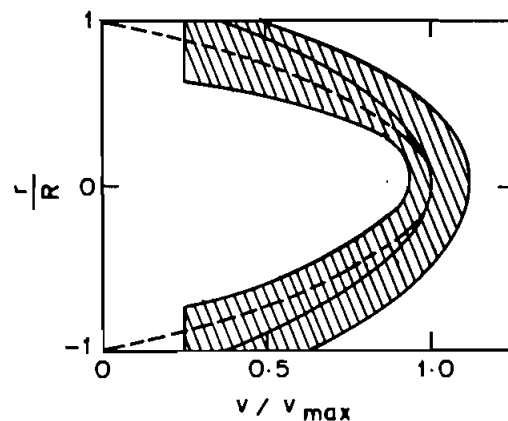


FIGURE 2 Velocity profile for blood flow through a 100 micron channel (Blackshear *et al.*, 1971), ---- parabola with no slip velocity, — best fit parabola. The shaded area represents the scatter of the experimental data.

THEORETICAL ANALYSIS

We consider the steady, laminar flow of blood through a tube or a channel where the walls consist of semipermeable membranes. Within the dialyser, solute is transferred from the blood to the dialysate stream through the membrane. This results in an axial concentration gradient along the length of the dialyser and we take  $c_i$  and  $c_o$  to be bulk solute concentrations of the blood stream at the inlet and outlet, respectively. The dialysate flow is considered turbulent as a result of which, the mass transfer coefficient can be considered to be rather high, thereby resulting in zero solute concentration on the dialysate side. Neglecting ultrafiltration and axial diffusion, the convective-diffusion equation describing the concentration of the solute in the dialyser can be written as:

$$v^* \frac{\partial c}{\partial x^*} = \frac{D}{r^a} \frac{\partial}{\partial r} \left( r^a \frac{\partial c}{\partial r} \right) \tag{1}$$

The boundary condition pertaining to uniform inlet concentration  $c_i$  is:

$$c = c_i, \quad x^* \leq 0, \quad \text{all } r \tag{2a}$$

with a centre line symmetry condition giving:

$$\frac{\partial c}{\partial r} = 0, \quad \text{all } x^*, \quad r = 0 \tag{2b}$$

and the solute removal rate at the wall is given by:

$$-D \frac{\partial c}{\partial r} = Pc, \quad \text{all } x^*, \quad r = R \tag{2c}$$

where  $a$ , the geometry indicator, is 0 or 1 depending on whether the geometry considered is planar or cylindrical in nature. Depending on the value of  $a$ ,  $R$  represents the radius for tubular geometry or half the channel height for planar configuration. If  $v_s$  is the slip velocity, that is velocity at the membrane surface at  $r = R$ , the velocity profile becomes:

$$v^* = v_s + V_m (1 - r^2/R^2) \tag{3}$$

where  $V_m = (\Delta PR^2/2\mu L)/2^a$ . Equation (3) implies that the non-Newtonian behaviour of blood has been neglected as its effect is negligible for typical haemodialyser operation (Kooijman, 1972).

The following dimensionless variables are now introduced:

$$\theta = c/c_i, \quad v = v^*/V_m, \quad \xi = r/R$$

$$x = \frac{x^*D}{V_m R^2}, \quad Sh_w = \frac{PR}{D}, \quad \alpha = \frac{v_s}{V_m}$$

In terms of these dimensionless variables, Eqs. (1) and (2) can be written as:

$$(\alpha + 1 - \xi^2) \frac{\partial \theta}{\partial x} = \frac{1}{\xi^\alpha} \frac{\partial}{\partial \xi} \left( \xi^\alpha \frac{\partial \theta}{\partial \xi} \right) \quad (4)$$

and

$$\theta = 1, \quad x \leq 0, \quad \text{all } \xi \quad (5a)$$

$$\frac{\partial \theta}{\partial \xi} = 0, \quad \text{all } x, \quad \xi = 0 \quad (5b)$$

$$\frac{\partial \theta}{\partial \xi} = -Sh_w \theta, \quad \text{all } x, \quad \xi = 1 \quad (5c)$$

Using separation of variables technique, a general solution to Eq. (4) can readily be written as:

$$\theta = \sum_{n=1}^{\infty} B_n Y_n(\xi) \exp(-\lambda_n^2 x) \quad (6)$$

where  $\lambda_n$  is the  $n$ th eigenvalue of the following equation:

$$\frac{d}{d\xi} \left( \xi^\alpha \frac{dY_n}{d\xi} \right) + \xi^\alpha (\alpha + 1 - \xi^2) Y_n = 0 \quad (7)$$

with the boundary conditions:

$$\frac{dY_n}{d\xi} = 0, \quad \xi = 0 \quad (8a)$$

$$\frac{dY_n}{d\xi} = -Sh_w Y_n, \quad \xi = 1 \quad (8b)$$

Equation (5a) implies that

$$1 = \sum_{n=1}^{\infty} B_n Y_n(\xi) \quad (9)$$

Multiplying Eq. (9) by  $\xi^\alpha (\alpha + 1 - \xi^2) Y_m$  and integrating with respect to  $\xi$  over the interval 0 to 1 gives:

$$B_n = \frac{\int_0^1 \xi^\alpha (\alpha + 1 - \xi^2) Y_n d\xi}{\int_0^1 \xi^\alpha (\alpha + 1 - \xi^2) Y_n^2 d\xi} \quad (10)$$

We now seek a solution for Eq. (7) in a power series form written as:

$$Y_n(\xi) = \sum_{k=0}^{\infty} b_{nk} \xi^{2k} \tag{11}$$

such that the condition of Eq. (8a) is satisfied. Substituting Eq. (11) in Eq. (7) and equating like powers of  $\xi$  gives the following relations:

$$\text{for } k \geq 2, \quad b_{nk} = \frac{\lambda_n^2(b_{n(k-2)} - (\alpha + 1)b_{n(k-1)})}{4k^2 + 2k(a - 1)} \tag{12}$$

and

$$b_{n1} = -\frac{(\alpha + 1)\lambda_n^2 b_{n0}}{2(1 + a)} \tag{13}$$

The boundary condition, given by Eq. (8b), implies that

$$2 \sum_{k=0}^{\infty} k b_{nk} + Sh_w \sum_{k=0}^{\infty} b_{nk} = 0 \tag{14}$$

We take  $b_{n0} = 1$ , then Eqs. (12), (13) and (14) permit the determination of the eigenvalues  $\lambda_n$ . Also, combining Eqs. (10) and (11) one gets:

$$B_n = \frac{\sum_{k=0}^{\infty} b_{nk} \frac{2 + \alpha(2k + a + 3)}{(2k + a + 1)(2k + a + 3)}}{\sum_{k=0}^{\infty} \sum_{l=0}^{\infty} b_{nk} b_{nl} \frac{2 + \alpha(2k + 2l + a + 3)}{(2k + 2l + a + 1)(2k + 2l + a + 3)}} \tag{15}$$

The bulk solute concentration, defined as:

$$\bar{\theta} = \frac{\int_0^1 \xi^a v \theta dr}{\int_0^1 \xi^a v dr} \tag{16}$$

can therefore be expressed as:

$$\bar{\theta} = \frac{(a + 1)(a + 3)}{2 + \alpha(a + 3)} \sum_{n=1}^{\infty} B_n \exp(-\lambda_n^2 x) \sum_{k=0}^{\infty} b_{nk} \frac{2 + \alpha(2k + a + 3)}{(2k + a + 1)(2k + a + 3)} \tag{17}$$

Equation (17) gives the average solute concentration in the blood as a function of the axial distance.



## RESULTS AND DISCUSSION

The eigenvalues,  $\lambda_n$ , were determined by numerically solving Eq. (14) in conjunction with the relations represented by Eqs. (12) and (13). Table I summarizes the first four eigenvalues for different slip conditions,  $\alpha$ , and wall Sherwood numbers,  $Sh_w$ . The no slip ( $\alpha = 0$ ) results are in excellent agreement with the values reported by Davis and Parkinson (1970) and Grimsrud and Babb (1966). For the calculations reported in the following, first four eigenvalues proved to be more than adequate for convergence of the infinite series. Higher eigenvalues (for small  $x$ ), if desirable, can readily be computed numerically but this exercise was not deemed necessary for the present work.

It should be mentioned that the dimensionless axial distance,  $x$ , involves the parameter  $V_m$ , which is not easily measurable in presence of slip. Therefore, we redefine the distance as  $\zeta = x \cdot D / 2^a R^2 \bar{V}$ , which now involves the mean velocity,  $\bar{V}$ . From the definition of  $x$  and  $\zeta$ , it follows that:

TABLE I  
Eigenvalues,  $\lambda_n$ , for different flow conditions

$\alpha$	$\lambda_1$	$\lambda_2$	$\lambda_3$	$\lambda_4$
Plane flow ( $a = 0$ ), $Sh_w = 0.96$				
0.00	0.98657	4.64474	8.55318	12.50853
0.01	0.97998	4.60162	8.47096	12.35708
0.10	0.92615	4.25940	7.82703	11.44400
0.25	0.85327	3.82589	7.02997	10.29211
0.50	0.76245	3.32835	6.13186	9.00070
Tube flow ( $a = 1$ ), $Sh_w = 1.0$				
0.00	1.64125	5.47831	9.43596	13.41524
0.01	1.62771	5.42593	9.34425	13.28424
0.10	1.51895	5.01428	8.63095	12.27272
0.50	1.20878	3.91951	6.77660	9.67124
1.00	1.00000	3.22784	5.60693	8.02086
$Sh_w = 7.6$				
0.00	2.46341	6.28404	10.17677	14.09912
0.10	2.31390	5.82912	9.39450	12.98190
0.25	2.13258	5.29738	8.49738	11.71857
0.50	1.90599	4.66154	7.44708	10.25958
0.75	1.73866	4.21023	6.71324	9.24792
1.00	1.60865	3.86893	6.16296	8.49124
5.00	0.88419	2.07326	3.29789	4.55130
Tube flow ( $a = 1$ ), $\alpha = 0.1$				
$Sh_w$				
0.00	0	4.63114	8.37476	12.05752
0.01	0.18219	4.63568	8.37758	12.07734
0.10	0.56556	4.67587	8.40271	12.09619
1.00	1.51895	5.01427	8.63095	12.27271
10.00	2.36608	5.91314	9.49720	13.09601
$\infty$	2.54940	6.23939	9.93635	13.63112

$$\zeta = \frac{x}{2^a} \frac{\bar{V}}{V_m} = \frac{x}{2^a} \left( \alpha + \frac{2}{a+3} \right) \quad (18)$$

Thus, knowing  $\zeta$  and  $\alpha$ ,  $x$  can readily be obtained which, in turn, allows  $\bar{\theta}(x)$  to be calculated from Eq. (17). Figure 3 illustrates the bulk concentration profiles for different wall Sherwood numbers. The geometry considered is tubular and the slip ve-

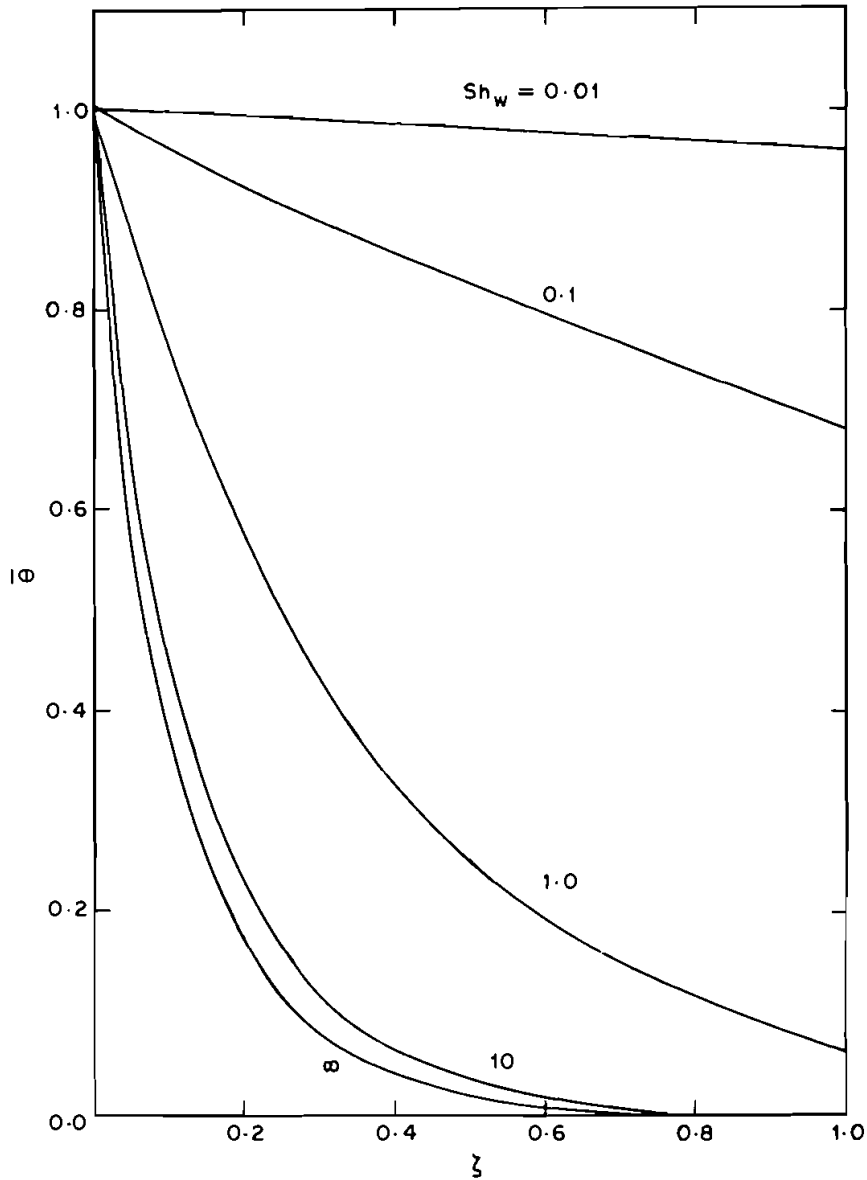


FIGURE 3 Effect of wall Sherwood number on concentration profiles,  $\alpha = 0.1$ ,  $a = 1$ .

locity is 9.1% of the maximum velocity since:

$$\frac{\text{Slip velocity}}{\text{maximum velocity}} = \frac{v_s}{v_s + V_m} = \frac{\alpha}{\alpha + 1} \quad (19)$$

It can be seen that the bulk concentration is a strong function of the wall Sherwood number, as might be expected. For a constant  $\zeta$ , as the membrane permeability increases, the solute removal is enhanced. This effect, however, is not in direct proportion to the magnitude of  $Sh_w$ . For example, the concentration profile for  $Sh_w = 10$  is not significantly different from  $Sh_w = \infty$  (constant wall concentration) case, but is considerably so from the  $Sh_w = 1$  profile. Moreover, the results imply that if the membrane permeability is steadily increased from 0 to  $\infty$ , there is a finite range within which mass transfer enhancement occurs at a maximum rate.

The effect of wall slip on the haemodialyser performance is shown in Figure 4 for a tubular geometry. Clearly, wall slip facilitates solute removal from the blood stream. Similar to the effect of wall Sherwood number, mass transfer augmentation is not directly proportional to the magnitude of wall slip. Also shown in Figure 4 are the experimental data reported by Davis and Parkinson (1970). Note that the results are for  $D = 1 \times 10^{-5}$  cm<sup>2</sup>/sec. A comparison of the theoretical predictions with the experimental data indicates the slip velocity to be of the order of 80% of the maximum velocity. This indeed is a large effect, but not unusual for blood flow through very narrow capillaries (Duffaux *et al.*, 1980). Next, we consider the data reported by Ramirez *et al.* (1971) using a parallel plate haemodialyser of 0.16 mm channel thickness. Figure 5 illustrates the experimental results along with the theoretical predictions of dimensionless bulk exit concentration,  $\theta$ , as a function of blood flow rate. Wall slip aids solute removal and therefore the exit concentration is lower than the corresponding no slip case. As before, by comparing the theoretical results with the experimental data, the slip velocity is estimated to be about 20% of the maximum velocity. Thus, it is evident that in typical haemodialysers, the slip velocity can vary over a considerably large range. This, however, is not entirely surprising as the cell-depleted layer, responsible for wall slip, is known to be a strong function of capillary dimension and geometry, blood flow rate, and blood composition (particularly the haematocrit content).

It should be noted that Davis and Parkinson's data are for  $\zeta < 0.2$ , whereas the Kiil dialyser employed by Ramirez *et al.* corresponds to  $\zeta$  values of the order of unity. Mashelkar and Dutta (1982) have analysed a number of instances of convective diffusion phenomena in non-homogeneous flows of heterogeneous fluids and shown that wall slip can cause as much as 100% to 200% enhancement in entrance region (small  $\zeta$ ) in the high Peclet number cases, whereas the corresponding effect for fully developed region (large  $\zeta$ ) is small. Therefore, it is likely that the measurements of Davis and Parkinson will be affected more as compared to those reported by Ramirez and coworkers.

In order to demonstrate the role of wall slip in practical haemodialyser design, it is useful to present a typical calculation for, say, a hollow fibre module. The various parameters necessary for this illustrative exercise are summarized in Table II. For  $\theta = 0.29$ ,  $\zeta$  is predicted to be 0.18 and 0.132 for  $\alpha$  values of 0 and 1, respectively. This cor-

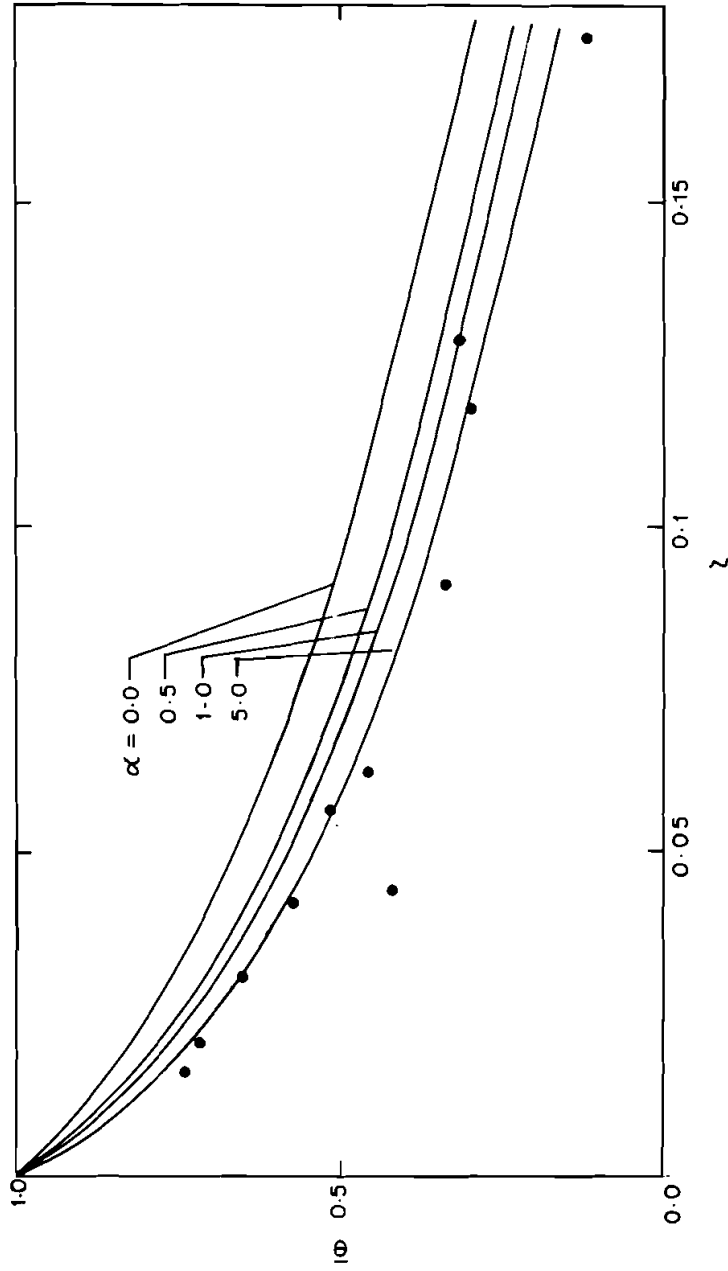


FIGURE 4 Influence of wall slip on concentration profiles,  $Sh_w = 7.6$ ,  $a = 1$ , (●) experimental data of Davis and Parkinson (1970).

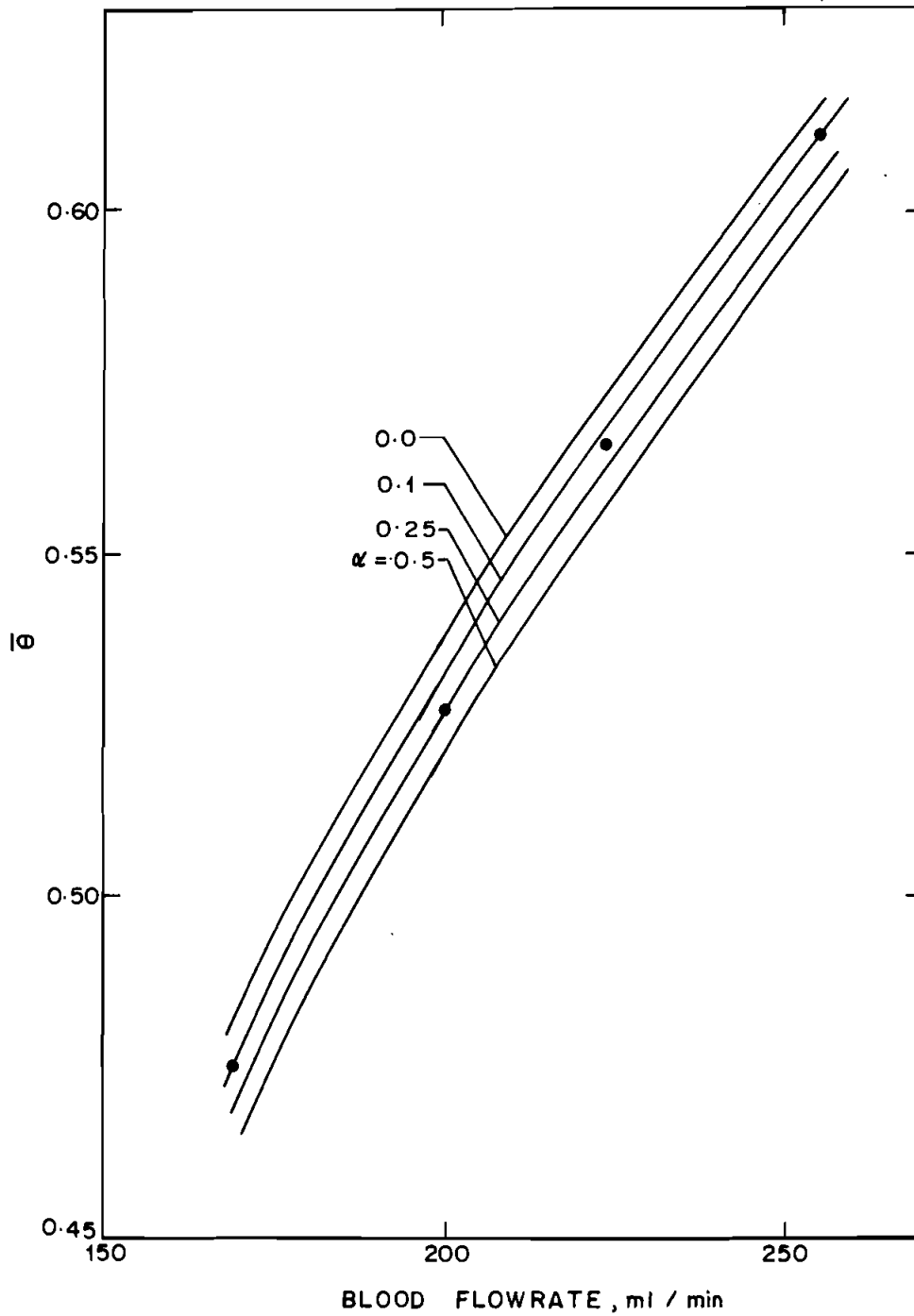


FIGURE 5 Exit concentration as a function of blood flow rate for different slip conditions,  $Sh_w = 0.96$ ,  $a = 0$ , (●) experimental data of Ramirez *et al.* (1971).

TABLE II

Parameters used in illustrative haemodialyser calculation

Geometry	Tubular ( $\alpha = 1$ )
Number of hollow fibres	1000
Fibre radius	100 microns
Total blood flow rate	200 cm <sup>3</sup> /min
Membrane permeability	$7.6 \times 10^{-3}$ cm/sec
Diffusivity of urea in blood	$1 \times 10^{-5}$ cm <sup>2</sup> /sec
Initial concentration of urea in blood	2 mg/cm <sup>3</sup>
Final concentration of urea in blood	0.58 mg/cm <sup>3</sup>

responds to dialyser lengths of 38.2 cm ( $\alpha = 0$ ) and 28 cm ( $\alpha = 1$ ). Therefore, about 25% reduction in length is possible for a slip velocity that is 50% of the maximum. Clearly, if slip effect is disregarded, the dialyser will be overdesigned.

Until now, it has been assumed that the slip velocity,  $v_s$ , is known *a priori*, but for design purposes, it will have to be determined independently. Assuming the shear rate to be constant within the cell free plasma layer, which is Newtonian in nature, the slip velocity can be calculated as:

$$v_s = \frac{\tau_w \delta}{\mu_p} \quad (20)$$

where  $\mu_p$  is the viscosity of plasma,  $\tau_w$  is the wall shear stress, and  $\delta$  is the layer thickness. Various correlations of  $\delta$  with blood composition is available in the literature (Bugliarello, 1970; Charm *et al.* 1968; Merrill *et al.* 1963). For example, Kried and Goldstein (1974) reported that:

$$\frac{H\delta}{2R} \sim f(Re) \quad (21)$$

where  $H$  is the haematocrit concentration,  $Re$  is the shear Reynolds number, and  $f(Re)$  is an experimentally determined function. Equation (21) or any other correlation for  $\delta$  can be used to estimate  $v_s$  from Eq. (20). Knowing  $v_s$ , the foregoing slip analysis can readily be utilized in order to assess the importance of wall slip in affecting practical haemodialyser performance.

## CONCLUSIONS AND REMARKS

For haemodialysis operations, the correspondence between theory and experiment is good for homogeneous sodium chloride solutions, but rather poor for blood, which is heterogeneous in nature. Effective wall slip, due to the formation of this plasma layer in blood flow, appears to be the most likely cause of this discrepancy. A theoretical analysis when combined with the available experimental data, suggests that

major slip effects are possible in typical haemodialysis operations. This corresponds to an improved performance of the dialysis unit and suggests potential savings in the cost of haemodialysis treatment.

Notwithstanding the plausible slip hypothesis proposed in this work to explain the augmentation in mass transfer rates in haemodialysers, it must be remarked that the actual problem in blood flow is much more complex, and there may be some other unknown factors which may also be contributing. Note also that the experimental data provided by Blackshear *et al.* (1971) essentially pertain to a haematocrit concentration that is twice that of most uremic blood and therefore there is a clear need for unambiguous and more accurate data on velocity distribution in the range of pragmatic interest. Until such data are available, the analysis provided therein can be viewed only as a potentially useful one when viewed in the context of many factors which might be contributing to the augmentation in mass transfer. The design considerations proposed here, therefore, are to be viewed in the same context.

#### NOTATION

$a$	geometry indicator
$c$	solute concentration in blood, $\text{mg}/\text{cm}^3$
$c_i$	inlet solute concentration, $\text{mg}/\text{cm}^3$
$c_o$	outlet solute concentration, $\text{mg}/\text{cm}^3$
$D$	diffusivity of solute in blood, $\text{cm}^2/\text{sec}$
$H$	haematocrit concentration, volume %
$L$	channel length, cm
$P$	wall permeability, $\text{cm}/\text{sec}$
$\Delta P$	pressure drop, $\text{dynes}/\text{cm}^2$
$r$	radial or transverse distance, cm
$R$	tube radius or half channel height, cm
$Re$	shear Reynolds number
$Sh_w$	wall Sherwood number, dimensionless
$v$	axial velocity, $\text{cm}/\text{sec}$
$v_s$	slip velocity, $\text{cm}/\text{sec}$
$V_m$	velocity contribution due to pressure flow, $\text{cm}/\text{sec}$
$\bar{V}$	average velocity, $\text{cm}/\text{sec}$
$x$	axial distance, dimensionless
$Y_n$	$n$ th eigenfunction, dimensionless

#### Greek Symbols

$\alpha$	slip parameter, dimensionless
$\delta$	marginal layer thickness, cm

$\lambda_n$	$n$ th eigenvalue, dimensionless
$\theta$	concentration, dimensionless
$\bar{\theta}$	bulk concentration, dimensionless
$\zeta$	axial distance, dimensionless
$\xi$	transverse distance, dimensionless
$\mu$	blood viscosity, poise
$\mu_p$	plasma viscosity, poise
$\tau_w$	wall shear stress, dynes/cm <sup>2</sup>

*Superscript*

\* dimensional

REFERENCES

- Blackshear, P.L., Forstrom, R.J., Dorman, F.D., and Voss, G.O., "Effect of flow on cells near walls," *Fed. Proc.*, **30**, 1600 (1971).
- Bugliarello, G., and Sevilla, J., "Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes." *Biorheology*, **7**, 85 (1970).
- Bugliarello, G., "Fluid Dynamics of blood circulation and respiratory flow," AGARD Conf. Proc. No. 65, Paper 2 (1971).
- Charm, S.E., Kurland, G.S., and Brown, S.L., "The influence of radial distribution and marginal plasma layer on the flow of red cell suspensions," *Biorheology*, **5**, 15 (1968).
- Colton, C.K., Smith, K.A., Stroeve, P., and Merrill, E.W., "Laminar flow mass transfer in a flat duct with permeable walls," *AIChE.J.*, **17**, 773 (1971a).
- Colton, C.K., Smith, K.A., Merrill, E.W., and Friedman, S., "Diffusion of urea in flowing blood," *AIChE.J.*, **17**, 800 (1971b).
- Cooney, D.O., Davis, E.J., and Kim, S.S., "Mass transfer in parallel plate dialysers—A conjugated boundary value problem," *Chem. Eng. J.*, **8**, 213 (1974).
- Davis, H.R., and Parkinson, G.V., "Mass transfer from small capillaries with wall resistance in the laminar flow regime," *Appl. Sci. Res.*, **22**, 20 (1970).
- Duffaux, J., Quemada, D., and Mills, P., "Velocity profile measurements by Laser-Doppler velocimetry (LDV) in plane capillaries. Comparison with theoretical profiles from a two-fluid model," Proc. 8th Intl. Congress on Rheology, Vol. 3, p. 561 (1980).
- Goldsmith, H.L., and Karino, T., "Mechanically induced thrombo emboli" in 'quantitative cardiovascular studies of engineering principles', Hwang, N.H.C., Gross, D.R., and Patel, D.J. (Eds.), Univ. Park Press, Balt. (1979).
- Grimsrud, L., and Babb, A.C., "Velocity and concentration profile for laminar flow of a Newtonian fluid in a dialyser," *Chem. Eng. Prog. Symp. Series*, **62**, 20 (1966).
- Kooijman, J.M., "The flow and mass transfer in haemodialysers: the influence on non-Newtonian blood flow," *Chem. Eng. J.*, **4**, 189 (1972).
- Kried, D.K., and Goldstein, R.J., "Measurement of velocity profiles in simulated blood by the Laser-Doppler technique," University of Minnesota Technical Report (1974).
- Leonard, E.F., and Dedrick, R.L., "The artificial kidney: problems and approaches for the chemical engineer," *Chem. Eng. Prog. Symp. Series*, **64** (84), 5 (1968).
- Mashelkar, R.A., and Dutta, A., "Convective Diffusion in Structured Fluids: A Need for New Analysis and Design Strategies," *Chem. Eng. Sci.*, in press (1982).
- Merrill, E.W., Gillard, E.R., Cokelet, G., Shin, A., Britten, A., and Wells, R.E., "Rheology of human blood near and at zero flow," *Biophys. J.*, **3**, 199 (1963).
- Ramachandran, P.A., and Mashelkar, R.A., "Lumped-parameter model for haemodialyser with application to simulation of patient-artificial-kidney system," *Med. Biol. Eng. Comput.*, **18**, 179 (1980).
- Ramirez, W.F., Mickley, M.C., and Lewis, D.N., "Mathematical modelling of a KIIL haemodialyser," *Chem. Eng. Prog. Symp. Series*, **114** (67), 116 (1971).