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BPS Nature of 3-String Junctions

Keshav Dasgupta[1] and Sunil Mukhi[2]

Tata Institute of Fundamental Research,
Homi Bhabha Rd, Mumbai 400 005, India

ABSTRACT

We study BPS-saturated classical solutions for the world-sheet theory of a D-string in the presence of a point charge. These solutions are interpreted as describing planar 3-string junctions, which arise because the original D-string is deformed by the presence of the inserted charge. We compute the angles of the junctions and show that the vector sum of string tensions is zero, confirming a conjecture of Schwarz that such configurations are BPS states.

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[1] E-mail: keshav@theory.tifr.res.in
[2] E-mail: mukhi@theory.tifr.res.in

D-branes[1] are those extended objects on which an open string can end. More precisely, a single D-brane is a state defined by having both ends of an open string end on it. If a single end of an open string ends on a D-brane (and the other end goes to infinity or terminates on a different brane) then in principle we face a problem with charge conservation: the charge of the open string appears to be lost. The resolution to this[2,3], which brought about an important conceptual advance in the understanding of branes in general, is that the world-volume theory of D-branes has certain couplings which render this termination consistent.

As a result, the 2-form charge carried by the open string gets converted into the gauge charge of the particle represented by its endpoint. This charge can be measured by enclosing the endpoint by a $(p-1)$-sphere within the p-brane. This has an important consequence when $p = 1$[4,5]. In this case there is a discontinuity in the charge measured on the 1-brane (D-string), so that it is not possible for both ends of the D-string to remain D-strings. If we denote a fundamental string (F-string) as a $(1,0)$ string, and a D-string as a $(0,1)$ string, then when these two meet, the third "outgoing" string must be a $(1,1)$ string. In general, 3-string junctions exist where three open string carrying charges (p_i, q_i), $i = 1,2,3$ meet at a point, and charge conservation requires $\sum_i p_i = \sum_i q_i = 0$. (We assume that the three strings are all oriented the same way with respect to the junction.)

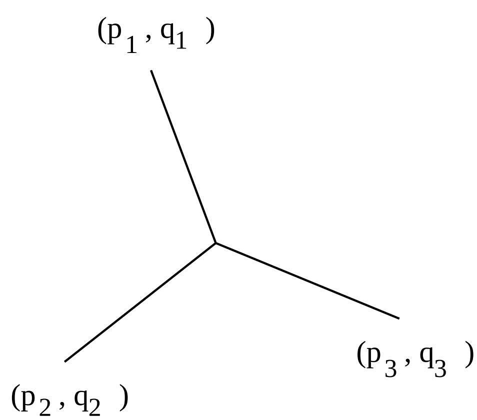

Fig.1: A 3-string junction.

It was conjectured in Ref.[5] that such 3-string junctions correspond to BPS saturated states in the theory, whenever the configuration consists of straight strings going off to infinity from the junction along definite angles. The angles are such that the vector sum of string tensions is zero. These 3-string junctions recently played an important role in

Ref.[6] where they were needed to explain the origin of exceptional symmetries in F-theory, following the proposal of Ref.[7].

In what follows, we provide evidence for the existence of BPS saturated 3-string junctions having precisely the properties described above. Instead of looking for them as solutions of the classical supergravity field equations, as suggested in Ref.[5], we will find them by examining worldsheet properties of the D-string.

The worldsheet field theory for a single D-string is given, in the linearized approximation, by a 2d supersymmetric $U(1)$ gauge theory. The bosonic fields are the gauge field and 8 scalars representing collective coordinates for the transverse motion of the D-string. Let us now consider a simple case where a point charge of unit magnitude is inserted onto a single D-string. This is a special case of the situation studied in Ref.[8], where point charges are inserted onto general D p-branes. We take the D-string to be oriented along the x^1 axis. Note that at present, the RR scalar $\widetilde{\phi}$ is not excited.

In the presence of this charge, Gauss' law in one space dimension states that

$$F_{01}^{+} - F_{01}^{-} = g \tag{1}$$

where g is the strength of the point charge, equal to the type IIB string coupling. This requires the scalar potential to be piecewise linear, with a fixed discontinuity in its slope. A simple solution of this equation is

$$\begin{aligned} A_0 &= -g x^1, \quad x^1 > 0 \\ &= 0, \qquad\quad\ \ x^1 < 0 \end{aligned} \tag{2}$$

This solution by itself is not BPS saturated but, as in Ref.[8], it can be made BPS by simultaneously exciting one of the transverse coordinates, say x^9, represented by the worldsheet field $X^9(x^0, x^1)$. Thus we choose

$$X^9(x^1) = A_0(x^1) \tag{3}$$

with A_0 given by the above equation.

Because of well-known properties of low-dimensional electrodynamics, the solution is linearly increasing away from the charge, in sharp contrast to the case for p-branes with $p \geq 3$. Indeed, the linear variation of X^9 as a function of x^1 suggests the followng interpretation: inserting the point charge at the origin of the D-string causes one half of the string to rigidly bend. The point charge itself, as in Ref.[8], is associated to the endpoint of an F-string coming in perpendicular to the original D-string (in contrast to the cases discussed in Ref.[8], our solution has no "spike" representing the F-string. However, a consistent interpretation certainly requires the F-string to be present, as it is the one which carries the inserted charge).

The resulting configuration is as in Fig 2, with the angle α determined by

$$\tan\alpha = \frac{1}{g} \tag{4}$$

The string that goes out from the junction towards the top left is neither an F-string nor a D-string, but must be thought of as a $(1,1)$ or $(-1,-1)$ string depending on the orientation.

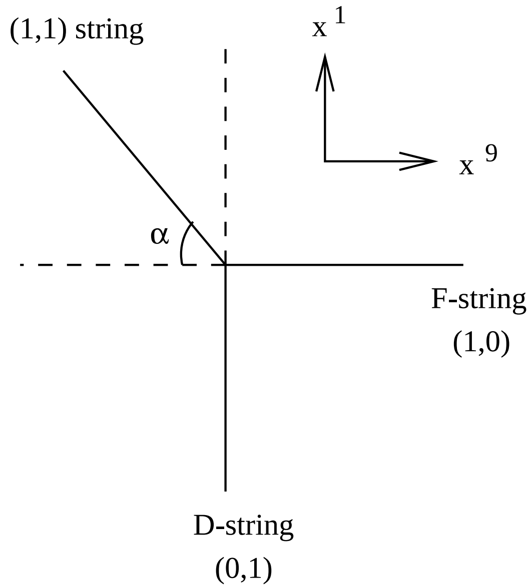

Fig.2: F-string ending on a D-string, with $\widetilde{\phi} = 0$.

Now, for this simple case, we can see that the force-balance property holds. The three strings, labelled by their (p,q) charges, have tensions given by[9]

$$T_{p,q} = \sqrt{p^2 + \frac{q^2}{g^2}}\, T_{1,0} \tag{5}$$

From Eqs.(4) and (5) it follows that

$$\begin{aligned} T_{1,1}\cos\alpha &= T_{1,0} \\ T_{1,1}\sin\alpha &= T_{0,1} = \frac{1}{g} T_{1,0} \end{aligned} \tag{6}$$

Thus, examination of the $U(1)$ gauge theory on the world-sheet of a D-string has yielded rather nontrivial information. Linear growth of the scalar potential in 1d translates, by the BPS condition, into a linearly rising deformation of the D-string on which the point charge is inserted. The angle is determined by the point charge, and satisfies the zero-sum condition on the tensions viewed as vectors.

More general 3-string junctions follow from the one considered above by application of S-duality transformations. However, so far we have dealt only with the case where the Ramond-Ramond scalar $\widetilde{\phi}$ is not excited on the D-string worldsheet. If we make an S-duality, in general a nonzero RR scalar background will be turned on. Therefore, we should start by considering the F-D junction (an F-string ending on a D-string), as above, but in the presence of an arbitrary RR scalar background. Then S-duality can be used to generate all the other cases. (It was emphasized in Ref.[6] that the most general 3-string junctions are the ones obtained by S-duality on the F-D junction, at least if one restricts attention to stable and not just marginally stable states.)

For nonzero $\widetilde{\phi}$, the D-string acquires a Chern-Simons coupling $\int \widetilde{\phi} \wedge F$ on its worldsheet[10]. This shifts the canonical momentum conjugate to A_1, and hence changes the quantization law by the replacement

$$F_{01} \to F_{01} + g\widetilde{\phi} \tag{7}$$

in Eq.(1) above. As a result, the classical solution Eq.(2) for the scalar potential is replaced by:

$$\begin{aligned} A_0 &= (\widetilde{\phi} - 1)\, gx^1, \quad && x^1 > 0 \\ &= \widetilde{\phi}\, gx^1, && x^1 < 0 \end{aligned} \tag{8}$$

We maintain the BPS nature of the solution by imposing Eq.(3).

The interpretation of this solution is as follows. The original D-string running along x^1 has been completely deformed, on both sides of the inserted charge. The inserted charge is, as before, associated with an F-string running along the X^9 axis. We have thus obtained the most general F-D junction, and this time (because the RR scalar is turned on) the F and D strings are not perpendicular. Thus the physical configuration, for $\widetilde{\phi} \neq 0$, is shown in Fig.3. From Eqs.(8) and (3), the angles α and β are given by

$$\begin{aligned} \tan\alpha &= \frac{1}{g(1-\widetilde{\phi})} \\ \tan\beta &= -\frac{1}{g\widetilde{\phi}} \end{aligned} \tag{9}$$

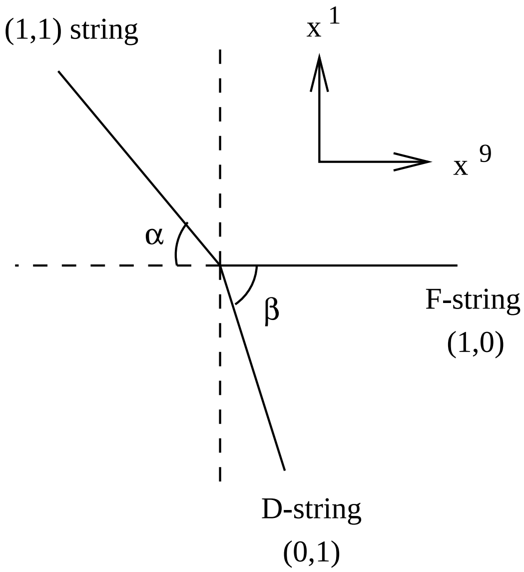

Fig.3: F-D junction for $\widetilde{\phi} \neq 0$.

The tension formula Eq.(5) is modified[9], in the presence of $\widetilde{\phi}$, to:

$$T_{p,q} = \sqrt{(p - q\widetilde{\phi})^2 + \frac{q^2}{g^2}}\, T_{1,0} \tag{10}$$

It is now again a straightforward matter to check that the force balance conditions hold:

$$\begin{aligned} T_{1,1} \cos\alpha &= T_{1,0} + T_{0,1} \cos\beta \\ T_{1,1} \sin\alpha &= T_{0,1} \sin\beta \end{aligned} \tag{11}$$

With these, we can make an SL(2,Z) transformation to the general case. Under a transformation with the matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ with $ps - qr = 1$, the F-string is mapped to a (p, r) string, and the D-string to a (q, s) string. The condition $ps - qr = 1$ is precisely the one proposed in Ref.[6] to characterize the allowed 3-string junctions.

The final configuration after the SL(2,Z) transformation is similar to that in Fig.3, with the above replacements for the string charges, and with angles given by

$$\begin{aligned} \tan\alpha &= \frac{1}{g\left((p - r\widetilde{\phi})((p+q) - (r+s)\widetilde{\phi}) + \frac{r^2}{g^2} + \frac{rs}{g^2}\right)} \\ \tan\beta &= -\frac{1}{g\left((p - r\widetilde{\phi})(s\widetilde{\phi} - q) - \frac{rs}{g^2}\right)} \end{aligned} \tag{12}$$

Thus we have found the most general BPS saturated, stable 3-string junction starting from very simple considerations. The tensions and angles of the junction satisfy the condition, which is intuitively rather obvious, that the vector sum of the tensions sums to zero. The stable junctions are those obtained by SL(2,Z) transformation of the F-D junction. To get all such stable junctions, we needed the F-D junction in the presence of an arbitrary RR scalar background.

To understand better the nature and role of 3-string junctions is an important problem. They are known to arise from M-theory by starting with a 2-brane in a "pants" configuration and wrapping each of the outgoing tubes on a different cycle of some 2-torus[5]. One may hope to get some insight by studying the 2-brane worldvolume along the lines of Ref.[8] and the present paper.

The 3-string junction could also be studied in the full Born-Infeld theory[8,11,12,13] rather than the linearized Maxwell case as we have done. The proposal of Ref.[5] to look for them as solutions of supergravity equations, remains to be carried out. It would be interesting to see if 3-string junctions can be recovered in the matrix approach to string theory[14]. Finally, it remains to investigate the proposal[6] that multi-pronged strings could play a fundamental role in nonperturbative string field theory.

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