

Stable Non-BPS States and Their Holographic Duals *

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Stable non-BPS states can be constructed and studied in a variety of contexts in string theory. Here we review some interesting constructions that arise from suspended and wrapped branes. We also exhibit some stable non-BPS states that have holographic duals.

1. Introduction

Type II string theory has stable, BPS Dp -branes with $p = 0, 2, 4, 6, 8$ in type IIA, and $p = -1, 1, 3, 5, 7, 9$ in type IIB. For the other values of p one finds unstable, non-BPS branes: $p = -1, 1, 3, 5, 7, 9$ in type IIA and $p = 0, 2, 4, 6, 8$ in type IIB theory.^{1,2} The spectrum on an unstable D-brane in superstring theory is the spectrum of a single open string, but without GSO projection. Hence there is a real tachyon.

The BPS branes are of course stable, while the non-BPS branes can decay, via tachyon condensation, into the vacuum, or into lower (BPS or non-BPS) branes. A pair of a BPS brane and its antibrane is also unstable and can decay similarly.

This is quite a general paradigm. In flat backgrounds, type IIA branes are either BPS and stable, or non-BPS and unstable. It is interesting to look for backgrounds which admit non-BPS but stable branes. In this situation, masses are not protected by BPS formulae. We can hope to disentangle effects of duality from effects of supersymmetry.

If the backgrounds are themselves non-supersymmetric then things rapidly become difficult. The most accessible situations are those where the backgrounds are supersymmetric, but the states that we study are not. Some examples are orbifolds, orientifolds and Calabi-Yau compactifications. Another class of examples is provided by suspended brane constructions.^{3,4} These all have lower supersymmetry than flat space, which helps to find stable non-BPS states.

In the following, we first investigate brane-antibrane configurations in the flat-space background of type II superstring theory and identify some stable non-BPS states. Next, we turn to AdS-type backgrounds and their holographically dual gauge theories.⁵ Here, we analyze stable, non-BPS configurations of branes wrapped over cycles in the $AdS_5 \times T^{1,1}$ background that is dual to 3-branes at a conifold.⁶ In the course of the discussion we will make extensive use of the conifold singularity and its brane-construction dual.⁷ ALE spaces will also play an auxiliary role.

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2. Singularities, Brane Duals and Fractional Branes

Let us start with type IIB on a Z_2 ALE singularity along the (6789) directions. Via T-duality along x^6 , the ALE singularity turns into a pair of parallel NS5-branes in type IIA string theory, extending along the (12345) directions and located at different points along the x^6 direction.⁸

The ALE singularity hides a 2-cycle Σ of zero size, which can be resolved to get an Eguchi-Hanson space. But at the orbifold point, the NS-NS B -field has a flux of $\frac{1}{2}$ through this 2-cycle.⁹ In the brane dual, the NS5-branes are symmetrically located along the x^6 circle. This duality extends beyond the orbifold point. Varying the B -flux in the ALE corresponds to varying the relative x^6 separations of the NS5-branes.¹⁰

If we bring a D3-brane into the plane of an ALE singularity, it can split into a pair of fractional D3-branes $f3, f3'$ of charge and tension α and $1 - \alpha$ where $\alpha = \int_{\Sigma} B$ is the B -flux.¹¹ The fractional branes are interpreted as:

$$\begin{aligned} f3 : & \quad \text{D5 wrapped on } \Sigma \\ f3' : & \quad \overline{\text{D5}} \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 1 \end{aligned}$$

In the dual brane construction, a D4-brane wrapped on x^6 can be brought in to touch the NS5-branes, where it can break into two pieces (Fig.1(a)). The gauge

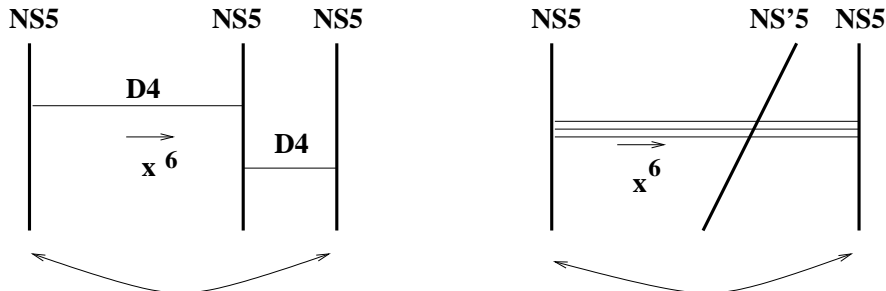


Fig. 1. (a) T-dual of D3-brane at an ALE singularity. (b) T-dual of D3-branes at a conifold.

group $U(1) \times U(1)$ and the presence of bi-fundamental matter is also evident from the brane construction.

An analogous relation holds for the conifold singularity along the (456789) directions. It is dual to a similar brane construction but with rotated NS5-branes (Fig.1(b)).⁷ This model too has bi-fundamental matter, but also a quartic superpotential as long as there is more than one D4-brane as shown in the figure.¹²

3. Fractional Branes and a Stable Non-BPS Configuration

An interesting class of non-BPS brane configurations is obtained from the system of an adjacent brane-antibrane pair.³ In some cases, this can be analyzed using

perturbative string theory, via duality to ALE or conifold singularities.

The configuration of interest contains a pair of parallel NS5-branes oriented as was just discussed. In the two intervals between the NS5-branes, we place a D4-brane and a $\overline{\text{D4}}$ -brane (Fig.2(a)). The NS5-brane configuration is T-dual to an

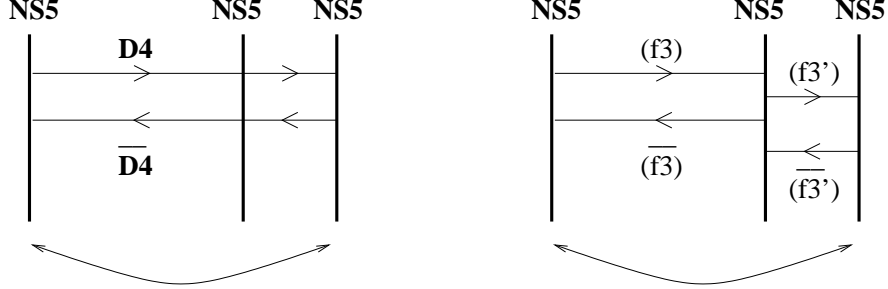


Fig. 2. (a) Brane-antibrane pair at an ALE. (b) The four types of (T-dual) fractional branes.

ALE singularity. The D4 and $\overline{\text{D4}}$ -brane in the intervals T-dualize into a fractional D3-brane and a fractional antibrane. Let us try to understand this correspondence in more detail.

A D3 – $\overline{\text{D3}}$ pair at a Z_2 ALE singularity splits into 4 distinct types of fractional branes, which we call $f3, f3', \overline{f3}, \overline{f3'}$. These are interpreted as follows:

$$\begin{aligned}
 f3 : & \quad \text{D5 wrapped on } \Sigma, \quad \int_{\Sigma} F = 0 \\
 f3' : & \quad \overline{\text{D5}} \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 1 \\
 \overline{f3} : & \quad \overline{\text{D5}} \text{ wrapped on } \Sigma, \quad \int_{\Sigma} F = 0 \\
 \overline{f3'} : & \quad \text{D5 wrapped on } \Sigma, \quad \int_{\Sigma} F = 1
 \end{aligned} \tag{1}$$

Introducing a D4 – $\overline{\text{D4}}$ pair in the brane construction, we see that it too can break into four distinct pieces: This is the Coulomb branch, and we can identify the four fractional branes as in Fig.2(b).

Since we are interested in studying an adjacent D4 – $\overline{\text{D4}}$ pair, we see that the dual fractional branes are $f3$ and $\overline{f3'}$. This system has a net D5-brane charge of +2, and a net D3-brane charge of $2\alpha - 1$.

The open strings connecting adjacent branes correspond in the ALE dual to the following Chan-Paton factors:

$$\begin{aligned}
 f3 - \overline{f3'} & : \quad \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
 \overline{f3'} - f3 & : \quad \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 - i\sigma_2) \\
 f3' - \overline{f3} & : \quad \frac{1}{2}(\sigma_3 - i\sigma_2) \otimes (\sigma_1 + i\sigma_2) \\
 \overline{f3} - f3' & : \quad \frac{1}{2}(\sigma_3 + i\sigma_2) \otimes (\sigma_1 - i\sigma_2)
 \end{aligned} \tag{2}$$

These are all odd under the ALE projection. Therefore the strings connecting $f3$ to $\overline{f3}$ have no tachyonic or massless bosonic states. In fact, these strings only give massless fermions.

Next we construct the boundary states corresponding to the fractional D3-branes, and use them to compute the force between the adjacent pair of interest. There are four independent consistent boundary states for D3, $\overline{D3}$, which can be identified with the four fractional branes $f3, f3', \overline{f3}, \overline{f3}$.¹³

$$\begin{aligned}
|D3, +\rangle &= \frac{1}{2} \left(|U\rangle_{NSNS} + |U\rangle_{RR} + |T\rangle_{NSNS} + |T\rangle_{RR} \right) & : f3 \\
|D3, -\rangle &= \frac{1}{2} \left(|U\rangle_{NSNS} + |U\rangle_{RR} - |T\rangle_{NSNS} - |T\rangle_{RR} \right) & : f3' \\
|\overline{D3}, +\rangle &= \frac{1}{2} \left(|U\rangle_{NSNS} - |U\rangle_{RR} - |T\rangle_{NSNS} + |T\rangle_{RR} \right) & : \overline{f3} \\
|\overline{D3}, -\rangle &= \frac{1}{2} \left(|U\rangle_{NSNS} - |U\rangle_{RR} + |T\rangle_{NSNS} - |T\rangle_{RR} \right) & : \overline{f3}
\end{aligned} \tag{3}$$

The amplitude of interest is:

$$\begin{aligned}
\int_0^\infty dl \langle \overline{D3}, + | e^{-lH_c} | D3, + \rangle &= \int_0^\infty \frac{dt}{2t} \text{tr}_{NS-R} \left(\frac{1-(-1)^F}{2} \frac{1-R}{2} e^{-2tH_0} \right) \\
&= \frac{v^{(4)}}{32(2\pi)^4} \int_0^\infty \frac{dt}{t^3} \left\{ \frac{f_3(\tilde{q})^8 + f_4(\tilde{q})^8 - f_2(\tilde{q})^8}{f_1(\tilde{q})^8} - 4 \frac{f_4(\tilde{q})^4 f_3(\tilde{q})^4 + f_4(\tilde{q})^4 f_3(\tilde{q})^4}{f_1(\tilde{q})^4 f_2(\tilde{q})^4} \right\}
\end{aligned} \tag{4}$$

This simplifies to:

$$\frac{v^{(4)}}{16(2\pi)^4} \int_0^\infty \frac{dt}{t^3} \frac{f_4(\tilde{q})^8}{f_1(\tilde{q})^8} \left[1 - 4 \frac{f_1(\tilde{q})^4 f_3(\tilde{q})^4}{f_2(\tilde{q})^4 f_4(\tilde{q})^4} \right] \tag{5}$$

The integrand is strictly negative, implying that the force between the $f3$ and $\overline{f3}$ is repulsive. Thus we find that the force between an adjacent suspended brane-antibrane pair is repulsive.

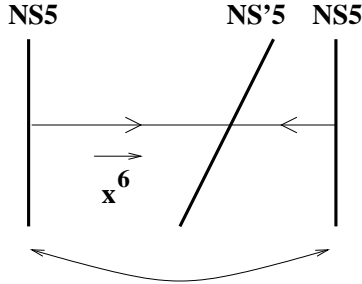


Fig. 3. Adjacent brane-antibrane pair at a T-dual conifold.

Now consider a “twist” on the configuration of adjacent brane-antibrane pairs that we discussed earlier.⁴ We rotate one NS5-brane as in Fig.3. Thus we now have

an NS5 and an NS5'-brane, making up the brane dual of the conifold. The adjacent D4-brane-antibrane pair is dual to fractional D3-branes at a conifold.

Physically, we expect a repulsive force between the adjacent brane and antibrane, as was shown earlier in the unrotated model. But there is also a classical attraction since the branes cannot separate without being stretched. This leads to a possibility of stable equilibrium at finite displacement. In fact we get a more complicated and interesting result exhibiting a phase transition as a function of the radius r of the compact x^6 direction.

The energy of the stretched D4-brane is

$$\mathcal{V} T_4 \sqrt{L^2 + 2r^2} \quad (6)$$

where \mathcal{V} is an (infinite) volume factor, T_4 is the tension of a BPS D4-brane, and L is the separation between the NS5 and NS5'-branes.

We assume that the repulsion is as for the ALE (unrotated) case, since it comes from strings connecting the D4 – $\bar{D}4$ pair across each NS5-brane. After a calculation,⁴ we find that the shape of the potential depends on the separation parameter L as shown in Figs.4(a) and 4(b) for small L and large L respectively. Here $y \sim r$ with a constant rescaling.

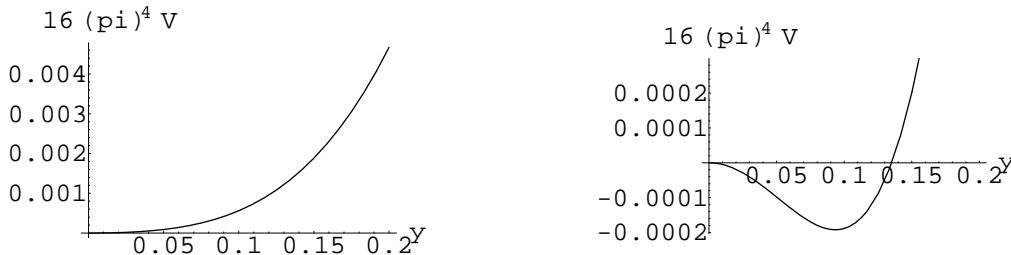


Fig. 4. (a) Brane-antibrane potential for small L . (b) Brane-antibrane potential for large L .

Hence the brane and antibrane are aligned for small L but they separate to a finite distance for large L . An estimate shows that the potential takes its minimum at $L_c \sim 0.28 g_s^{-1}$.

4. Branes at a Conifold and Non-BPS States in AdS_5

If we bring N D3-branes to a conifold singularity and take the large- N limit, we end up with a $\frac{1}{4}$ -supersymmetric background of type IIB: $AdS_5 \times T^{1,1}$ where $T^{1,1}$ is a particular Einstein 5-manifold.⁶ If we T-dualize the conifold we get a model of rotated NS5-branes. N D3-branes at the conifold become N D4-branes wrapped round the x^6 circle, as described above.⁷

The adjacent brane-antibrane model that we have described above does not have an AdS dual. If we add N D4-branes to it, then the $\bar{D}4$ will annihilate against a

fractional D4-brane, leaving $N - 1$ whole D4-branes plus two fractional D4-branes, as shown in Fig.5. Let us now describe a stable non-BPS brane construction that,

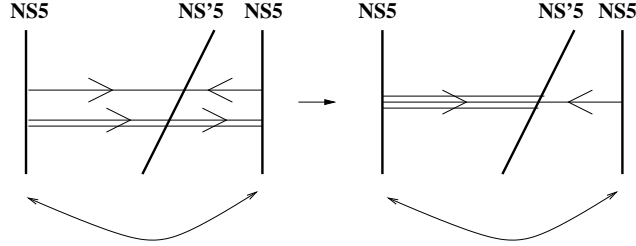


Fig. 5. Adjacent brane-antibrane pair at a T-dual conifold.

instead, does have an AdS dual.

Take N D4-branes as before and introduce a D2-brane in the first interval, as in Fig.6. In the conifold geometry, this corresponds to the introduction of a

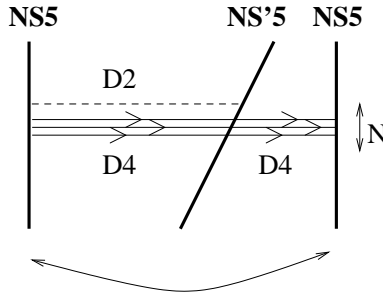


Fig. 6. T-dual conifold with D2 and D4-branes.

fractional D-string in the plane of the singularity. This configuration is clearly non-supersymmetric. For example, the strings joining a D2-brane and N D4-branes in the interval will be tachyonic. This part of the configuration will decay into a stable bound state of the D4-branes and the D2-brane. While this is BPS by itself, the neighbouring interval still has only D4-branes, as in Fig.7. The $(D2, D4)$ bound state and the D4-branes preserve incompatible supersymmetries. Hence the whole system is non-BPS, much as for an adjacent brane-antibrane pair. In the conifold geometry, we have a fractional D-string bound to N $f3$ -branes and coincident with N $f3'$ branes.

This system is stable, and we can take the large N limit. In this limit, the conifold geometry is replaced by its 5-manifold base, the Einstein space $T^{1,1}$. Topologically,

$$T^{1,1} \sim S^2 \times S^3$$

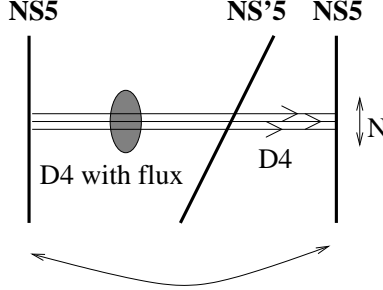


Fig. 7. T-dual conifold with D4-branes and flux.

The S^2 is the same 2-cycle that was of vanishing size before taking the large- N limit. The fractional D-string was actually a D3-brane wrapped on this S^2 .^{14,10} Hence, in the large N limit, the fractional D-string can be identified with a “fat string” obtained by wrapping a D3-brane on S^2 . We will analyze this fat string further in the following section.

At this point, it is instructive to list all the unwrapped and wrapped branes of this model:

Dimension	Unwrapped	S^2	S^3	$S^2 \times S^3$
-1	$D(-1)$	$D1$	$UD2$	$UD4$
0	$UD0$	$UD2$	$D3$	$D5$
1	$D1$	$D3$	$UD4$	$UD6$
2	$UD2$	$UD4$	$D5$	$D7$
3	$D3$	$D5$	$UD6$	$UD8$
4	$UD4$	$UD6$	$D7$	$D9$

In this table, the prefix “U” indicates an unstable brane. The remaining branes are stable. Some of these wrapped branes have been studied previously.^{14,10} For example, the D5 wrapped on S^2 is known to be a domain wall that augments the gauge group: $U(N) \times U(N) \rightarrow U(N+1) \times U(N)$, while the D3 wrapped on S^2 is our fat string. We would like to understand its holographic dual description. The Euclidean D-string wrapped on S^2 gives rise to an instanton, while the (unstable) $UD2$ on S^2 is an unstable D0-brane.^a We will comment on their holographic duals too.

5. Properties of the Fat String

The nature of the fat string depends on the B-flux through S^2 . In general we have

$$\int_{S^2} B_{NS,NS} = \alpha, \quad \int_{S^2} B_{RR} = \beta$$

^aThese are distinct from the standard type IIB D-instantons and D0-branes, whose holographic duals have been studied previously.^{15,16}

The $U(N) \times U(N)$ gauge theory on the 3-branes has couplings and θ -angles given by:¹⁷

$$\begin{aligned}\tau_1 &= \beta + \alpha\tau_s \\ \tau_2 &= -\beta + (1 - \alpha)\tau_s\end{aligned}\tag{7}$$

where $\tau_s = \frac{\chi_{RR}}{2\pi} + \frac{i}{g_s}$.

The fat string carries D-string charge α and F-string charge β , by virtue of the Chern-Simons coupling

$$\int B_{NS,NS} \wedge B_{RR} \rightarrow \alpha \int B_{RR} + \beta \int B_{NS,NS}$$

on a D3-brane. It is convenient to choose $\beta = 0$. The tension of the fat string can be estimated from integrating the DBI action of a D3-brane over S^2 :

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{\det g + (B_{NS,NS})^2}$$

In the flat space limit, the S^2 is of zero size and this becomes

$$T_{\text{fat}} \sim T_3 \alpha$$

which shows that it is BPS. On the other hand at large N the dominant contribution comes from

$$T_{\text{fat}} \sim T_3 \int_{S^2} \sqrt{g} \sim \frac{N}{(g_s N)^{\frac{1}{2}} \alpha'}$$

As with fractional branes, there are really two complementary fat strings, the second one being an anti D3-brane wrapped over S^2 and having a magnetic flux $\int F = 1$ over the cycle. We call this a fat' string. It has a D-string charge $(1 - \alpha)$. The non-BPS nature of fat strings, and their charges, imply that a fat string and a fat' string can annihilate with loss of energy into a D-string.

Recall how a D-string is understood in holography.¹⁸ In $AdS_5 \times S^5$, a D-string parallel to the boundary corresponds to a magnetic flux tube. As the string falls towards the horizon, the flux tube fattens and in the limit becomes a constant flux. The same result holds for a D-string in $AdS_5 \times T^{1,1}$, but the flux is in the diagonal of the $U(N) \times U(N)$ gauge group.

The fat string is similarly a flux tube in the boundary theory, but this time the flux is only in one $U(N)$ factor. This is consistent with its non-BPS nature. On a 3-brane we have nonlinearly realized supersymmetry that acts on the gauginos as:

$$\delta^* \lambda_\alpha^{(1)} = \frac{1}{4\pi\alpha'} \eta_\alpha^*, \quad \delta^* \lambda_\alpha^{(2)} = \frac{1}{4\pi\alpha'} \eta_\alpha^*$$

and linearly realized supersymmetry:

$$\delta \lambda_\alpha^{(1)} = F_{23}^{(1)} \sigma_\alpha^{23\beta} \eta_\beta, \quad \delta \lambda_\alpha^{(2)} = F_{23}^{(2)} \sigma_\alpha^{23\beta} \eta_\beta$$

If and only if the fluxes are diagonal: $F^{(1)} = F^{(2)} = F$, there is a surviving set of linearly realized supersymmetries, described by choosing

$$\eta_\alpha^* = -4\pi\alpha' F_{23}\sigma_\alpha^{23\beta} \eta_\beta$$

For non-diagonal fluxes, no supersymmetry is preserved.

One can also compute the potential experienced by the non-BPS fat string, and also study Wilson/'t Hooft loops in the *AdS* context.¹⁹

Let us comment briefly on some of the other wrapped branes. The euclidean D1 wrapped on S^2 is a new ‘‘D-instanton’’. It is expected to be dual to a Yang-Mills instanton in the first factor of $U(N) \times U(N)$. It has its own associated sphaleron, the D2-brane of type IIB wrapped on S^2 . The relation between the two is parallel to the one between unwrapped D-instantons and D0-branes, studied recently.^{20,16}

6. Conclusions

The stable brane-antibrane construction that we have exhibited should describe an interesting non-SUSY model field theory. Microscopically it has a pair of branes separated by a finite calculable distance. Such constructions might be useful in making brane-world type models. It would be interesting to understand the spectrum and interactions of the effective low-energy field theory on these branes, which have so far not been worked out in complete detail.

Recall that the conventional BPS brane constructions are most useful when we can use S-duality (in type IIB) or the duality with M-theory (in type IIA). What do we learn from these dualities about brane-antibrane constructions? A stable non-BPS brane configuration in type IIA theory, such as the one we have exhibited, must have a well-defined M-theory limit as an M5-brane wrapping a 2-cycle. Because the configuration is not BPS, the 2-cycle will not be holomorphic, so a novel approach would be required to determine it.

In the present discussion, ‘‘fat’’ objects were associated to one $U(N)$ factor while ‘‘thin’’ objects are diagonal in $U(N) \times U(N)$. This is quite general. The study of branes at more complicated singularities gives rise to product gauge groups involving many factors, and one should then be able to construct large numbers of non-BPS objects associated to one or more of these factors. Each one will be a ‘‘fat’’ object, related to a fractional brane at the generalized singularity.

The stable non-BPS configurations described here are particularly suitable for investigation using the AdS/CFT correspondence. This should help us to generalize many of the notions of holography to situations without supersymmetry, nevertheless retaining some control over the dynamics.

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