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Anomalous transport phenomena in rapid external flows of viscoelastic fluids

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With 3 figures

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Notation

A, B	numerical constants
A_1, A_2	surface areas
C_D	drag coefficient
D	cylinder diameter
F	hoop force
h	heat transfer coefficient
k	thermal conductivity
M	molecular weight
Nu	Nusselt number
R	gas constant
T	absolute temperature
u	x-component of the velocity
U	free stream velocity
x, y	Cartesian coordinates

Greek letters

$\dot{\gamma}$	shear rate
δ	boundary layer thickness
δ_0	elastic boundary layer thickness
θ	relaxation time
μ	viscosity
ν	kinematic viscosity
$[\eta]$	intrinsic viscosity
ϱ	density
σ	normal stress difference
τ	shear stress

1. Introduction

External flows of dilute polymer solutions past obstacles show some interesting effects which in general have been lacking a convincing interpretation. One of the more interesting phenomena can be described as follows. When a dilute polymer solution flows past a bluff body (such as a cylinder) the drag coefficient and the heat transfer coefficient follow the normal behaviour at low velocities. But then, at a particular point, they become independent of the free stream velocity (*James* and *Acosta* (1), *James* and *Gupta* (2), *Metzner* and *Astarita* (3), *Ultman* and *Denn* (4)). The constant value of the normalized transport coefficient appears to depend only upon the cylinder diameter and the fluid properties. This observation has some important practical implications (e.g. in hot wire anemometry) and is obviously interesting from a fluid mechanical viewpoint.

In this paper we consider the problem in the framework of the formation of an "elastic" boundary layer and attempt to explain the anomalous transport phenomena in the light of some plausible mechanistic considerations. No "exact" solution is found, but the heuristic approach is nevertheless helpful in clarifying the possible physical phenomena.

2. The concept of a boundary layer dominated by normal stresses

We shall assume that the fluid obeys a Maxwell equation with a viscosity μ and a relaxation time θ. Thus in a shear flow with a shear rate $\dot{\gamma}$, the steady state values of the shear and normal stresses are given by

$$\tau = \mu \dot{\gamma}, \quad [1]$$

$$\sigma = 2\mu\theta\dot{\gamma}^2. \quad [2]$$

We shall first consider the laminar boundary layer which forms on a flat plate immersed in a parallel uniform stream having a velocity U. We shall use a Cartesian coordinate system with the x-axis in the direction of the stream, the y-axis perpendicular to the plate and the origin at the leading edge.

One of the difficulties in dealing with a viscoelastic fluid is that the stresses are usually not at their steady state in a Lagrangian sense.

For the case at hand, one may expect this to be particularly true in the zone of the leading edge where the fluid is abruptly decelerated by the effect of the plate. However, the fluid which is very close to the wall has a very small velocity i.e., for $y \to 0$, the residence time tends to infinity so that, at least in that limit, Eqs. [1] and [2] must hold true even at $x = 0$.

A consequence of the above consideration and of Eq. [2] is that the boundary layer, contrary to the purely viscous case, must have a non-zero thickness δ_0 at the leading edge. In fact, a momentum balance between a section somewhere upstream of the leading edge and the leading edge section itself gives

$$\varrho U^2 \delta_0 = \int_0^{\delta_0} (\varrho u^2 + \sigma)\, dy \qquad [3]$$

where ϱ is the fluid density and $u(y)$, $\sigma(y)$ are the x-velocity component and normal stress distributions in the leading edge section, respectively.

Of course, we do not know these distributions in any detail but we know that, close to the wall, Eq. [2] must hold true with a value of $\dot{\gamma}$ proportional to U/δ_0:

$$\dot{\gamma} \propto \frac{U}{\delta_0}. \qquad [4]$$

Thus Eq. [3] becomes

$$\varrho U^2 \delta_0 = A^2 \mu \theta \frac{U^2}{\delta_0} \qquad [5]$$

with A^2 an unknown numerical factor which is given by

$$A^2 = \frac{\int_0^1 \frac{\sigma}{\mu \theta U^2/\delta_0^2}\, d(y/\delta_0)}{1 - \int_0^1 (u/U)^2 d(y/\delta_0)}. \qquad [6]$$

Eq. [5] is obviously incompatible with a zero value of the thickness. From Eq. [5], the value of δ_0 is obtained as

$$\delta_0 = A \sqrt{\nu \theta} \qquad [7]$$

where $\nu = \mu/\varrho$ is the kinematic viscosity.

A similar argument can be developed to calculate the thickness $\delta(x)$ of the boundary layer along the plate[1]). A momentum balance between two sections at a distance dx gives in fact

$$\varrho U^2 \frac{d\delta}{dx} = \frac{d}{dx} \int_0^{\delta} (\varrho u^2 + \sigma)\, dy + \tau \qquad [8]$$

where τ, the shear stress at the wall, is given by Eq. [1].

If one assumes a "similarity solution", i.e. the velocity and normal stress distributions over the boundary layer dimensionless thickness y/δ are independent of x, Eq. [8] becomes

$$\varrho U^2 \frac{d\delta}{dx} = A^2 \mu \theta U^2 \frac{d}{dx}(1/\delta) + B\mu \frac{U}{\delta} \qquad [9]$$

where A^2 and B are numerical factors. A^2 is the same as in Eq. [6] (with δ in place of δ_0), while B is given by

$$B \propto \frac{1}{1 - \int_0^1 (u/U)^2 d(y/\delta)}. \qquad [10]$$

Integrating Eq. [9] with the condition that $\delta = \delta_0$ at $x = 0$, gives

$$\frac{\delta^2}{\delta_0^2} - 1 + \ln \frac{\delta^2}{\delta_0^2} = \frac{2B}{A^2} \frac{x}{U\theta} \qquad [11]$$

where use has been made of Eq. [7]. Eq. [11] shows that, if $x/U\theta \ll 1$, δ is of order δ_0 whereas for $x/U\theta \gg 1$, Eq. [11] reduces to the purely viscous result

$$\delta = \sqrt{2B \frac{\nu x}{U}}. \qquad [12]$$

The results so far obtained can be summarized as follows: In the region close to the leading edge of the plate, a boundary layer is formed which is dominated by normal rather than shear stresses. The thickness of this "elastic" boundary layer, given by Eq. [7], is independent of the mainstream velocity U and only depends on fluid properties ν and θ. With respect to the purely viscous case, one may speak of a boundary layer thickening due to normal stresses. One may surmise that the disturbance to the main flow also extends somewhere upstream of the leading edge, but no easy calculations of this effect seem possible. Moving forward along the plate, the contribution of shear stresses becomes increasingly

[1]) The authors are indebted to Prof. *H. Giesekus* for suggesting this calculation when reviewing a previous version of the paper.

important so that a viscous boundary layer is eventually reestablished with a thickness given by the classical result, Eq. [12].

It may be noted here that the concept of a boundary layer thickening is not new and was indeed suggested by *Metzner* and *Astarita* (3). However, their argument was based on the idea that in high Deborah number flows a solid like behaviour is to be expected. They thus obtained the result $\delta_0 \sim U\theta$ which is completely different from Eq. [7] above. The calculations developed in this work are rather in the line indicated briefly by *Astarita* and *Marrucci* (5).

Considering now the case of a bluff body, the following qualitative argument can be developed. We call D the characteristic dimension of the body and assume that the Deborah number $U\theta/D$ is large. By comparison with the results for the flat plate, where the characteristic dimension was x, we may expect that for large values of the Deborah number the boundary layer over the whole body is dominated by normal stresses so that its thickness is of order δ_0 up to separation. The normal stresses which exist in the boundary layer act as hoop stresses on the body surface and, in view of the asymmetry which exists between the front and rear zones, they significantly contribute to the drag.

An illustration of this concept is shown schematically in Fig. 1, with reference to a

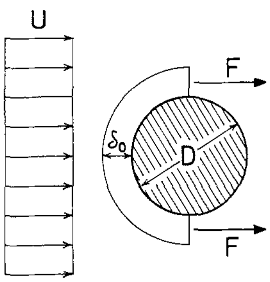


Fig. 1. Schematic representation of the elastic boundary layer in front of a bluff body

cylindrical body having a circular cross section. In the front zone a boundary layer with a thickness δ_0 is depicted. We do not know what happens in the rear zone but it seems plausible that, similarly to the viscous case, the boundary layer will detach from the surface and grow into a wake where normal stresses relax to zero. An estimate of the contribution of normal stresses to the drag is then obtained by considering the hoop force per unit length of the cylindrical body, indicated as F in Fig. 1. This is given by

$$F = \int_0^{\delta_0} \sigma \, dy = \frac{A^2}{B} \mu \theta \frac{U^2}{\delta_0} \qquad [13]$$

where A^2 and B have the same meaning as in Eqs. [6] and [10].

The corresponding drag coefficient is then obtained as

$$C_D = \frac{2F}{D} \frac{1}{\varrho U^2/2} = \frac{4A^2}{B} \frac{\nu \theta}{D \delta_0}. \qquad [14]$$

Finally, substituting for δ_0 from Eq. [7], we obtain

$$C_D = \frac{4A}{B} \frac{\sqrt{\nu \theta}}{D}. \qquad [15]$$

Of course, Eq. [15] is expected to apply only when the effect of normal stresses is the dominant one, i.e. when C_D as predicted by Eq. [15] is larger than that obtained by the usual correlations for purely viscous fluids.

We conclude this section by considering that for dilute polymeric solutions θ and ν are typically of order 10^{-4} sec and 10^{-2} cm^2/sec respectively so that $\sqrt{\nu\theta}$ is of order 10^{-3} cm. For small objects such as the cylindrical wire of diameter 5×10^{-3} cm used by *James* and *Acosta* (1), Eq. [15] would predict values of C_D of order 1 or larger which are at least comparable with those predicted in the viscous case for Reynolds numbers of order 100. It may be noted that in such a case, the boundary layer thickness itself is of the same order as the body dimension.

Thus anomalous effects due to normal stresses are to be expected for the case of small bodies such as hot film sensors, pitot tubes, etc. whereas these effects should be minor in case of larger bodies. The results by *Acrivos* et al. (6), *Weil* (7) and *Acharya* et al. (8) on large cylinders or spheres indeed do not show anomalous effects contrary to those obtained by *James* and *Acosta* (1) as well as *James* and *Gupta* (2) on tiny cylinders.

3. Comparison of Eq. [15] with the anomalous drag coefficients by *James* and *Gupta*

James and *Gupta* (2) have reported experimental data of drag coefficients for small circular cylinders in a variety of dilute polymer solutions. In almost all cases reported by them the drag coefficient lies well above that predicted by the Newtonian correlation and is virtually independent of the fluid velocity. Using a dimensional analysis, these drag coefficients were then correlated with the group $D^2/\nu\theta$. The residual differences which are found among the various solutions were attributed to the uncertainty in estimating θ, a fluid property which is strongly dependent on the spread of the polymer molecular weight distribution.

While agreeing with this viewpoint, we further note that in the present context θ is unequivocally defined by Eqs. [1] and [2] which, by eliminating $\dot{\gamma}$, give

$$\theta = \frac{\mu\sigma}{2\tau^2}. \qquad [16]$$

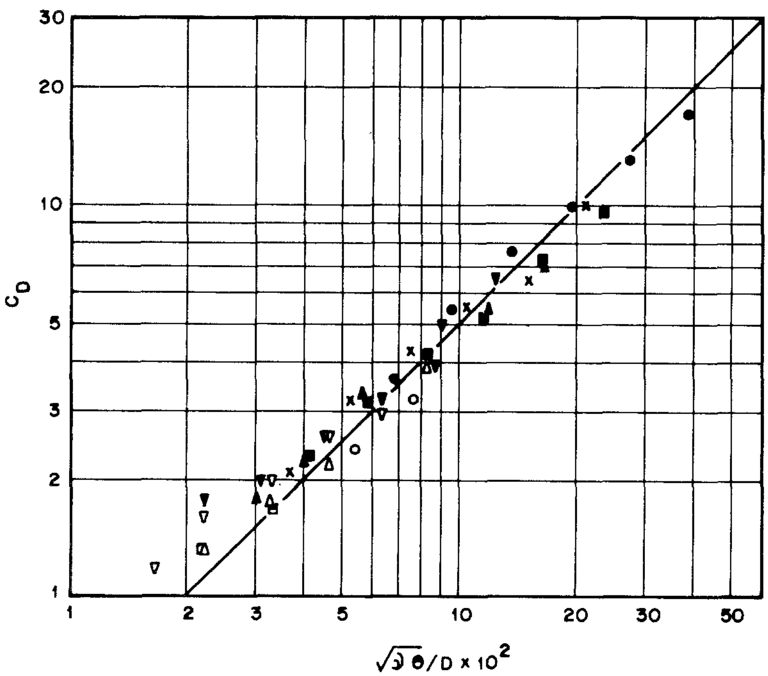
However, direct measurements of normal stresses in dilute solutions are often unavailable so that recurse is made to molecular models which allow to rewrite Eq. [16] in the form

$$\theta = \frac{\mu[\eta]^2 Mc}{RT} \qquad [17]$$

where $[\eta]$ is the intrinsic viscosity, M the molecular weight and c the concentration by weight of the polymer. Use of Eq. [17] only requires a knowledge of the molecular weight and intrinsic viscosity, which are more readily available than normal stress coefficients. However, the numerical factor in Eq. [17] is open to question as it depends on the model which is used (*James* and *Acosta* (1) and *James* and *Gupta* (2) use a numerical factor 2/5) and even more questionable is the extension of Eq. [17] to a polydisperse situation.

Returning to the data by *James* and *Gupta*, we note that the results reported in Fig. 10 of ref. (2) can be roughly grouped in two separate sets each of them presenting a moderate dispersion. We have then chosen one of the groups, that which covers the widest range of C_D values. The group comprises the following polymers: Polyox FRA ($[\eta]$ = 25.8 dl/g, $M_w = 8.35 \times 10^6$); Polyox coagulant ($[\eta]$ = 17.8 dl/g, $M_w = 5.18 \times 10^6$); Polyox WSR N3000 ($[\eta]$ = 4.1 dl/g, $M_w = 0.79 \times 10^6$). The data were obtained on three cylinders with diameters 0.006, 0.010, 0.014 inches.

The data have been reworked by using Eq. [17] for calculating θ and plotted in Fig. 2 in the form suggested by Eq. [15], i.e. as C_D vs. $\sqrt{\nu\theta}/D$. As shown by Fig. 2, a straight line of slope 1 correlates the data reasonably well. The largest departures are found at low values of C_D



Symbols are as follows:

Polyox FRA
● 0.006 inches dia cylinder
■ 0.010
▲ 0.014

Polyox coagulant
× 0.006 inches dia cylinder
▼ 0.010
▽ 0.014

Polyox WSR N3000
○ 0.006 inches dia cylinder
△ 0.010
□ 0.014

Fig. 2. Drag coefficient correlation.

where the dominance of normal stress effects is more questionable. The straight line in Fig. 2 has the equation

$$C_D = 50\,\frac{\sqrt{\nu\theta}}{D}. \qquad [18]$$

The numerical factor which is found, ~50, is surprisingly large. By considering the meaning of A and B in Eq. [15], one would have guessed a numerical factor of order 1 or 10 at most. It looks more plausible that we are grossly underestimating θ by using Eq. [17] in place of Eq. [16].

Whatever its origin, the large value of the numerical factor implies that a fairly thick boundary layer is formed in the conditions of these experiments, definitely of the same order as the (small) body dimension or even larger. This would have the meaning that the body is surrounded by a quasi-stagnant thick layer of fluid. Direct or semi-direct evidence of such phenomena are found in a few works. For example, *Leider* and *Lilleleht* (9) studied the flow behaviour of a solution of polyisobutylene in a stagnation flow formed by a T-shaped channel. Determination of the velocity distribution indicated that a nearly stagnant region exists at the stagnation point contrary to the Newtonian case. They observed a cusp-shaped zone lying along the centerline and found that the thickness of this zone is *not dependent upon the velocity* in a certain velocity range. These observations appear to be consistent with the idea of a velocity independent elastic boundary layer.

Also, *Davis* (10) took photographs of lines of hydrogen bubbles produced at a known frequency (isochrones) upstream of bodies of various shapes and dimensions and detected the first point of isochrone distortion. While for a Newtonian fluid the upstream distance for isochrone distortion was zero, for a viscoelastic fluid this distance was finite and independent of both the free stream velocity and the shape or dimension of the object. Finally *James* and *Acosta* (1), though mainly concerned with heat transfer, made photographic observations of the flow field by dye injection and noted significant differences between Newtonian and viscoelastic fluids. In the latter case, there was clear evidence of an upstream influence of the cylinder and of an increased width of the wake.

It is interesting to note that if one takes the simplistic approach of considering the immersed object plus the quasi-stagnant layer of thickness δ_0 around it as a whole body immersed in a purely viscous fluid, Eq. [15] is again obtained. In fact, since the drag would be proportional to the overall dimension $D + 2\delta_0$, one would obtain

$$C_D = \frac{D + 2\delta_0}{D}\, C_{D,N} \qquad [19]$$

where $C_{D,N}$ is the Newtonian drag coefficient calculated at an appropriate value of the Reynolds number. For δ_0 larger than D, Eq. [19] becomes equivalent to Eq. [15]. *Abraham* (11) and *van Dyke* (12) have explored similar ideas, i.e. the notion of virtual bodies, when looking for an approximate description of the flow of a Newtonian fluid past a sphere.

4. Anomalous heat transfer coefficients

Encouraged by the results discussed in the previous section, we attempt here a very simple interpretation of the anomalous heat transfer results reported by *James* and *Acosta* (1). Similarly to the drag behaviour, they find that above a certain free stream velocity the heat transfer coefficient becomes essentially velocity independent and thus remains smaller than that predicted by the Newtonian correlation. The corresponding values of the Nusselt number are then correlated with the group $D^2/\nu\theta$ for a variety of polymeric solutions and (tiny) cylinder diameters.

If one assumes that a virtually stagnant fluid layer of thickness δ_0 exists around the body, the heat transfer coefficient h is calculated by simply considering a steady conduction through this layer. One obtains

$$h = \frac{k}{\delta_0}\,\frac{\dfrac{A_2}{A_1} - 1}{\ln(A_2/A_1)} \qquad [20]$$

where k is the fluid thermal conductivity and A_1, A_2 are the surface areas of the body and of the outer fluid shell at a distance δ_0, respectively. In the case of a cylinder $A_2/A_1 = (D + 2\delta_0)/D$ and Eq. [20] becomes

$$Nu = \frac{2}{\ln\left(1 + 2A\,\dfrac{\sqrt{\nu\theta}}{D}\right)} \qquad [21]$$

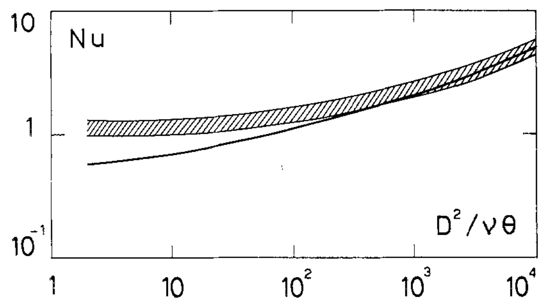


Fig. 3. Heat transfer correlation. The shaded area represents the data by *James* and *Acosta*

where Nu is the Nusselt number and use has been made of Eq. [7].

Fig. 3 compares the data by *James* and *Acosta* with the predictions obtained by using Eq. [21] with a value of $A = 25$. This value was assigned by imposing $Nu = 5$ at $D^2/\nu\theta = 10^4$, i.e. by fitting the data at the extreme right of the diagram. (Note that the value of θ used by *James* and *Acosta*, which differs by a factor 2/5 from that of Eq. [17], was not changed in this figure.)

The shape of the curve predicted by Eq. [21] is not too different from that indicated by the data. The difference by a factor of ~2 in the Nusselt number which is observed at low values of $D^2/\nu\theta$ could be attributed either to natural convection or to a different behaviour of the front and rear zones of the cylinder. At large values of $D^2/\nu\theta$, the thickness of the boundary layer is relatively small and the front zone is virtually the only zone which contributes to the heat transfer. Conversely, at small values of $D^2/\nu\theta$, the thickness is so large that the two zones become equivalent in their thermal behaviour.

It should finally be noted that the magnitude of the numerical factor in the heat transfer correlation proves to be about the same as that obtained in the drag coefficient correlation which was discussed in the previous section.

Summary

The concept of an "elastic" boundary layer is proposed to explain certain anomalous transport phenomena which occur during rapid external flows of viscoelastic fluids past immersed objects. Reported experimental observations are interpreted by using models based on this concept. Particularly, data on velocity independent drag and heat transfer coefficients for flow of dilute polymer solutions past tiny cylinders are satisfactorily correlated.

Zusammenfassung

Es wird das Konzept einer „elastischen" Grenzschicht entworfen, um gewisse anomale Transportphänomene zu erklären, welche bei schnellen Strömungen viskoelastischer Flüssigkeiten um eingetauchte Körper auftreten. Die berichteten experimentellen Beobachtungen werden mit Hilfe von Modellen interpretiert, die auf diesem Konzept basieren. Insbesondere werden Daten über geschwindigkeitsunabhängige Widerstands- und Wärmeübertragungs-Koeffizienten bei der Strömung verdünnter Polymerlösungen um dünne Zylinder befriedigend korreliert.

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