F-theory at Constant Coupling

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ABSTRACT

The subspace of the moduli space of F-theory on K3 over which the coupling remains constant develops new branches at special values of this coupling. These values correspond to fixed points under the $SL(2, Z)$ duality group of the type IIB string. The branches contain points where K3 degenerates to orbifolds of the four-torus by Z_3, Z_4 and Z_6. A singularity analysis shows that exceptional group symmetries appear on these branches, including pure $E_8 \times E_8$, although $SO(32)$ cannot be realised in this way. The orbifold points can be mapped to a kind of non-perturbative generalization of a IIB orientifold, and to M-theory orbifolds with non-trivial action on 2-brane wrapping modes.

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1. Introduction

Compactifications of the type IIB string in which the complex coupling varies over a base are generically referred to as "F-theory". The simplest such construction corresponds to elliptically fibred K3, equivalent to type IIB on P^1 with 24 7-branes[1]. Other compactifications of F-theory, and many miraculous properties of this class of models, have been investigated in Refs.[1][2][3][4][5][6][7].

Recently some new insights into the K3 compactification have been obtained[8] by considering a limit in which the coupling is constant over the base. In this limit, the elliptic fibre always has a constant, arbitrary complex-structure modulus τ, but over 4 points of the base it degenerates to the singular fibre T^2/Z_2. The entire K3 then degenerates to an elliptically fibred orbifold T^4/Z_2. The advantage of considering this limit is that one can explicitly map the problem to an orientifold of the type IIB string. It turns out that physical effects which are nonperturbative and hence invisible in the orientifold picture are nicely captured by F-theory.

The limit considered in Ref.[8] has a 2 complex dimensional moduli space, of which 1 complex dimension corresponds to the arbitrary value of the coupling constant (there is also one real modulus, the size of the base, which we will not refer to explicitly since it is always present). Thus, there is a region of weak IIB coupling in this moduli space.

In this note, we observe that there are two other branches of the F-theory moduli space on K3 where the coupling is constant over the base. On these branches, the constant coupling is fixed to lie at one of the fixed points of the moduli space of the elliptic fibre, namely $\tau = i$ or $\tau = \exp(i\pi/3)$. The corresponding moduli spaces have 5 and 9 complex dimensions respectively. At special points within these branches, the base becomes T^2/Z_n and the whole K3 becomes the orbifold T^4/Z_n, with the fibre becoming T^2/Z_p over fixed points of order p (where p divides n) on the base. Here $n = 4$ in the first branch, and $n = 3, 6$ in the second branch.

At these orbifold points, a singularity analysis predicts to various enhanced-symmetry groups containing E_6, E_7 and E_8 factors. Also, following Ref.[8] one can map the theory to a kind of orientifold[9][10] of the type IIB string. However, in this case the orientifold group includes nonperturbative symmetries of the string theory.

2. F-theory and K3 Orbifolds

Elliptically fibred K3 surfaces can be defined by the family of elliptic curves

$$y^2 = x^3 + f(z)x + g(z) \tag{1}$$

where z is a coordinate on the P^1 base, and f, g are polynomials of degree 8,12 respectively in z. The modular parameter $\tau_f(z)$ of the fibre is given in terms of the j-function by

$$j(\tau_f(z)) = \frac{4(24f(z))^3}{4f(z)^3 + 27g(z)^2} \tag{2}$$

The case of constant modulus corresponds to a situation where $f^3 \sim g^2$, in which case one can write

$$f(z) = \alpha(\phi(z))^2$$
$$g(z) = (\phi(z))^3 \tag{3}$$

with $\phi(z) = \prod_{i=1}^{4}(z - z_i)$ an arbitrary polynomial of degree 4 (whose overall coefficient can be scaled to 1). The constant α determines the modular parameter by

$$j(\tau_f) = \frac{55296\alpha^3}{4\alpha^3 + 27} \tag{4}$$

Thus we have obtained a subspace of the moduli space on which the elliptic fibre has constant modulus τ_f. Besides this modulus, the subspace also has a complex parameter giving the location of one of the zeroes of $\phi(z)$ (the other three locations can be fixed by the $SL(2, C)$ invariance of P^1), and a real parameter corresponding to the size of the base.

It has been argued in Ref.[8] that this subspace represents a K3 that has degenerated to T^4/Z_2, with the base P^1 having become T^2/Z_2. The one free complex parameter in $\phi(z)$ represents the complex structure of this base, which we call τ_b, while the size of the P^1 remains the real Kähler modulus of T^2/Z_2. In the duality between F-theory on K3 and the heterotic string on T^2[1], this region can be mapped quite explicitly: the F-theory side has gauge symmetry $SO(8)^4 \times (U(1))^4$, which means the heterotic string has Wilson lines in the 9 and 10 directions breaking $SO(32)$ or $E_8 \times E_8$ to $SO(8)^4$, while the complex-structure moduli τ_f and τ_b become the complex- and Kähler-structure moduli τ and ρ of the T^2 on which the heterotic string is compactified. Finally, the size of the base in the F-theory picture gets mapped to the heterotic string coupling.

It is also worth remarking that, in accordance with the analysis in Ref.[11], the heterotic string cannot develop any enhanced symmetry in the presence of these Wilson lines,

so there is no prediction of any further enhanced gauge symmetries on the F-theory side as we vary τ_b and τ_f.

Two other branches of moduli space over which τ_f remains constant emerge, very simply, in the limits $\alpha \to \infty$ and $\alpha \to 0$. The former limit corresponds (after a rescaling) to taking $g(z) = 0$, so that $j(\tau_f) = 13824$, from which $\tau_f = i$. The latter limit gives instead $f(z) = 0$, from which $j(\tau_f) = 0$ and $\tau_f = \exp(i\pi/3)$. In these limits, the parametrization implied by Eq.(3) is no longer required, so the theory develops a new branch in its moduli space. (It should be stressed that these are not new branches of F-theory, but rather new branches of the subspace of F-theory moduli space over which τ_f is constant.)

On the first branch, which we call branch (I) from now on, $f(z)$ is an arbitrary polynomial of degree 8, while on the second (branch (II)), $g(z)$ is an arbitrary polynomial of degree 12. After subtracting an overall scaling and 3 $SL(2,C)$ parameters, one finds that these branches of moduli space have complex dimension 5 and 9 respectively. Moreover, none of these parameters is the IIB coupling, since that remains fixed on each branch. The size of the base of course continues to be a real modulus.

Let us analyze the structure of the base on these branches. On branch (I), Eqn.(1) has the discriminant

$$\Delta(z) = \prod_{i=1}^{8}(z - z_i)^3 \tag{5}$$

where z_i are the 8 zeroes of $f(z)$. Thus, generically, the base is a P^1 with 8 singular points, at each of which there are 3 F-theory 7-branes, producing a deficit angle of $\pi/2$. These cannot be thought of as orbifold singularities, since the deficit angle has to be of the form $\frac{n-1}{n}2\pi$ for a fixed point of order n.

Suppose we go to the special point in this moduli space where the 8 zeroes of f have coalesced into 3 zeroes of order 3, 3 and 2. In this case,

$$\Delta(z) = (z - z_1)^9(z - z_2)^9(z - z_3)^6 \tag{6}$$

The deficit angles are now $3\pi/2, 3\pi/2$ and π at the three points. Thus we have two orbifold points of order 4, and one of order 2. This means the base has turned into T^2/Z_4, for which the element of order 4 fixes 2 points and the element of order 2 fixes another pair, which form a doublet under the Z_4 generator and count as one point.

All the 5 moduli on this branch have to be fixed to achieve this, so in this situation the base is completely fixed apart from its size. In fact, the Z_4 quotient of a 2-torus is

only defined if its modular parameter is i. Thus we have an elliptically fibred K3 whose base and fibre both have modulus $\tau_f = \tau_b = i$. Under a monodromy around a fixed point of order 4 in the base, we have

$$f \sim (z - z_1)^3 \rightarrow e^{6\pi i} f \tag{7}$$

The equation defining the K3 is invariant under this only if we also transform

$$\begin{aligned} x &\rightarrow e^{3\pi i} x = -x \\ y &\rightarrow e^{\frac{9\pi i}{2}} y = iy \end{aligned} \tag{8}$$

from which we conclude that the fibre above such a fixed point has degenerated to T^2/Z_4. Altogether, this means that the K3 has degenerated to T^4/Z_4.

The monodromy in the fibre corresponds to the element

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{9}$$

which is of order 4 in $SL(2, Z)$. This monodromy operation transforms the axion-dilaton pair $\tau = \widetilde{\phi} + ie^{-\phi}$ by

$$\tau \rightarrow -\frac{1}{\tau} \tag{10}$$

and the NS-NS and R-R 2-forms $(B, \widetilde{B})$ of the type IIB string by

$$\begin{aligned} B &\rightarrow -\widetilde{B} \\ \widetilde{B} &\rightarrow B \end{aligned} \tag{11}$$

The modulus $\tau_f = i$, which is the background value of the axion-dilaton, is invariant under precisely this element, as expected.

Note that in the T^4/Z_2 orbifold limit studied in Ref.[8], the corresponding monodromy of the fibre was given by $diag(-1, -1)$, which lies in the duality group $SL(2, Z)$ but acts as the identity in the $PSL(2, Z)$ subgroup which is the nonperturbative part. In the present case, the monodromy S is nontrivial (and of order 2) in $PSL(2, Z)$, and does not correspond to a perturbative symmetry of the type IIB string.

Consider now the branch (II) in which $f(z) = 0$. Here, we have $\tau_f = \exp(i\pi/3)$ and there are generically 12 singular points on the base, each with deficit angle $\pi/3$. This time,

we can find two interesting orbifold limits of the base. Consider first the case where the 12 zeroes of $g(z)$ coalesce into three zeroes of order 5,4 and 3. Then,

$$\Delta(z) = (z - z_1)^{10}(z - z_2)^8(z - z_3)^6 \tag{12}$$

and the deficit angles are $5\pi/3, 4\pi/3$ and π. We can think of these as fixed points of order 6,3 and 2 respectively. This is precisely the structure of a T^2/Z_6 orbifold, whose modular parameter is $\tau_b = \exp(i\pi/3)$. A monodromy about a fixed point of order 6 transforms the fibre by the order-6 element of SL(2,Z):

$$ST = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \tag{13}$$

while around the point of order 3 we find $(ST)^2$, and around the point of order 2, $(ST)^3 = S^2 = diag(-1, -1)$. Thus, this point is just the limit where K3 has become T^4/Z_6.

Another interesting point in this space has the 12 zeroes coalescing into three identical ones of order 4 each. For this case, it is easy to see that

$$\Delta(z) = (z - z_1)^8(z - z_2)^8(z - z_3)^8 \tag{14}$$

and we have a base T^2/Z_3, while the K3 has become T^4/Z_3.

The only other points where the base can be thought of as an orbifold are the ones for which f develops 4 zeroes of order 2 each (in branch (I)) or g develops 4 zeroes of order 3 each (in branch (II)). These are precisely the points where the base becomes T^2/Z_2 and where these branches join onto the branch of moduli space studied in Ref.[8].

At generic points on branch (I), where f vanishes linearly, one can check that the fibre above the singularity is again T^2/Z_4, while at generic points of branch (II), where g vanishes linearly, the fibre is T^2/Z_3.

To summarise, our picture of the two branches is that on branch (I), we have $\tau_f = i$ and a 5 complex dimensional moduli space, on which the F-theory 7-branes can move in units of 3, while on branch (II), $\tau_f = \exp(i\pi/3)$ and a 9 complex dimensional moduli space on which the 7-branes move in units of 2.

3. Enhanced Gauge Symmetries

From the F-theory point of view, gauge symmetries arise from the singularity type of the fibration. At a zero of the discriminant, one uses Tate's algorithm to find the singularity type, and this has been the subject of extensive study in recent months[2].

The cases of most interest to us are the ones where the base has orbifold points of order 2,3,4 or 6. For order 2, near a zero at z_1 we have

$$
\begin{aligned}
f(z) &\sim (z - z_1)^2 \\
g(z) &\sim (z - z_1)^3 \\
\Delta(z) &\sim (z - z_1)^6
\end{aligned}
\tag{15}
$$

from which the singularity is of type $D_4 \sim SO(8)$. This is the case relevant to Ref.[8].

For order 3, we have

$$
\begin{aligned}
f(z) &= 0 \\
g(z) &\sim (z - z_1)^4 \\
\Delta(z) &\sim (z - z_1)^8
\end{aligned}
\tag{16}
$$

for which the singularity type is E_6. For orbifold points of order 4, we get

$$
\begin{aligned}
f(z) &\sim (z - z_1)^3 \\
g(z) &= 0 \\
\Delta(z) &\sim (z - z_1)^9
\end{aligned}
\tag{17}
$$

and the singularity type is E_7. Finally, for order 6 we get

$$
\begin{aligned}
f(z) &= 0 \\
g(z) &\sim (z - z_1)^5 \\
\Delta(z) &\sim (z - z_1)^{10}
\end{aligned}
\tag{18}
$$

for which the singularity type is E_8. It is rather remarkable that the three exceptional groups arise precisely for the three allowed types of orbifold points beyond Z_2.

Putting this together with the analysis of the previous section, we find the following enhanced gauge symmetry groups for the three orbifold limits of K3:

$$
\begin{aligned}
T^4/Z_3 &: \quad E_6 \times E_6 \times E_6 \\
T^4/Z_4 &: \quad E_7 \times E_7 \times SO(8) \\
T^4/Z_6 &: \quad E_8 \times E_6 \times SO(8)
\end{aligned}
\tag{19}
$$

Note that these groups are all of rank 18, unlike the case of T^4/Z_2 where the nonabelian part of the group has rank 16, and indeed can only be $SO(8)^4$. Moreover, as explained earlier, one can deform the above theories while preserving constancy of the coupling. For example, for the branch with $\tau = \exp(i\pi/3)$, on which the Z_3 and Z_6 points lie, one has a generic situation with $f(z) = 0$, $g(z) \sim (z - z_1)$ and $\Delta(z) \sim (z - z_1)^2$, for which according to Tate's algorithm there is no singularity and hence we expect the gauge group to be completely Abelian. On the branch with $\tau = i$, we have at generic points $g(z) = 0$, $f(z) \sim (z - z_1)$ and $\Delta(z) \sim (z - z_1)^3$, for which the singularity is of A_1 type and hence an $SU(2)$ gauge symmetry appears there.

Finally, let us note that among the various gauge groups that can appear, we have the possibility of realising pure $E_8 \times E_8$. For this, on branch (II) take

$$
\begin{aligned}
g(z) &= (z - z_1)^5 (z - z_2)^5 (z - z_3)(z - z_4) \\
\Delta(z) &\sim (z - z_1)^{10} (z - z_2)^{10} (z - z_3)^2 (z - z_4)^2
\end{aligned}
\tag{20}
$$

Then the two zeroes of $g(z)$ of order 5 give an E_8 factor each, while as we have just seen, the simple zeroes give no singularity[1] Moreover, merging the zeroes at z_3 and z_4 will produce an $SU(3)$ singularity there, so we will get $E_8 \times E_8 \times SU(3)$. The only other allowed nonabelian gauge group of the heterotic string on T^2 with unbroken $E_8 \times E_8$ has the extra factor $SU(2) \times SU(2)$, but this does not live on this branch of F-theory moduli space. It cannot live on branch (I) either, since there E_8 singularities cannot appear. Thus this vacuum evidently cannot be realised by F-theory with constant coupling, and must lie somewhere else in the full F-theory moduli space, where the coupling varies. This is also true of the $SO(32)$ gauge group (and its enhancements by $SU(2) \times SU(2)$ or $SU(3)$). Indeed, it is easy to see that no D_n group other than $D_4 = SO(8)$ can arise in the moduli space of constant coupling.

Let us briefly comment on the relationship of all this with the heterotic string. First of all, the three Z_n orbifold limits with $n = 3, 4, 6$ all give rank 18 gauge groups, hence in the heterotic string they are special points in the Narain moduli space in which the compactification torus is nontrivially mixed with the $E_8 \times E_8$ torus. Hence the complex structure and Kähler structure moduli τ_h and ρ_h of the two-torus on which the heterotic string is compactified should be fixed, and indeed they are, since one has $\tau_h = \tau_f$ and

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[1] In the first reference of [2], a somewhat similar limiting configuration was suggested to give $E_8 \times E_8$.

$\rho_h = \tau_b$. As we have seen, these are both fixed to be i on branch (I) and $\exp(i\pi/3)$ on branch (II). On the other hand, on branch II we can have the gauge group $E_8 \times E_8$, for which the heterotic string is just compactified on the two-torus with no Wilson lines. This should admit a decompactification limit to 10 dimensions, corresponding to $\rho_h \to i\infty$. To find this, one has to identify the appropriate modulus on branch (II) to take to infinity. One can only roughly think of this as the parameter τ_b, since now the base of IIB is not a T^2 orbifold but a more general singular P^1. However, it is clear that the configuration of 24 branes grouped in units of 10,10,2 and 2 possesses precisely one modulus, which has to be the relevant one for decompactification.

4. Relation to Orientifolds

In Ref.[8], it was shown that F-theory on T^4/Z_2 maps to the orientifold of type IIB on T^2 modded out by the Z_2 group $\{1, \Omega(-1)^{F_L} I_{910}\}$ where Ω is reversal of world-sheet orientation, F_L is the spacetime fermion number of world-sheet left-movers, and I_{910} is inversion of the 9th and 10th dimensions. Since all the symmetries appearing in this Z_2 are perturbatively realised, one can at least perturbatively define and study this orientifold using conventional string theory techniques. The vacuum contains 16 Dirichlet 7-branes, occurring 4 at each fixed point of Z_2 on the base T^2. By a mechanism similar to that studied in Ref.[11], moving the D-branes away from the fixed points forces the coupling constant to vary over the base.

When this happens, perturbative considerations in type IIB with D-branes lead to an inconsistent description of the moduli space, while the original F-theory formulation captures the nonperturbative effects required to make this description consistent and correct. This relationship between the perturbative IIB orientifold and F-theory turns out to be identical[8] to that between the perturbative and nonperturbative pictures of the moduli space in 4d N=2 supersymmetric $SU(2)$ gauge theory with 4 hypermultiplets in the fundamental[12].

Something similar can be attempted on the branches of moduli space that we have been studying, though a description through weakly coupled D-branes will not be possible even in a limit. This is just as well, since exceptional groups are not produced by D-branes at weak coupling. So we will confine ourselves to a phenomenological description of the situation.

Given the monodromies of the fibre at fixed points in the base, as studied in Section 2, we can map F-theory on T^4/Z_n to orientifolds as follows. First, consider $n = 4$. Over

a fixed point of order 4, the monodromy is given by the matrix S in Eq.(9). This acts on the spacetime fields as described in Eqs.(10) and (11). Denoting this action by S, and defining $R^{(4)}$ to be the anticlockwise $\pi/2$ rotation of the base T^2 ($z \to iz$), we can write this F-theory background as the "generalised orientifold" type IIB on T^2 quotiented by

$$Z_4 = \{1, S.R^{(4)}, (S.R^{(4)})^2 = \Omega(-1)^{F_L}I_{910}, (S.R^{(4)})^3\} \tag{21}$$

As far as the action on the T^2 base is concerned, one has two fixed points of order 4 and one doublet of order 2. Concentrating on the order 2 point first, it is fixed under the element $\Omega(-1)^{F_L}I_{910}$, hence this is a conventional orientifold situation and should give rise, as usual, to 4 Dirichlet 7-branes at the fixed point. This gives rise to the $SO(8)$ factor in the gauge group (see Eq.(19)).

We must now look for the $E_7 \times E_7$ factor in the group. Each E_7 must be associated with the "twisted sector" with respect to the element $S.R^{(4)}$. This is precisely what we cannot describe by conventional means. We can, however, indirectly argue some properties of this sector. Suppose it is made of some new dynamical objects ("E-branes"?). This configuration, which has 9 coincident F-theory 7-branes, can split in F-theory into a configuration localised at upto three distinct points, each with 3 7-branes. This is because we are in branch (I), where units of 3 7-branes must move around together. The completely split case gives the gauge group $(SU(2))^3$, while the case where one point splits from the other two gives $SO(8) \times SU(2)$. Thus, any future understanding of such "E-branes", perhaps as strongly coupled D-branes, will have to incorporate these properties.

Similarly, on branch (II) at the T^4/Z_6 point, we can map the F-theory to an orientifold of IIB on T^2/Z_6 where the Z_6 is generated by the element $ST.R^{(6)}$ where $R^{(6)}$ is a rotation of order 6. In this case, the point of order 2 again gets 4 conventional Dirichlet 7-branes, giving $SO(8)$, while the point of order 3 gets an E_6 gauge symmetry coming from 8 coincident F-theory 7-branes. Splitting of this point can now give $SO(8) \times U(1)$, $SU(3) \times SU(3)$, $SU(3) \times U(1)^2$ and $U(1)^4$ depending on whether the 8 branes split into (6,2), (4,4), (4,2,2) or (2,2,2,2). Note that in this process the rank of the gauge group is not preserved, unlike the behaviour of conventional D-branes. It would be very interesting to understand better, if this is at all possible when the coupling is of order 1, the behaviour of these unusual dynamical objects.

The generalized orientifolds described above can also be mapped to other orientifolds of IIB and M-theory by T-duality. Suppose first that we T-dualise in the directions 9 and

10. We concentrate on the T^4/Z_4 case for definiteness. The order-2 element $\Omega(-1)^{F_L}$ of the Z_4 group gets mapped to Ω, following the observations in Ref.[8] for the T^4/Z_2 case. As for the order 4 element, it must map to something which is a square root of Ω. This can be deduced from the action of Ω on spacetime fields. We have

$$\Omega: \quad \begin{aligned} B &\to -B \\ \widetilde{\phi} &\to -\widetilde{\phi} \\ \widetilde{D} &\to -\widetilde{D} \end{aligned} \tag{22}$$

where B is the NS-NS 2-form and $\widetilde{\phi}$, $\widetilde{D}$ are the R-R 0-form and 4-form. Under compactification to 8 dimensions these fields give:

$$\widetilde{\phi} \to \widetilde{\phi}$$
$$B_{MN} \to B_{\mu\nu},\ B_{\mu 9},\ B_{\mu 10},\ B_{910} \tag{23}$$
$$\widetilde{D}_{MNPQ} \to \widetilde{D}_{\mu\nu\lambda 9},\ \widetilde{D}_{\mu\nu 910}$$

Here we have not listed $\widetilde{D}_{\mu\nu\lambda 10}$ and $\widetilde{D}_{\mu\nu\lambda\rho}$ since, by self-duality of the original $\widetilde{D}$ in 10 dimensions, these components are related to the ones listed above.

Thus we have a pair of scalars, a pair of 1-forms, a pair of 2-forms and a single 3-form. Now the action of the square root of Ω must preserve 8-dimensional Lorentz invariance. Thus scalars must be mapped to linear combinations of scalars, 1-forms should be mapped to 1-forms and so on. This uniquely determines the square root of Ω to be the transformation

$$\begin{aligned} \widetilde{\phi} &\to B_{910} & B_{\mu 9} &\to B_{\mu 10} & B_{\mu\nu} &\to \widetilde{D}_{\mu\nu 910} \\ B_{910} &\to -\widetilde{\phi} & B_{\mu 10} &\to -B_{\mu 9} & \widetilde{D}_{\mu\nu 910} &\to -B_{\mu\nu} \end{aligned} \tag{24}$$

along with a duality on the 3-form $\widetilde{D}_{\mu\nu\lambda}$ (which, as usual, means we take the Poincaré dual of its field strength). The square of this duality in 8 dimensions is (-1), and clearly the transformations in the equation above square to (-1) as well.

Calling this combined transformation U, we see that it is part of the U-duality group of type IIB in 8 dimensions. Thus after T-duality we find that this orientifold is type IIB on T^2 modded out by the Z_4 group given by $\{1, U, U^2 = \Omega, U^3\}$. So even after T-duality, the Z_4 group still depends on compactifying the theory to 8d, unlike the Z_2 case where one found that all reference to the compactified dimensions disappeared after T-dualising.

Now suppose instead that we had T-dualised only in one direction. In the Z_2 case, this would take us to type IIA on the dualised orientifold. However, in the present case two things change: on the one hand, the presence of an S-duality transformation in the orbifolding means that the T-dualised theory must be thought of as M-theory rather than type IIA. In M-theory the S-duality is realised as an interchange of the 11th and 10th directions. Second, the rotation of order 4 on the compactification torus becomes, after one T-duality, a rotation between momentum modes of IIA in the 9-direction and winding modes in the 10-direction. In M-theory language, this orbifold group therefore interchanges modes of the 2-brane that wind on a 2-torus, with modes that wind on a circle and propagate on the other circle. Such an orbifold may not be out of reach of analysis, and should provide further insight into F-theory and its relation to M-theory, but we will not pursue it here.

5. Discussion and Conclusions

We have shown that the simplest compactification of F-theory, on K3, has various regions of moduli space where the F-theory coupling remains constant over the base. One branch of this region has been utilised by Sen[8] to map to a problem involving conventional D-branes, and to show that the moduli space of this problem is governed by the Seiberg-Witten analysis of the moduli space of certain 4-dimensional N=2 supersymmetric gauge theories. These gauge theories, in turn, have recently been interpreted in terms of the worldbrane actions for Dirichlet 3-branes used as probes[13][2]. Even more recently it has been argued that this framework is a powerful tool for analysing the dynamics of supersymmetric gauge theories[14].

Clearly, one should ask which 4d gauge theory, if any, relates to our case. However, precisely because the orientifold related to our theory is not of conventional type, one cannot in any obvious way introduce 3-branes as probes. A related fact is that the IIB coupling (which would turn into the gauge coupling on the brane worldvolume), cannot be taken small in the region that we study.

If a gauge theory description of this moduli space nevertheless exists (and realises the E-series gauge symmetries of F-theory as global symmetries) then this would be evidence that the concept of 3-branes as probes could make sense beyond the context of conventional

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[2] This reference also suggested the possible relevance of Z_n orbifolds to the problem of getting exceptional groups.

Dirichlet 3-branes. Alternatively, it might suggest that strongly coupled D-branes exhibit unusual behaviour, including the possibility of producing exceptional gauge groups.

Finally, it has been speculated[4] that at $\tau = i$ or $\tau = \exp(i\pi/3)$, the type IIB string might exhibit some new properties, analogous to tensionless strings. Since these points coincide with the region of F-theory moduli space discussed in this note, one might hope to see these new properties by examining the dual heterotic description.

Acknowledgements

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