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Mode of development of sigmoidal en echelon fractures

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Abstract. The solution of stress distribution for a multicrack system and model experiments confirm that en echelon cracks mutually interact with each other during their growth. Such a mechanical interaction deviates the crack-tip stress axes orientations from that of the bulk stress field and leads to a continuous change in propagation direction of tension cracks, initially at a right angle to the bulk tension direction. The sigmoidal shape of en echelon fractures evolve through rotation and crack length increments with changing orientations. The theoretical analysis shows that the instantaneous fracture-tip stress orientation is a function of initial crack spacing, orientation of crack array with respect to the principal axes of far-field stress.

Keywords. En echelon cracks; sigmoidal fractures; progressive deformation.

1. Introduction

Sigmoidal en echelon fractures (veins) are characteristically common in rocks which have deformed in brittle-ductile regime. These veins occur either in a single set or conjugate sets. Field data (Shainin 1950; Roering 1968) show a wide range of variation in the dihedral angle of conjugate en echelon veins. Initial fractures in conjugate sets may or may not be mutually parallel to each other (Beach 1975). It has been interpreted (Pollard *et al* 1982; Segall 1984; Ramsay and Huber 1983, 1987) that en echelon fracture formation may involve indilation of the rocks. Ramsay and Huber (1987) have shown that individual fracture orientation in the array will be at an angle of less than and greater than 45° for positive and negative dilation respectively. However, Olson and Pollard (1991) have argued that arrays of any orientation may develop in response to a small positive dilation.

En echelon fractures, in majority, are believed to initiate as tension cracks along potential planes of shear in response to either primary or secondary stress field (Hancock 1972) and to propagate at a right angle to the principal tension direction. However, en echelon fractures are sometimes parallel to the principal shear plane (Roering 1968). Olson and Pollard (1991) have shown that initial cracks in a series itself may be locales of stress intensification and act as a mechanically favoured zone of en echelon fracture growth.

In the history of progressive development of sigmoidal en echelon fractures, as postulated by Ramsay and Huber (1987), a set of tension cracks initiate at an angle of 135° with the shear direction. The crack length defines the shear zone width at that instant. With continued deformation the cracks rotate and open out, and simultaneously propagate at its initial orientation. The cracks eventually attain sigmoidal, spindle-shaped geometry. However, the growth history of fractures that occur in a series is much more complex (Olson and Pollard 1991) where inter-crack interaction can be a factor.

With the help of model experiments and theory of stress distribution for multicrack system, the present paper substantiates that the mutual interaction among en echelon cracks is important in the development of sigmoidal en echelon fractures. En echelon cracks, initially at a right angle to the bulk tension direction will necessarily redirect their propagation path during the growth. It is shown that the initial crack spacing, normalized to crack length determines the degree of deviation of the propagation path.

2. Experimental study

2.1 *Experimental method*

Fractures were simulated in plasticine layers lying above a viscous substratum. Plasticine is somewhat ductile in room temperature (30°C) condition. Raw talc powder was added in a volume ratio of 3:1 and its ductility reduced. A 3 mm thick plasticine layer was stuck on the top of viscous (pitch) block. A series of cuts were then induced in the layer with the cut array at an angle of 45° with the layer's boundary (figure 1). Two wooden bars were fixed at the two lateral faces of the pitch block parallel to the induced cuts. The wooden bars were pulled apart to flow the pitch and create a tension in the overlying plasticine layer. The movement direction was such that the overall tension was at a right angle to the cut. A number of experiments were conducted with different initial cut spacing.

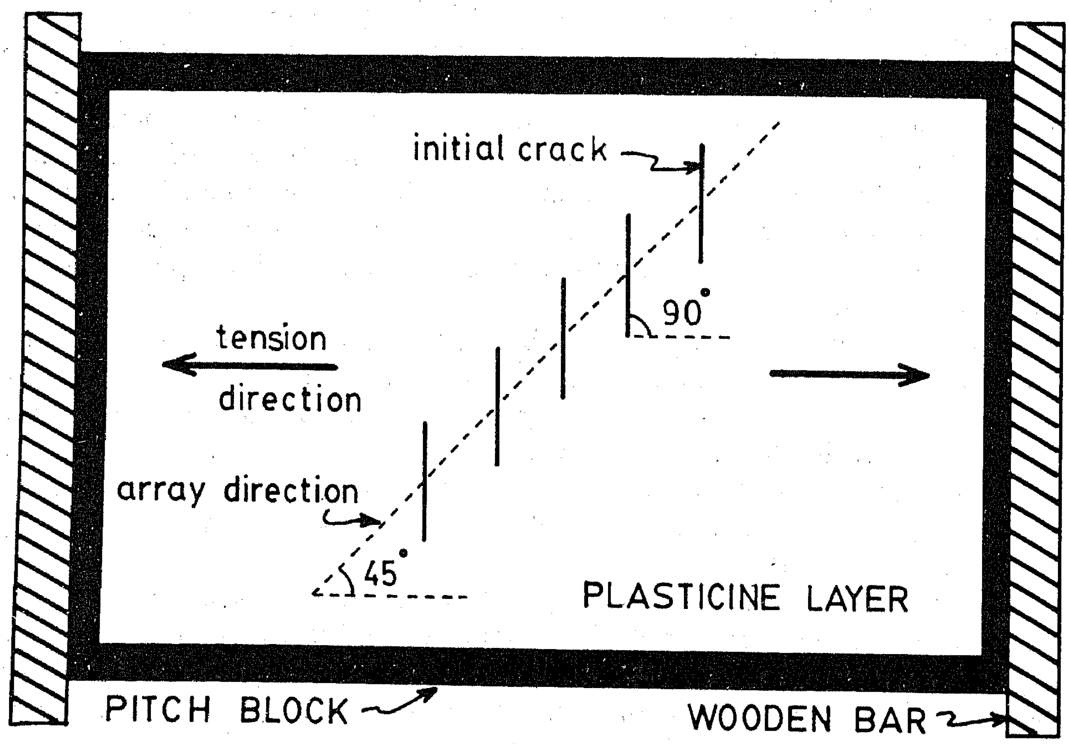

Figure 1. A schematic plan view of experimental setup.

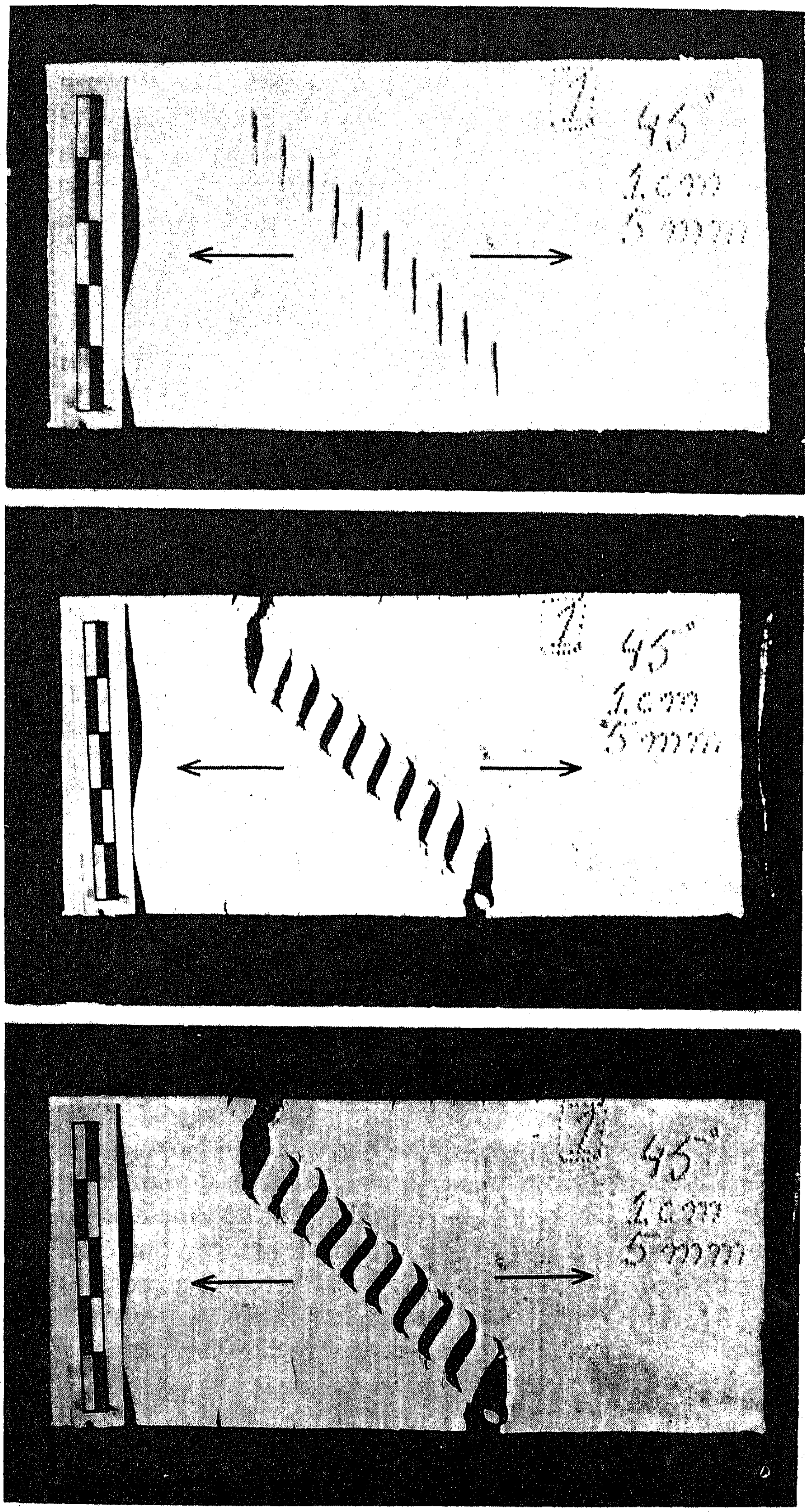

Figure 2. Successive stages of development of en echelon sigmoidal fracture in plasticine layers. Initial cut spacing 0·5 cm, individual cut length 1 cm and cut array at 45° with bulk tension direction.

2.2 *Experimental results*

Under tension, cuts immediately propagated but not at a right angle to the overall tension direction and fracture tips had a tilt from their initial orientation (figures 2 and 3). The propagation path continuously changed its direction and made a lower angle with the cut array. With this, the cuts rotated in consistency with the shear sense along the cut array as well as opened out. They finally acquired a sigmoidal spindle-shaped geometry.

Both the degree of rotation and deviation of propagation path depended on the initial cut spacing when all other parameters like cut length, array orientation were identical (figure 4). With increase in the initial cut spacing the rate of change in propagation direction reduced. This is because of lesser mutual interaction of the local stresses around cuts as their spacing increased. When the spacing was extremely large, the cuts propagated undeviated for a large extent. However, as the fracture length increased to a large value (comparable to the cut spacing) they started to interfere with each other and redirected their propagation path (figures 3 and 4b).

The rotation magnitude of individual cuts also decreased with the increase in initial cut spacing (figures 2b and 4) while they underwent a large amount of opening for a given bulk extension (figure 5a). The rotation of widely-spaced en echelon cuts became important after they increased in length in progressive extension.

En echelon fracture zone widened enough only when the initial cracks had a large spacing (figure 5). For a lower spacing the fracture propagation path rapidly turned to the array direction and yielded to a limited widening of the fracture zone. For a very close crack spacing the path deviated for a large angular span. The fractures ultimately coalesced with each other and formed a single fracture along the array (figure 5c). The bridge-segments of the en echelon fractures rotated bodily within the 'fault zone' and occurred as 'fault breccia'.

3. Theoretical analysis

In this section we shall see that, when a set of cracks are in en echelon arrangement and are close to each other, the principal axes of crack-tip stress significantly deviate from those of the far-field stress, and we can predict the changing propagation path of experimental en echelon fractures. The present analysis is based on the general theory of stress distribution around crack in infinitely extended medium.

Let A_0B_0, A_1B_1, A_2B_2 be any three cracks in an echelon array in $o_0x_0y_0$ space where individual cracks are parallel to x_0 axis while the array makes an angle of θ with x_0. Two other references $o_1x_1y_1$ and $o_2x_2y_2$, parallel to $o_0x_0y_0$ be chosen at the centres of cracks A_1B_1 and A_2B_2 (figure 6). We shall first derive the stress field for each crack independently with respect to their respective references but with the same boundary conditions. We shall then overlap these quantities in $o_0x_0y_0$ space and obtain the stress field of A_0B_0 in the en echelon arrangement. In the following deductions we shall use a common index $o_jx_jy_j$ $(j = 0, 1, 2)$ for the references.

From the plane theory of elasticity, stress components at a point are obtained as

$$\sigma_{x_j} + \sigma_{y_j} = [\phi'(z_j) + \overline{\phi'(z_j)}], \tag{1a}$$

$$\sigma_{y_j} - \sigma_{x_j} + 2i\sigma_{x_j}\sigma_{y_j} = 2[\overline{z_j}\phi''(z_j) + \psi(z_j)] \tag{1b}$$

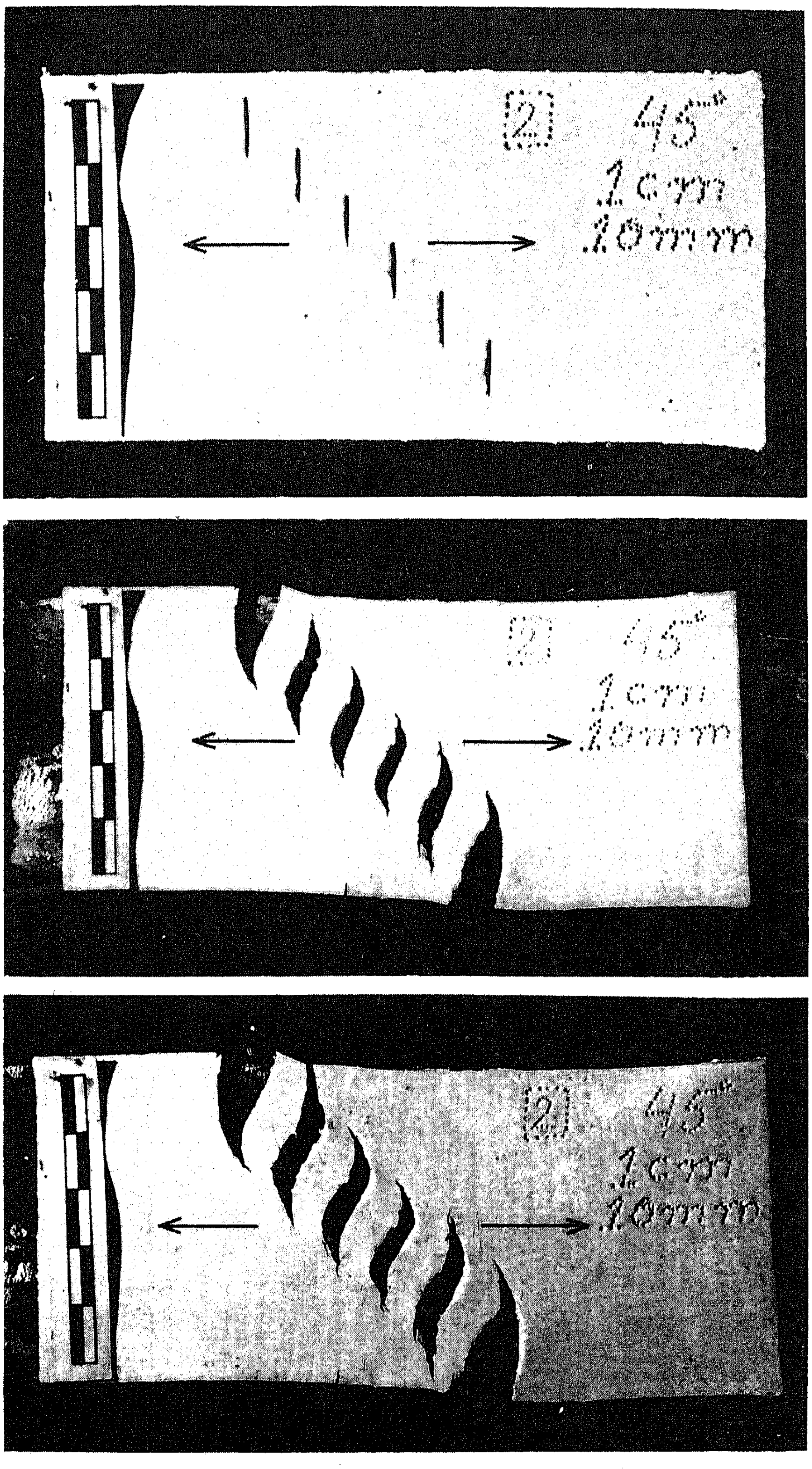

Figure 3. Successive stages of development of en echelon sigmoidal fractures. All parameters were same as in the experiments shown in figure 2 but with initial cut spacing 1 cm.

(Mushkhelishvili 1953), where $\phi(z_j)$ and $\psi(z_j)$ are functions of a complex variable $z_j(z_j = x_j + iy_j)$. Single and double primes indicate first order and second derivatives of the functions. For the stress analysis of a crack in infinite plane the functions

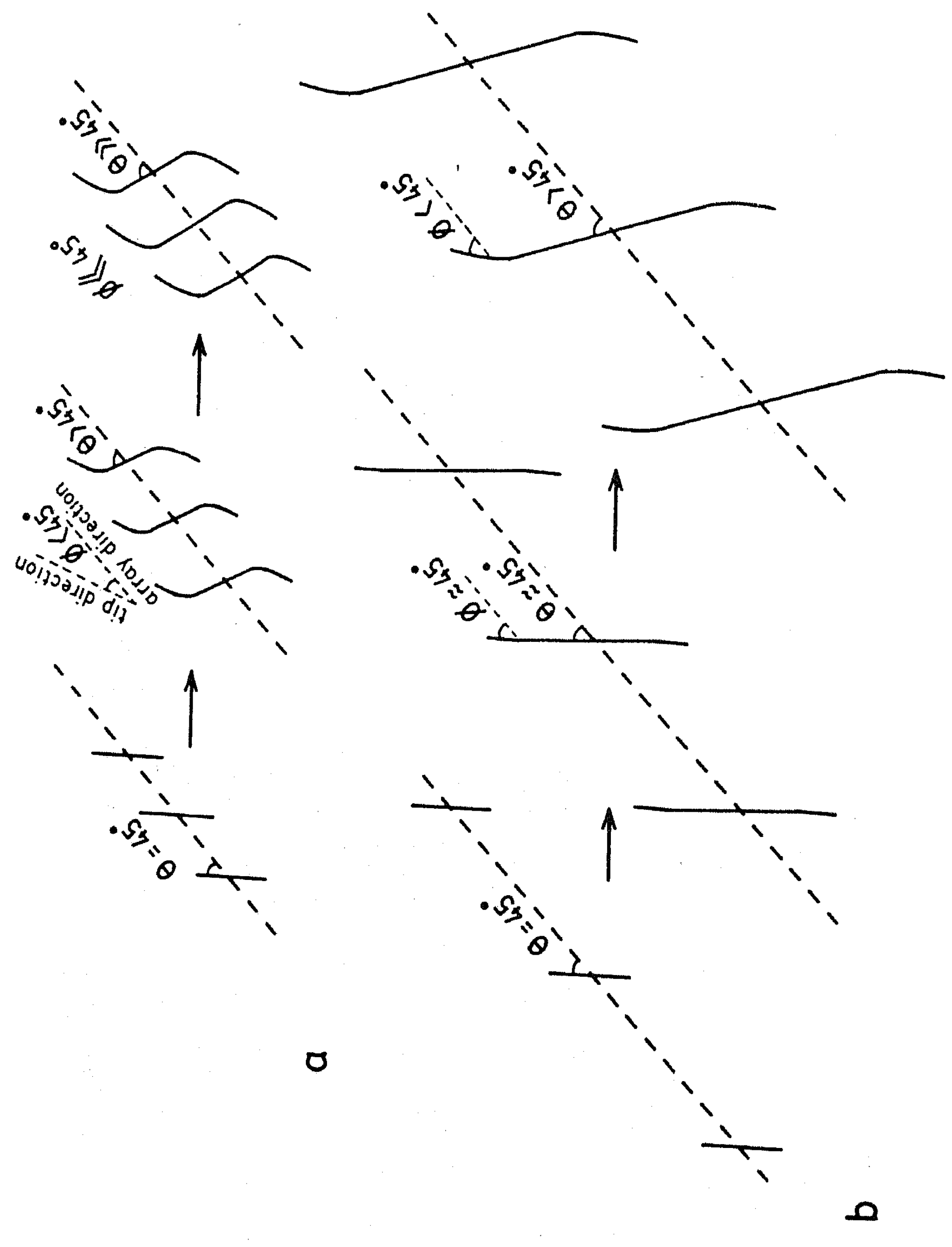

Figure 4. Progressive rotation and changes in propagation path of en echelon cracks in closed-spaced (**a**) and wide-spaced (**b**) arrangements.

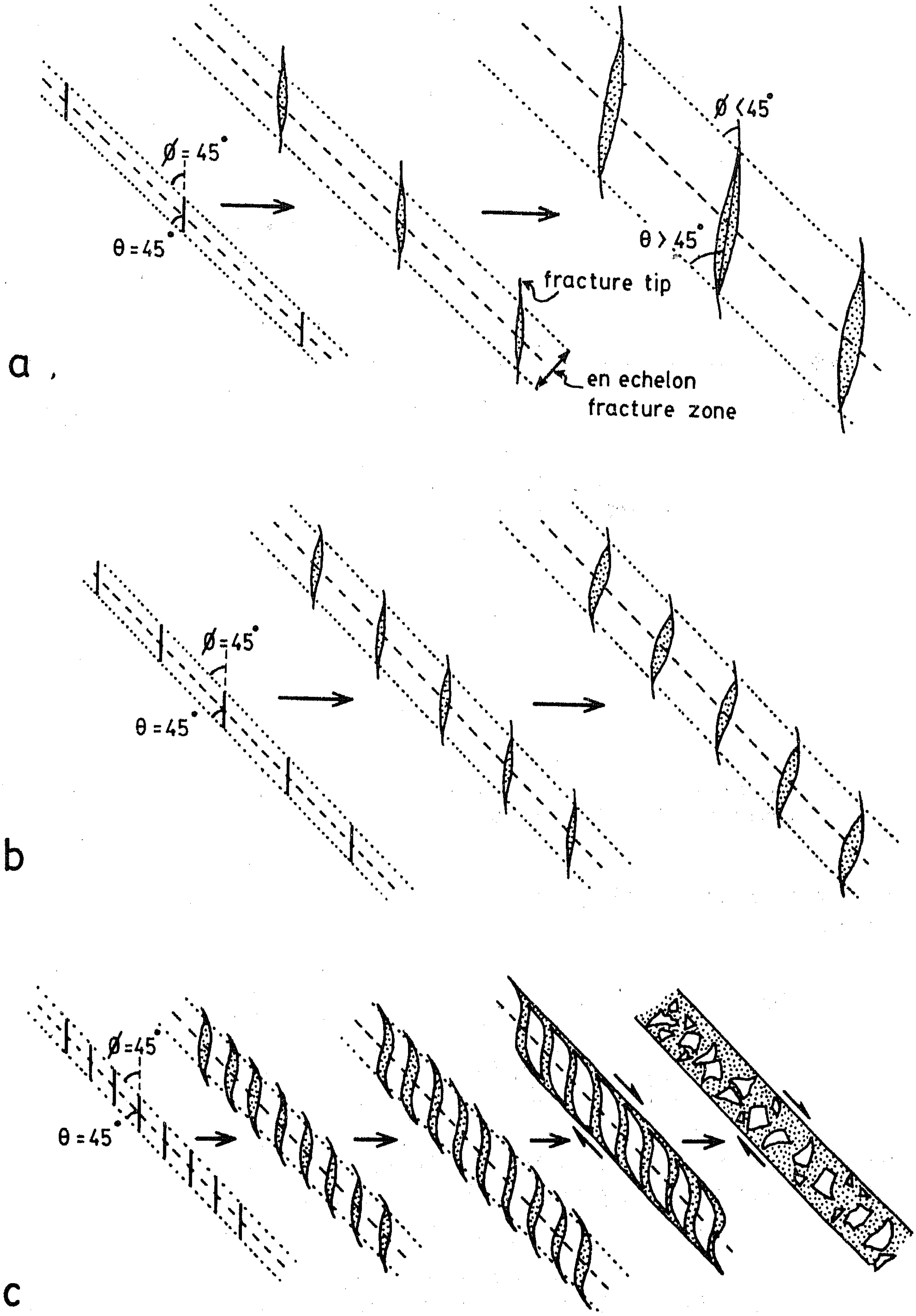

Figure 5. Widening of en echelon fracture zone, and propagation and opening of individual fractures with increasing initial crack spacing in (**a**), (**b**), and (**c**). Note that widening of fracture zones decreases from (a) to (c). (c) For a very closed crack spacing individual fractures coalesce to form a continuous fracture zone in which fracture-bridges occur as 'fault breccia'.

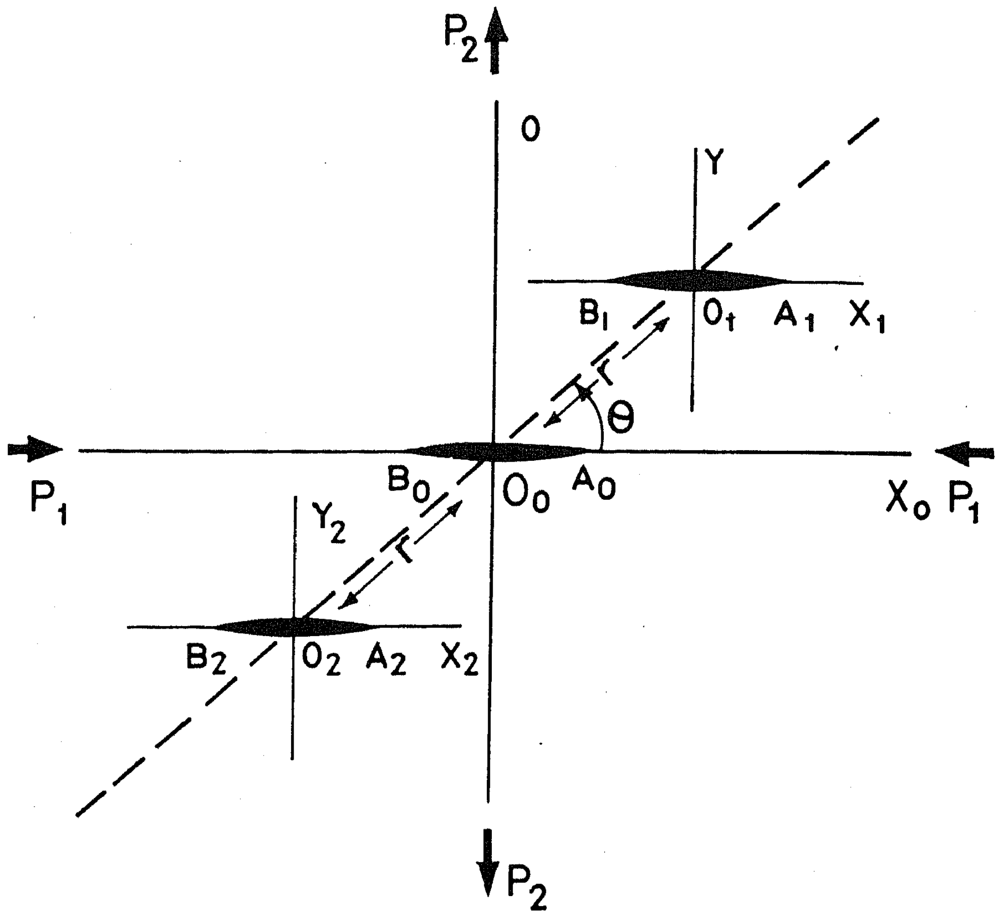

Figure 6. Consideration of reference axes for the theoretical analysis of stress distribution around cracks in en echelon arrangement.

can be expressed, after modifications of equations 2 and 3 of Mandal and Karmakar (1989) as

$$\phi(z_j) = -p_i R/4\left[\frac{z_j}{2R} + \frac{z_j^2 - 4R^2}{2R} + 2R\frac{2e^{2i\alpha} - 1}{z_j + (z_j - 4R^2)^{1/2}}\right], \tag{2a}$$

$$\psi(z_j) = p_i R/4\left[\frac{z_j + (z_j^2 - 4R^2)^{1/2}}{2R} + \frac{2R}{z_j + (z_j - 4R)}\right], \tag{2b}$$

where p_i is the principal stress, α is the angle of p_i with x_j axis and $R = a/2$, a is half the crack length. For the present problem of biaxial stresses with $\alpha = 0$ and $90°$ with $R = 1$ we have

$$\phi(z_j) = (p_2 - p_1)\frac{z_j + (z_j - 4R^2)^{1/2}}{8} + 2\frac{(p_1 + 3p_2)}{z_j + (z_j - 4R^2)^{1/2}}, \tag{3a}$$

$$\psi(z_j) = \frac{(p_2 + p_1)}{4}\left[z_j + (z_j^2 - 4)^{1/2} + \frac{1}{2}(z_j + (z_j^2 - 4)^{1/2})\right]$$
$$- 2p_2[z_j + (z_j^2 - 4)^2]/[\{z_j + (z_j^2 - 4)^{1/2}\} - 4]. \tag{3b}$$

From equations (1) and (3), and after separating the real and the imaginary parts we get

$$\sigma_{x_j} + \sigma_{y_j} = p_2\left[\frac{1}{2}(1 - R_s)(1 + A_j) + 2K_j(3 + R_s)\right], \tag{4a}$$

$$\sigma_{y_j} - \sigma_{x_j} = p_2\left[\frac{1}{2}(1 + R_s)(1 + A_j + B_j - 4K_j) + 4(E_j - F_j)\right], \tag{4b}$$

$$\sigma_{x_jy_j} = p_2\left[\frac{1}{2}(B_j - 4L_j) + 4(Q_j - G_j)\right], \tag{4c}$$

where $R_s = p_1/p_2$. The expressions of $A_j, B_j, \ldots$, parameters being functions of the co-ordinates of the point of interest, are given in table 1.

The orientations of the principal axes of stresses at a point in the neighbourhood of A_0B_0 are

$$\operatorname{Tan} 2\alpha' = 2\sum_{j=0}^{2} \sigma_{x_jy_j}/(\sigma_{x_j} - \sigma_{y_j}), \tag{5}$$

where α' is inclination of one of the principal axes of stresses with x_0 axis. Substituting the stress components from equations (4) in (5) we have,

$$\operatorname{Tan} 2\alpha' = \frac{\sum(1 + R_s)(B_j - 4L_j) + 8(Q_j - G_j)}{\sum(1 + R_s)(1 + A_j + B_j - 4K_j) + 8(E_j - F_j)}. \tag{6}$$

The orientations of the principal axes of stress (α') at the crack tip (A_0) gives us the direction of crack propagation assuming that the crack propagates in mode I. The co-ordinate-dependent parameters are determined using the following transformations:

$$x_0 = 2, \quad y_0 = 1$$

$$x_1 = 2 - r\cos\theta, \quad y_1 = -r\sin\theta$$

$$x_2 = 2 + r\cos\theta,\ y_2 = r\cos\theta. \tag{7}$$

Table 1. Parameters given in equations 4a, 4b and 4c.

$A_j = (x_jc_j + y_jd_j)/(c_j^2 + d_j^2);\ B_j = (y_jc_j - x_jd_j)/(c_j^2 + d_j^2)$
$E_j = x_jc_j(c_j^2 - 3d_j^2) - y_jd_j(d_j^2 - 3c_j^2)/(c_j^2 + d_j^2)^3$
$F_j = x_jc_j(c_j^2 - 3d_j^2) + y_jd_j(d_j^2 - 3c_j^2)/(c_j^2 + d_j^2)^3$
$Q_j = x_jd_j(d_j^2 - 3c_j^2) + y_jc_j(c_j^2 - 3d_j^2)/(c_j^2 + d_j^2)^3$
$G_j = x_jd_j(d_j^2 - 3c_j^2) - y_jc_j(c_j^2 - 3d_j^2)/(c_j^2 + d_j^2)^3$
$K_j = (x_jc_j - y_jd_j + a_j)/(x_jc_j - y_jd_j + a_j)^2 + (x_jd_j + y_jc_j + b_j)^2$
$L_j = -(x_jd_j + y_jc_j + b_j)/(x_jc_j - y_jd_j + a_j)^2 + (x_jd_j + y_jc_j + b_j)^2$
$c_j = \frac{1}{2}(\sqrt{a_j^2 + b_j^2} + a_j)^{1/2};\ d_j = \frac{1}{2}(\sqrt{a_j^2 + b_j^2} - a_j)^{1/2}$
$a_j = x_j^2 - y_j^2 - 4;\ b_j = 2x_jy_j$

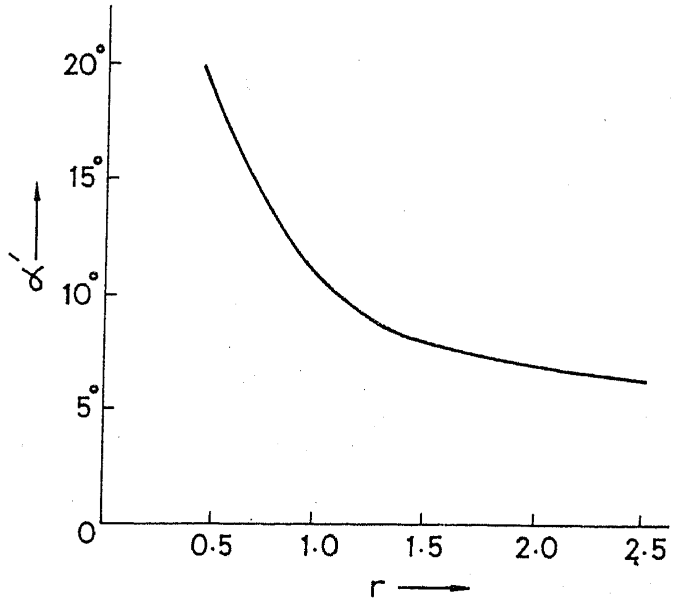

Figure 7. Calculated variation of the orientation of principal axes of stress at crack-tip (α') with initial crack spacing (r). $R = 1$, $R_s = 0$.

From equations (6) and (7) we find that the orientations of crack tip stresses (α') depend on the crack spacing (r), the orientation of crack array (θ) and the ratio of far-field principal stresses (R_s). $\alpha' = 0$ only when $\theta = 0$ or 90°. In other cases, $0° < \theta < 90°$, $\alpha' \neq 0$, that indicates that the crack propagation will deflect from its initial orientation. The deviation (α') decreases with increase in initial crack spacing (r) (figure 7) when all other parameters are kept identical. En echelon cracks, for a certain spacing, must grow to a critical length (e.g. in the present case $r = 1$ unit, $R = 1$, i.e. $a = 2$ units) at which they will interact with each other and behave as a multicrack system. At this stage they start to redirect their propagation path and describe a sigmoidal geometry.

4. Discussion

In the experiments en echelon cracks were induced at a desired spacing with an arbitrary array orientation. This approach does not give us any information about the mode of crack initiation and the factors that control the crack spacing. However, from the present study we can predict that, after initiation of en echelon cracks at right angle to the bulk tension direction the cracks will mutually interact with each other and redirect their propagation path in the course of their growth. Consequently, the sigmoidal geometry of en echelon fractures evolves by a combination of rotation and continuously changing propagation path of the initial cracks.

Both the theory and experiments show that cracks that are initially very closed-aced describe a large change in their propagation path and subsequently coalesce h each other and form a continuous fracture zone along the en echelon array. In al situations en echelon sigmoidal veins are likely to develop when the cracks are ly somewhat widely-spaced. Otherwise, if the condition of deformation is such he cracks nucleate with a high density distribution along the potential shear a narrow fault zone will be the ultimate product.

Rotation of en echelon fractures became significant when the cracks increased to a critical length and started to interact with each other. During the deformation of rocks in brittle-ductile regime incipient cracks may nucleate along potential zone of shear. However, shear failure along that zone will not occur until the cracks increased their length comparable to the initial spacing. Before this stage the cracks will only open up and propagate at a right angle to the bulk tension direction, without giving the form of a shear zone along the en echelon crack array.

The present theoretical analysis has been made with a conversion of the crack length factor $R(a/2) = 1$ for simplification of mathematical deductions. However, crack length will be a parameter determining the local stress distribution around a crack. As the crack length continually changes, the neighbourhood stress field will change accordingly. The present theoretical approach can be utilized only to determine the stress distribution for a certain stage of shear zone development. Another constraint of the present analysis is that, as it is based on linear crack system the theoretical result will not hold good when the cracks, in the course of propagation are significantly curved. The prime aim of the present analysis, however, is not to trace out the entire course of crack propagation but to show that the mechanical interaction is an important process in redirecting the propagation path of en echelon fractures.

From the present study we can go to the following conclusions. Mutual interactions among en echelon cracks are an important phenomenon in the development of sigmoidal fractures. En echelon fractures grow in response to local stresses, that continuously redirect their propagation path. For the development of sigmoidal en echelon fractures initial incipient cracks must have a critical range of spacing, below which individual fractures will coalesce with each other to form a single fracture zone and above which en echelon cracks will not mutually interact with each other and form sigmoidal shape. The maximum opening of a fracture in progressive deformation is also determined by the initial crack spacing.

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