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A Note on Low-Dimensional String Compactifications

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ABSTRACT

We study supersymmetric compactifications of type II strings on eightfolds to two dimensions. It is demonstrated that the type IIB string on an eightfold is free of gravitational anomalies. T-duality requires that this theory when further compactified on a circle must have a vacuum momentum; this is explicitly shown to be present and to have the right value. A subtlety in the relation of IIB compactifications and M-theory orientifolds to two dimensions is pointed out.

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1. Introduction

Compactifications of string theory to 2 spacetime dimensions are interesting for various reasons, one of which is the relationship to M and F theory in 3 and 4 dimensions respectively (this holds for the type IIA string). Certain consistency conditions arise when we require the compactification to satisfy the equations of motion[1][2]. The type IIB string on an 8-fold leads to chiral theories in two dimensions. Since type IIA and IIB are T-dual to each other after compactification on a circle, there are various interconnections between the above properties. This will be the subject of the present paper.

2. Type IIA on eightfolds

Compactification of the type IIA string on an 8-fold is potentially destabilised by a term of the form $\int B \wedge I_8$ which arises at 1-loop level[3]. Here B is the 2-form gauge field, and I_8 is a linear combination of the Pontryagin classes p_2 and p_1^2, whose integral on an eightfold gives the Euler characteristic χ. (A similar term $\int C \wedge I_8$ arises in M-theory[4], where this time C is the M-theory 3-form field.) A naive compactification of type IIA on an eightfold thus gives a tadpole term $\int B$ in two dimensions (and analogously, M-theory gives a tadpole $\int C$ in three dimensions), proportional to the value of χ.

It was observed in Ref.[1] that for M-theory compactifications, this tadpole also receives contributions from the classical term $\int C \wedge dC \wedge dC$ if we allow a background value of the 3-form field on the eightfold. This contribution is proportional to $\int dC \wedge dC$ over the eightfold. Clearly the analogous result holds also for type IIA compactifications. We will discuss signs and factors presently.

Finally, in Ref.[2] it was noted that the presence of branes in the vacuum, filling spacetime, contributes to the tadpole as well. (Thus we need 1-branes for IIA and 2-branes for M-theory). Each brane modifies the tadpole by an integer value.

Combining the above results, the condition for a consistent type IIA compactification on an eightfold are:

$$\frac{\chi}{24} - \frac{1}{8\pi^2} \int dC \wedge dC - n = 0 \tag{1}$$

where n is the number of branes[1].

To check the signs and factors in this expression, let us compare type IIA on $K3 \times K3$ with the heterotic string on $T^4 \times K3$. These two theories are dual to each other, from

[1] Note that we differ from Ref.[1] in the sign of the second term above. We are grateful to E. Witten, K. Becker and M. Becker for correspondence on this point.

six-dimensional string-string duality. For the heterotic string on $K3$, we have a condition relating the instanton number of the background gauge fields on $K3$ and the number of M-theory 5-branes in the vacuum, which can arise if we view the heterotic string as M-theory on S^1/Z_2[5]. This condition may be written[6]:

$$24 - \frac{1}{16\pi^2}\int \mathrm{tr} F \wedge F - n = 0 \qquad (2)$$

where the second term is integrated over $K3$. n is the number of M-theory 5-branes, but in the present context they are wrapped on T^4 to become 1-branes. Clearly the number of 1-branes is bounded above by 24. Moreover, the sign of the second term (the instanton number) is such that instantons, rather than anti-instantons, contribute a negative integer to the equation.

Equation (1) integrated over $K3 \times K3$ gives precisely this result, since the Euler of $K3 \times K3$ is $(24)^2$, while the second term gives rise (generically) to $U(1)$ gauge fields on one $K3$ and their Pontryagin class is then integrated over the second $K3$. Note that this term is integer in Eq.(1) as long as $[\frac{dC}{2\pi}]$ is an integral cohomology class, which (as shown in Ref.[7]) is true whenever $\frac{\chi}{24}$ is integral. Finally, the last term is the number of IIA 1-branes, which are M-theory 2-branes wrapped on the 11th dimension. So the last terms are electric-magnetic dual between the two theories, as expected.

This confirms Eq.(1), and shows that the number of branes in the vacuum is bounded above by the Euler characteristic of the eightfold (this would not have been true if the second term had the opposite sign)[2].

Note that the second term of Eq.(1) takes values congruent to $\frac{1}{4}$ mod integers if $[\frac{dC}{2\pi}]$ is a half-integral cohomology class. In this case, $\frac{\chi}{24}$ must also be congruent to $\frac{1}{4}$ mod integers.

3. Analogous issues for type IIB

On compactifying type IIB on a circle, it becomes equivalent to IIA on a circle under T-duality. Thus it must possess the dimensional reductions of both the classical term $\int B \wedge dC \wedge dC$ and the one-loop term $\int B \wedge I_8$. After reducing on a circle, the B-field of type IIA becomes a 1-form A which measures the winding charge with respect to that circle. Under T-duality this, in turn, becomes the Kaluza-Klein 1-form arising by reduction

[2] An independent check of the sign in Eq.(1) can be made by examining Eq.(4.4) of Ref.[7]. This gives the same result.

of the 10D metric of IIB on the circle. Thus we must look for terms in type IIB which reduce to $\int A \wedge dC \wedge dC$ and $A \wedge I_8$ in 9 dimensions.

To find the first of these is reasonably straightforward, except for a familiar subtlety: for nonzero values of the self-dual 4-form D^+, the type IIB string does not have a covariant action. Yet we need precisely terms depending on D^+, since that field reduces to the 3-form C in 9d. One way out is to make use of the so-called non-self-dual (NSD) [8] action for type IIB in 10d. In this formulation, one starts by "forgetting" the self-duality condition on D. An action can then be written down, with the property that its equations of motion reduce to the correct ones after imposition of the self-duality constraint by hand.

In this formalism the D field has a kinetic term $\int dD \wedge *dD$. This term involves the metric via the operation of taking the Poincare dual. However, "morally speaking" it is topological, since the 5-form dD is eventually set equal to its dual. Thus on compactification to 9d, it can give rise to a topological term. The D field reduces to a 3-form in 9d, while one of the index contractions requires the metric component $g_{\mu 10}$ which is just the KK gauge field A_μ. (we label the dimensions $(x^1, x^2, \ldots x^{10})$ where x^1 is the time). As a result, one gets the desired term $\int A \wedge dC \wedge dC$ in 9d.

The one-loop term is far more subtle. It is known that in 10 dimensions there is no one-loop correction in type IIB analogous to the term $\int B \wedge I_8$ in type IIA. Moreover, one can easily convince oneself directly that there is no purely gravitational term that one can write down in 10d which reduces to $\int A \wedge I_8$ in 9d with A being the KK gauge field. In fact, we will argue that no modification is required in 10d to the type IIB action, but as soon as one compactifies on a circle, however large, there is a radius-dependent term of the desired form in 9d. We will return to this point below.

4. Type IIB on eightfolds: the anomaly

Consider the supersymmetric compactifion of type IIB on an eightfold. We will consider manifolds of holonomy $spin(7)$, $SU(4)$ and subgroups of $SU(4)$. Joyce eightfolds[9] of $spin(7)$ holonomy lead to $(0,2)$ spacetime supersymmetry in 2 dimensions, while Calabi-Yau eightfolds of $SU(4)$ holonomy give $(0,4)$ supersymmetry. Other interesting cases are $K3 \times K3$ leading to $(0,8)$ supersymmetry, and T^8/Z_2 (an orbifold which cannot be blown up to a smooth manifold, but gives consistent string compactifications all the same) where the supersymmetry is $(0,16)$. In each case, the supersymmetry is counted in terms of one-component Majorana-Weyl spinors.

The number of scalars from a CY eightfold in 2 dimensions is counted as follows: the 10d metric g_{MN} gives rise to $h_{11} + 2h_{31}$ non-chiral scalars, while the two 2-form fields B_{MN} and $\widetilde{B}_{MN}$ produce $2h_{11}$ non-chiral scalars. Two more come from the 10 dimensional scalars of type IIB. Finally, the self-dual 4-form D^+_{MNPQ} leads to $(b_4^-) = h_{22}^- + 2h_{31}^-$ scalars of + chirality and $(b_4^+) = 2 + h_{22}^+ + 2h_{31}^+$ scalars of − chirality.

Combining, we have $n(\phi^+) = 3h_{11} + 2h_{31} + 2 + b_4^-$ and $n(\phi^-) = 3h_{11} + 2h_{31} + 2 + b_4^+$. With $(0, N)$ supersymmetry, the supergravity multiplet is $(g_{\mu\nu}, \phi, N\psi_\mu^-, N\psi^+)$. Let us assume that we have some number n_+/N of chiral matter multiplets $(N\phi^+, N\psi^+)$ (thus, n_+ counts the number of individual scalars, which is more convenient), along with n_-^ϕ anti-chiral scalars ϕ^-, and n_-^ψ anti-chiral Majorana-Weyl fermions ψ^- which are supersymmetry singlets.

Since one scalar goes into the gravity multiplet, we have $n_+ = n(\phi^+) - 1$, $n_-^\phi = n(\phi^-) - 1$. The difference is given by

$$n_+ - n_-^\phi = b_4^- - b_4^+ = -\tau \tag{3}$$

where τ is the signature of the eightfold.

The gravitational anomaly from the supergravity multiplet, combining contributions from the gravitinos and the spin-$\frac{1}{2}$ fermions, is N times the anomaly polynomial. Chiral scalars contribute $\frac{1}{12}$ in the same units, and Majorana-Weyl fermions $\frac{1}{24}$. Thus the total anomaly is proportional to

$$N + \frac{n_+ - n_-^\phi}{12} + \frac{n_+ - n_-^\psi}{24} \tag{4}$$

The fermions are counted as follows: the spin-$\frac{3}{2}$ particles in 2 dimensions come from the gravitinos in 10 dimensions, and their number is given by the Dirac index of the internal manifold. An equal number of spin-$\frac{1}{2}$ particles in two dimensions come from the spin-$\frac{1}{2}$ fermion in 10 dimensions. All these go into the supergravity multiplet. Additional spin-$\frac{1}{2}$ fermions arise from the 10d gravitinos, and their number is given in terms of the Rarita-Schwinger index. Therefore

$$n_-^\psi - n_+ = 2\ ind(\not{D}_{3/2}) \tag{5}$$

where the 2 on the RHS comes from the fact that there are 2 gravitinos in 10 dimensions.

In terms of Pontryagin classes, we have

$$ind(\not{D}_{3/2}) = \frac{37}{720}p_1^2 - \frac{31}{180}p_2 \tag{6}$$

where

$$p_1 = -\frac{1}{2} tr R^2$$
$$p_2 = -\frac{1}{4} tr R^4 + \frac{1}{8}(tr R^2)^2 \tag{7}$$

Using the additional relations

$$\chi = \frac{p_2}{2} - \frac{p_1^2}{8}$$
$$ind(\not{D}_{1/2}) = \frac{N}{2} = \frac{1}{1440}(\frac{7}{4}p_1^2 - p_2) \tag{8}$$

we find that

$$p_2 = 120N + \frac{7\chi}{3}$$
$$p_1^2 = 480N + \frac{4\chi}{3} \tag{9}$$

¿From these relations, it follows that

$$ind(\not{D}_{3/2}) = 4N - \frac{\chi}{3} \tag{10}$$

The anomaly thus becomes

$$N - \frac{\tau}{12} - \frac{8N - 2\chi/3}{24} \tag{11}$$

and the condition for anomaly cancellation reduces to

$$24N - 3\tau + \chi = 0 \tag{12}$$

This is actually an identity, as can be easily seen by replacing each term by its value in terms of Pontryagin classes, using Eq.(8) and the Hirzebruch signature theorem

$$\tau = \frac{1}{45}(7p_2 - p_1^2) \tag{13}$$

Thus we have demonstrated explicitly that the type IIB string on any eightfold preserving some supersymmetry is anomaly-free in 2d.

5. Type IIA and IIB on eightfolds: the spectrum

We can easily write down the complete spectrum of type IIA and IIB on an eightfold. For type IIA, the supersymmetry is labelled $(\frac{N}{2}, \frac{N}{2})$ and the supermultiplets are the supergravity multiplet $(g_{\mu\nu}, \phi, \frac{N}{2}\psi^+_\mu, \frac{N}{2}\psi^-_\mu, \frac{N}{2}\psi^+, \frac{N}{2}\psi^-)$. We assume a number $(2n_+/N)$ of (non-chiral) matter multiplets $(\frac{N}{2}\phi^+, \frac{N}{2}\phi^-, \frac{N}{2}\psi^+, \frac{N}{2}\psi^-)$. Then it is easy to see that for a CY

$$\frac{n_+}{2} = h_{11} + h_{21} + h_{31} = \frac{\chi}{6} - 8 + 2h_{21} \tag{14}$$

where the last equality follows from the identity[2]:

$$h_{11} - h_{21} + h_{31} = \frac{\chi}{6} - 8 \tag{15}$$

For type IIB, we need to use the index calculations of the previous section in addition to the cohomology of the eightfold. We have seen at the beginning of the previous section that n_+ and n^ϕ_-, the number of + and − chirality scalars are given by

$$\begin{aligned} n_+ &= 3h_{11} + 2h_{31} + 1 + b_4^- \\ n^\phi_- &= 3h_{11} + 2h_{31} + 1 + b_4^+ \end{aligned} \tag{16}$$

Following Eq.(15), this can be rewritten

$$\begin{aligned} n_+ &= 4h_{21} + \frac{2\chi}{3} - 8N \\ n^\phi_- &= 4h_{21} + \chi \end{aligned} \tag{17}$$

Notice that, as χ is divisible by 6 for manifolds of $SU(4)$ holonomy, n_+ is divisible by 4, which should be the case since 4 scalars fit into a supermultiplet. On the other hand, χ is not divisible by 4 except under more stringent conditions such as elliptic fibration[2], so that n^ϕ_- is not necessarily divisible by 4.

Since, by virtue of supersymmetry, n_+ also counts the number of + chirality fermions, it only remains to count those of minus chirality, which of course follows from the index theorem of the previous section. The result is

$$n^\psi_- = 4h_{21} \tag{18}$$

Observe that the IIA and IIB spectra written above are invariant under mirror symmetry, which for eightfolds maps h_{pq} to $h_{4-p,q}$. This map preserves both χ and h_{21}, which are the only invariants that determine the spectrum.

This completes the general analysis of the spectrum for type IIB on an eightfold. We now describe a few examples, before turning to a discussion of the relationship with type IIA after further compactification on a circle.

(i) Joyce Manifolds

This is the case of $spin(7)$ holonomy, and $(0,2)$ spacetime supersymmetry in 2d. The Joyce eightfolds[9] are blown-up orbifolds of the eight torus, T^8/Γ, where Γ is a suitable discrete group. It was shown by Joyce that there are essentially five types of singularity in the space T^8/Γ which are to be blown up to construct the manifold.

The Joyce manifolds have $\chi = 144$ and $\tau = 64$, verifying Eq.(12) with $N = 2$. The spectrum is

$$n_+ = 2 + 2b_2 + b_4 - 64, \qquad n_-^\phi = 2 + 2b_2 + b_4, \qquad n_-^\psi = 2b_3 \tag{19}$$

(ii) Borcea eightfolds

These are Calabi-Yau 4-folds of the form $(K3 \times K3)/Z_2$ with $SU(4)$ holonomy, and lead to $(0,4)$ supersymmetry for Type IIB compactifications[10]. These manifolds are labelled by a set of integers $(r_1, a_1, \delta_1; r_2, a_2, \delta_2)$. The Euler characteristic and signature are

$$\begin{aligned} \chi &= 6(r_1 - 10)(r_2 - 10) + 288 \\ \tau &= 2(r_1 - 10)(r_2 - 10) + 128 \end{aligned} \tag{20}$$

verifying Eq.(12) with $N = 4$. From the Hodge diamond one finds

$$h_{21} = 5(r_1 + r_2) - 6(a_1 + a_2) - \frac{1}{2}(r_1 r_2 - a_1 a_2) + 22 \tag{21}$$

These data suffice to determine the spectrum. The mirror transformation acts, for these manifolds, as $r_i \to 20 - r_i$ with a_i unchanged, and it is evident that the above data are invariant under this.

(iii) K3 $\times$ K3

This is the case of $SU(2) \times SU(2)$ holonomy and $(0,8)$ supersymmetry. The relevant invariants are

$$\chi = 576, \qquad \tau = 256, \qquad h_{21} = 0 \tag{22}$$

verifying Eq.(12) with $N = 8$. Thus we have

$$n_+ = 320, \qquad n_-^\phi = 576, \qquad n_-^\psi = 0 \tag{23}$$

This reproduces the result of Ref.[11] for $K3 \times K3$.

(iv) T^8/Z_2

For this singular space, the holonomy is Z_2 and the supersymmetry is $(0, 16)$. The topological invariants have to be computed in the orbifold sense, and one finds[12]:

$$\chi = 384, \qquad \tau = 256, \qquad h_{21} = 0 \tag{24}$$

Thus the spectrum is

$$n_+ = 128, \qquad n_-^\phi = 384, \qquad n_-^\psi = 0 \tag{25}$$

It has been noted in Ref.[13] that this compactification is dual to the orientifold of M-theory on T^9/Z_2 (this and related cases have been studied in more detail in Ref.[14]). However, in the M-theory case one finds the spectrum in the form:

$$n_+ = 128, \qquad n_-^\phi = 128, \qquad n_-^\psi = 512 \tag{26}$$

which is equivalent to the previous expression only after bosonization of the 512 chiral fermions.

The different forms of Eqs.(25) and (26) are not accidental, but closely related to the geometry of the corresponding compactifying spaces. In the case of IIB on T^8/Z_2 we have $2^8 = 256$ fixed points, and a chiral boson is obtained as the twisted sector for each one. For M-theory on T^9/Z_2 there are instead $2^9 = 512$ fixed points, and a chiral fermion arises as the twisted sector for each one. Since M-theory does not possess a 1-brane, these twisted sector multiplets must appear symmetrically at the fixed points.

It is amusing that M-theory seems to "know" about bosonization. This also has a nontrivial consequence which we will point out in the following.

6. T-duality and the vacuum momentum

Suppose we compactify type IIA and IIB on the same eightfold, and then further on a circle to $0 + 1$ dimensions. T-dualizing along the circle maps one theory to the other. Now we have an apparent puzzle: type IIA has a 2-form tadpole in 2d, which will become a 1-form tadpole in 1d, and this is proportional to the Euler characteristic χ of the eightfold. However, we have found no inconsistency for type IIB on the eightfold to two dimensions, so the inconsistency required by T-duality must arise upon compactifying one further dimension. Moreover, it must take the form of a tadpole for the KK 1-form $A = g_{12}$.

Recently it was pointed out[15], in the particular case of $K3 \times K3$ compactification, that this can be understood in terms of the nonzero vacuum values of L_0 and $\overline{L}_0$ arising from free fields on a cylinder. Here we will prove that the correct vacuum momentum is obtained in the general case, and will examine this point in a little more detail.

For 2d field theories on a cylinder, the generator of translations along the compact direction is $L_0 - \overline{L}_0$. Thus, a nonzero value of this operator in the vacuum implies that, from a 2d point of view, there is a nonzero momentum in the vacuum state. Under T-duality, this will turn into a nonzero winding charge of the vacuum, precisely what we would expect in a theory which has a 2-form tadpole in 2d. The tadpole must have the precise value $\frac{\chi}{24}$.

Given the spectrum of type IIB in 2d, we use the fact that a free periodic boson has vacuum energy $-\frac{1}{24}$ while a free periodic fermion has energy $\frac{1}{24}$. Thus, associating L_0 to what we have earlier called + chirality, we have, for type IIB on an eightfold,

$$(L_0)_{\rm vac} = 0, \qquad (\overline{L}_0)_{\rm vac} = \frac{1}{24}(n_-^\psi - n_-^\phi) \tag{27}$$

where the first equation is a consequence of the chiral supersymmetry in 2d. From Eqs.(17) and (18), it follows that

$$(L_0 - \overline{L}_0)_{\rm vac} = \frac{\chi}{24} \tag{28}$$

as desired.

Note that if the circle becomes large and we are effectively in two noncompact dimensions, this effect goes away. The reason is that the operator L_0 as conventionally defined in conformal field theory has a zero-point contribution $-\frac{1}{24}$ for a free boson only if the radius of the circle (the range of the σ coordinate) is fixed to be 2π, as is conventionally done. For a circle of radius $2\pi R$, the zero-point contribution is actually $-\frac{1}{24R}$, so that it goes away in the limit $R \to \infty$. This explains why there is no corresponding one-loop term in the effective action of type IIB theory in 2 (or 6 or 10) dimensions, and yet the prediction of T-duality with type IIA is satisfied.

7. An M-theory subtlety

The example of IIB on T^8/Z_2, and its M-theory dual discussed in the previous section, now poses a slight problem. We saw that M-theory produces the same spectrum but in a fermionized form. From Eq.(26), the vacuum momentum for M-theory on T^9/Z_2 compactified on a further circle is actually $-\frac{\chi}{24}$, equal in magnitude but opposite in sign to that of type IIB on T^8/Z_2. The sign can be changed by redefining the coordinate x^2 to $-x^2$, but this choice of convention is no longer available once we have fixed the chirality of the spacetime supersymmetry. In other words, type IIB on T^8/Z_2 produces a theory where the supersymmetry has $+$ chirality and the vacuum momentum is positive, or else by a change of convention, the supersymmetry has $-$ chirality and the vacuum momentum is negative. On the other hand, M-theory on T^9/Z_2 gives a theory with supersymmetry of $+$ chirality and negative vacuum momentum, or the other way around. This suggests that the equivalence between the two theories is more subtle than was previously thought, as long as the eightfold has nonzero χ.

This may seem surprising given that the two theories are related by bosonization, but the reason is that the fermion contribution to the zero-point values for L_0 and $\overline{L}_0$ has been calculated for periodic boundary conditions on the fermions around the circle. But bosonization does not equate periodic bosons with periodic fermions.

One may expect a larger class of dualities between IIB on orbifolds of the form $\mathcal{M}_8/Z_2$ and M-theory on corresponding orientifolds $(\mathcal{M}_8 \times S^1)/Z_2$. In these cases, the latter theory will always have twice the number of fixed points as the former, since the extra S^1 contributes a pair of fixed points for each one of the original orbifold. Assuming a symmetric distribution of twisted-sector states, it would seem that M-theory should always give the fermionized form of the IIB compactification, as in the case we have discussed. This would be interesting to confirm.

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