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Quark mass corrections to the perturbative thrust and its effect on the determination of α_s

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Abstract. We consider the effects of quark masses to the perturbative thrust in e^+e^- annihilation. In particular we show that perturbative power corrections resulting from non-zero quark masses considerably alters the size of the non-perturbative power corrections and consequently, significantly changes the fitted value of α_s.

Keywords. QCD; jets; power corrections.

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One of the cleanest signatures of perturbative QCD comes from jet cross sections in e^+e^- annihilation. In such processes, it is possible to define infra-red safe event shape variables which can be calculated order by order in perturbative QCD and compared subsequently with experiment. However, in order to carry out these comparisons, a method has to be evolved to parametrize non-perturbative effects which though expected to be small at present Q^2 values at LEP, actually turn out to be substantial ($\sim$ 25%) even at $Q \sim m_Z$. One of the reasons for this is that these non-perturbative effects are actually suppressed by Q rather than Q^2. In addition, it is also possible that these power corrections could be comparable to $\mathcal{O}(\alpha_s^2)$ at present LEP energies.

In order to address these issues the Milan group of Dokshitzer *et al* [1] drawing on the earlier work of Webber [2], Korchemsky and Sterman [3] and others, presented a systematic approach for handling power corrections using perturbation theory. Very briefly, they studied the consequences of assuming that α_s has a low energy effective form which does not grow at low scales but has an infra-red regular form. The moments of α_s are integrated only over the infra-red region. Various non-perturbative parameters are then parametrized and the form and magnitude of power corrections are determined.

However, before one uses the approach of the Milan group in order to get a handle on power corrections and subsequently determine α_s by a fit to the data, it is important to isolate power corrections coming from a purely perturbative region. The Milan approach neglects the masses of all the quarks but instead uses a gluon mass as a 'trigger' to differentiate the perturbative from the non-perturbative region. We find however that the masses of the quarks, particularly the **c** and the **b** quarks, even at present LEP energies, can contribute significantly (of the order of about 25%). In fact, if we go beyond the top quark threshold (which is expected, perhaps in the future NLC) the perturbative contribution to power contributions due the top quark mass is even larger. We will have more to say on this later in the paper.

In this paper, we consider the example of one such event shape variable – the thrust – and show the significance of the effect of quark masses which need to be folded in before estimating the non-perturbative contribution to power corrections. We present explicit expressions to $\mathscr{O}(\alpha_{\rm s})$ of quark mass corrections expanded to $\mathscr{O}(m)$. We also show the effect of keeping the full mass contribution to $\mathscr{O}(\alpha_{\rm s})$ which unfortunately does not have a simple analytic form like the former and needs to be calculated numerically. Using these expressions we then fold in the power corrections of the Milan type and use this full expression to estimate both α_0 and $\alpha_{\rm s}$ and compare it with estimates that exist in the literature without taking quark masses into account.

The first paper which calculated the effect of quark masses to $\mathscr{O}(\alpha_{\rm s})$ was published about 16 years ago by one of the authors [4]. For completeness, in what follows, we quote those results from that paper which we need for our analysis here. The thrust, as defined traditionally, is given by

$$T = 2\frac{\max\sum_{i\in h}(p_i\cdot\hat{n})}{\sum_i |p_i|}, \tag{1}$$

where the denominator runs over all observed particles and the numerator runs over all particles in a hemisphere. $\hat{n}$ is a unit vector chosen in a direction that maximizes the numerator and defines the jet axis.

While this definition is appropriate for all massless particles, to include mass effects in the definition of the thrust, we modify the above definition slightly and write

$$T = 2\frac{\max\sum_{i\in h}(p_i\cdot\hat{n})}{W}, \tag{2}$$

where $W^2 = s$. Of course the denominator equals $\sum_i |p_i|$ when all the particles are massless. This normalization with the total energy is also what is used by the Milan group in their analysis though in their case the massive gluon eventually decays into massless quarks and gluons.

For a three-particle final state, the thrust, as we define it, is given by

$$T = \max\left[(x_1^2-\xi)^{1/2}, (x_2^2-\xi)^{1/2}, x_3\right], \tag{3}$$

where $x_i = 2E_i/W$, E_i being the energy of the ith particle in the final state in the c.m. frame and $\xi = 4m^2/W^2$, m being the mass of the quarks. Note that in the two-jet limit $T = T_0 \equiv \sqrt{1-\xi}$.

The average value of the thrust is defined by

$$\langle T\rangle = \frac{\left[\int T\frac{{\rm d}\sigma}{{\rm d}T}{\rm d}T\right]}{\left[\int \frac{{\rm d}\sigma}{{\rm d}T}{\rm d}T\right]}. \tag{4}$$

The numerator of the above is given up to $\mathscr{O}(\alpha_{\rm s})$ and to $\mathscr{O}(\xi)$ by ($\sigma_0 = (4\pi\alpha_2/s)e_i^2$ is the total cross section for $e^+e^- \to q_i\bar{q}_i$) [4]

$$\begin{aligned}\frac{1}{\sigma_0}\int T\frac{\mathrm{d}\sigma}{\mathrm{d}T} = 1-\frac{\xi}{2}+\frac{4\alpha_\mathrm{s}}{3\pi}\Bigg\{ & \frac{137}{16}\xi+\frac{5}{4}\xi\ln 2-\frac{1}{2}\xi^{1/2}+\frac{7}{9}\\ & +\frac{1}{4}\xi\ln^2\xi-\xi\ln 2\ln\xi+\frac{\pi^2}{6}-\frac{\xi\pi^2}{6}\\ & -\frac{1}{8}\xi\ln\xi-\frac{1}{2}\xi\ln 3\ln 2-\frac{9}{2}\xi\ln 3-\ln^2 3+\frac{3}{8}\ln 3\\ & -\frac{1}{3}\xi\left[\mathrm{Li}_2(1-\xi^{1/2}+\xi/2)-\mathrm{Li}_2\left(\frac{1}{3}+\frac{1}{2}\xi\right)\right]\\ & -2\mathrm{Li}_2\left(\frac{2}{3}-\frac{1}{2}\xi\right)+\xi\mathrm{Li}_2\left(\frac{2}{3}-\frac{1}{2}\xi\right)\\ & -\frac{1}{2}\xi\mathrm{Li}_2\left(\frac{1}{3}-\frac{1}{4}\xi\right)+\xi\ln^2 2\Bigg\},\end{aligned} \tag{5}$$

where $\mathrm{Li}_2(x)$ is the dilogarithm function. In the $\xi\to 0$ limit this gives, for the average thrust,

$$\langle T\rangle = 1+\frac{4\alpha_\mathrm{s}}{3\pi}\left[\frac{1}{36}+\frac{\pi^2}{6}-\ln^2 3+\frac{3}{8}\ln 3-2\mathrm{Li}_2\left(\frac{2}{3}\right)\right], \tag{6}$$

which works out to, for the perturbative thrust in the massless limit,

$$\langle 1-T\rangle = 1.05\frac{\alpha_\mathrm{s}}{\pi}, \tag{7}$$

as quoted in numerous places in the literature.

Several points here are worthy of note. The leading term in the $O(m)$ expansion above is $\xi^{1/2}$. Thus the leading mass correction goes as $1/Q$. To the best of our knowledge, this fact was noticed for the first time in [4] and subsequently in [2] and [1] who have traced it to appear from the soft phase space boundary. We would like to stress that this $1/Q$ behavior is a pure perturbative higher-twist effect to the thrust and not related to any non-perturbative contribution. Thus, it seems clear, that the coefficient of $1/Q$ in the full expression for the thrust would include contributions both from the perturbative as well as the non-perturbative sectors. This aspect will become more quantitative, when we do our fits later.

The second point to note is a calculational one. Since, in the two-jet limit, the thrust is equal to $T_0=\sqrt{1-\xi}$, in order to make the virtual contributions vanish we need to calculate, not as in the usual case $\langle 1-T\rangle$, but $\langle T_0-T\rangle$. It is then a trivial matter to add a term $\langle 1-T_0\rangle$ to obtain $\langle 1-T\rangle$ to compare with experiment.

In order to compare with experiment, however, and to redo the fits for α_s and α_0 we have used not only the $\mathcal{O}(m)$ contribution above but also the full massive contribution to $\mathcal{O}(\alpha_\mathrm{s})$ albeit evaluated numerically. In addition we have also compared the $\mathcal{O}(\alpha_\mathrm{s}^2)$ massless corrections to the $\mathcal{O}(\alpha_\mathrm{s})$ massive correction to try and estimate how much of the $1/Q$ corrections can be mimicked by higher orders in the coupling constant.

Figure 1 shows $\langle 1-T\rangle$ as a function of the center of mass energy computed with $\alpha_\mathrm{s}(m_\mathrm{Z})=0.12$. As one sees from the curve the contribution of the second-order terms are large over the entire energy region ($\sim$ 55% at $Q=12$ GeV going down to 33% at 200 GeV). On the other hand, the effect of quark masses, evaluated only to first-order in

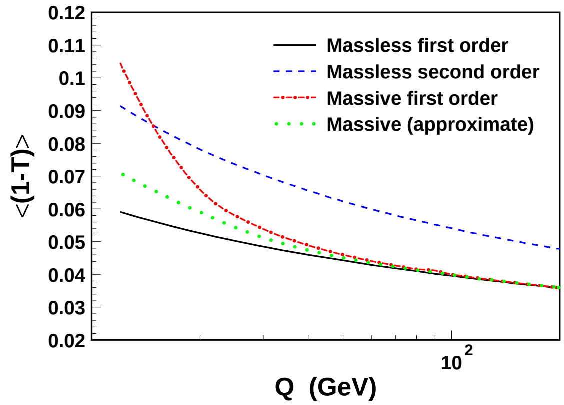

Figure 1. Effect of mass correction and higher order in the mean thrust value. The solid and the dashed lines are respectively the first- and second-order calculations of $\langle 1-T\rangle$ without mass corrections. The dashed–dotted and dotted lines are respectively the complete and approximate ($\mathscr{O}(m)$) first-order calculations with quark mass effects.

α_s, is even larger at low center of mass energy ($\sim$ 76% at $Q = 12$ GeV). This is clearly a $1/Q$ power law effect and hence dies off faster, becoming 2.5% at $Q \sim m_Z$ and negligible at 200 GeV. It is clear from the figure that one needs to take the full massive correction rather than the $\mathscr{O}(m)$ contribution, because it accounts only 60% (30%) of the mass correction at 20 GeV (12 GeV).

In order to compare the theoretical predictions with the measurements done at different center of mass energies [5] at PETRA, PEP, TRISTAN, SLC and LEP, we add the non-perturbative contribution *a la* the Milan group [1] to the perturbative contribution. In this paper, we use only the $\mathscr{O}(\alpha_s)$ calculation of $\langle 1-T\rangle$ and a more detailed comparison with a $\mathscr{O}(\alpha_s^2)$ calculation is under preparation [6]. We will have more to say on this later. The non-perturbative contribution, as is well known, is given by an additive contribution $\langle 1-T\rangle_{\text{pow}}$:

$$\langle 1-T\rangle_{\text{pow}} = 2\frac{4C_F}{\pi^2}\mathscr{M}\frac{\mu_I}{Q}\left[\alpha_0(\mu_I) - \alpha_s(Q) - \beta_0\frac{\alpha_s^2(Q)}{2\pi}\left(\ln\frac{Q}{\mu_I} + \frac{K}{\beta_0} + 1\right)\right] \tag{8}$$

where μ_I is an infra-red matching scale (taken as 2 GeV), $K = (67/18 - \pi^2/6)\cdot C_A - 5N_f/9$ and $\mathscr{M}$ is the Milan factor (determined to be 1.49) [7].

Figure 2 shows the experimental values of $\langle 1-T\rangle$ together with the two fits which use respectively the massless and the massive forms (both to $\mathscr{O}(\alpha_s)$ for the perturbative contribution). These fits have been carried out with two free parameters $\alpha_s(m_Z)$ and α_0. Both massless and massive formulation of the perturbative component give reasonable fits to the data with χ^2 of 126.6 and 116.6 respectively for 53 degrees of freedom. However, they do differ in the final values of $\alpha_s(m_Z)$ and α_0 as can be seen in table 1. In these fits the scale parameter is chosen to be 1.0.

Some issues have to be kept in mind here. The effect due to masses comes from two sources. The changed (new) definition of the thrust (eq. (2)) that we have used here (by dividing by the total energy) gives a factor $\xi/2$ independent of α_s in eq. (5).

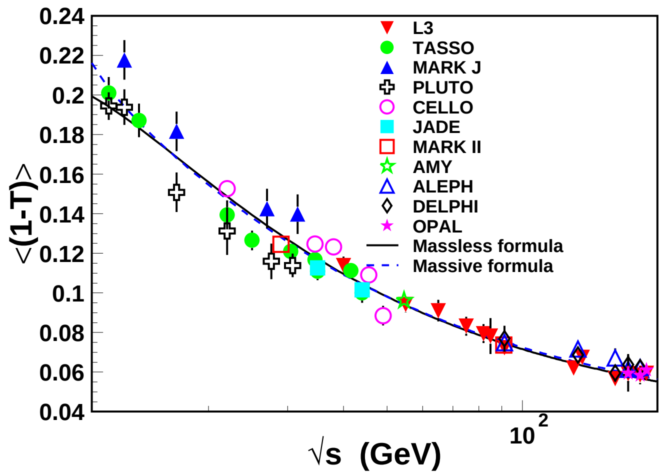

Figure 2. Results of the fit of $\langle 1-T\rangle$ to the first-order calculations with and without quark mass effects using eq. (2) as the definition of the thrust. The points indicate measured $\langle 1-T\rangle$ values from different experiments and the solid and dashed lines are fits without and with mass corrections.

Table 1. Results of the fits of the $\langle 1-T\rangle$ distribution to perturbative and power law terms when the quark mass effects in the perturbative term is ignored or included, using the new definition of the thrust, eq. (2).

Fit type	α_0	$\alpha_s(m_Z)$
Massless quarks	0.8980 ± 0.0047	0.1547 ± 0.0015
Massive quarks	0.8712 ± 0.0051	0.1639 ± 0.0014

Table 2. Results of the fits of the $\langle 1-T\rangle$ distribution to perturbative and power law terms when the quark mass effects in the perturbative term is ignored or included, using the standard definition of the thrust, eq. (1).

Fit type	α_0	$\alpha_s(m_Z)$
Massless quarks	0.7267 ± 0.0045	0.1487 ± 0.0016
Massive quarks	0.6969 ± 0.0049	0.1577 ± 0.0015

However this is an artificial dependence of ξ introduced by our definition. We have therefore extracted this dependence (which amounts to about 4.9% at 12 GeV, 0.92% at 91.2 GeV and drops to 0.5% at 189 GeV) which is equivalent to using the old definition of the thrust and this then gives the results shown in table 2. The remaining dependence on ξ which is multiplied by α_s is then the genuine mass dependence of the thrust and which appears as $\xi^{1/2}$ as the leading term in eq. (5). (For completeness, the older definition of thrust (eq. (1)) gives χ^2 as 133.4 and 123.2 for massless and massive quarks respectively, for 53 degrees of freedom).

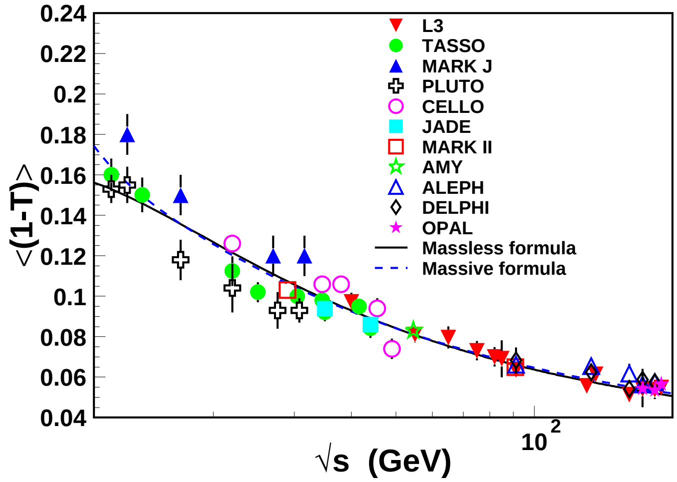

Figure 3. Same as in figure 2 but with the old definition of the thrust, eq. (1).

Figure 2 shows the fit using the new definition of the thrust consistent with the values in the first table. A similar fit using the old definition of the thrust (corresponding to eq. (1) and table 2) would, as we have stated already, show only minor variations, that too at the lowest energy (12 GeV) and this is shown in figure 3.

The errors quoted in table 1 are experimental errors obtained from the minimization procedure. We can also estimate the theoretical uncertainties on these quantities by varying the scale parameter. If we vary the scale parameter between 0.5 and 2.0, we obtain uncertainties in α_s and α_0 to be ± 0.010 and ± 0.12 respectively. The value of $\alpha_s(m_Z)$, obtained from the fits, when quark mass effects are included or ignored, differ by 0.009 which is much larger than the experimental uncertainty of about 0.001 on the α_s value and comparable in fact to the theoretical uncertainty.

It is thus clear from the preceding analysis that an estimate of the power corrections due to the non-zero masses of the quarks is crucial in getting better and more realistic estimates on the strong coupling constant and indeed, in general, on power corrections. The next obvious step would be to calculate mass corrections to $\mathcal{O}(\alpha_s^2)$. Some results in this direction have been obtained by Nason and Oleari [8] which could be used to carry out a similar analysis to the one presented above. We are, at present, in the process of extending our analysis to second order in the strong coupling using the results of [8].

We would now like to comment on our work as compared to similar work in the literature. The α_s estimation from event shape distributions is done mainly in the following two ways: (1) some event shape distributions are fitted to second order or matched second order with resummed NLL calculations; (2) fit the moments of a few event shape distributions to second order + power law corrections. In both these methods the QCD calculations used are for massless quarks. In method (1) hadronization corrections are introduced using different Monte Carlo models – JETSET, HERWIG, ARIADNE etc. There, in addition to hadronization models, some quark mass effects go in through model parameters.

Unlike papers in the literature on this topic (some of which we have cited) we have given an explicit analytic expression for the mass correction to the thrust. This expression explicitly also demonstrates the typical $\xi^{1/2}$ behavior that is also predicted for power corrections in the dispersive approach of Dokshitzer *et al*. This effect is, in fact, what is responsible for the changing of the usual α_s value from the value obtained from the massless case and gives a sizable effect at low energies.

In various projected Linear Collider scenarios (like, for example, the NLC) energies upwards of 500 GeV are expected. In such a region, the effect of the top quark would be dramatic and significant. The combination of the large mass of the top quark and a charge squared of 4/9 implies that the usual massless expressions for the thrust would not work. We have estimated that the difference between choosing a massless formula for describing the thrust beyond the top quark threshold and using the (more appropriate) massive formula changes the value of the thrust by about a factor of 5 near the threshold. Most of this contribution comes, in fact, from the top quark mass. In table 3 we give an estimate of the change that would occur between choosing all quarks massless and massive above the top quark threshold. It is obvious that the effect is spectacularly large, particularly near the threshold. Mass effects in the resummation of event shape variables are also expected to be significant and this is presently being studied.

Thus, it is imperative that in order that reliable estimates be made of the thrust at these energies, we have available, calculations to higher orders in α_s of e^+e^- scattering with massive quarks in the final state. This would also give us a handle on the relative magnitudes of power corrections to the thrust to a particular order in α_s and the magnitude of the next order term in α_s [7,9]. For example, NNLO effects might be capable of mimicking the $1/Q$ behavior. Mass effects in the resummation of event shape variables are also expected to be significant and this is presently being studied.

To summarize, we have, in this paper, made some significant departures from earlier work in this area, viz.,

- There are no Monte Carlo ‘artefacts’ in any of our analysis.
- We have shown that the masses of quarks (in addition to the power corrections of the non-perturbative kind) also give an m/Q effect rather than a m^2/Q^2 effect.
- That this effect even at lowest order is sufficient to change the central value of α_s. Higher order corrections would change this presumably even further but the effect is already visible. This is why we have not in fact called it a ‘new’ determination of α_s. That would have to wait for the full NLO analysis.
- Even though mass correction calculations to the perturbative thrust and other jet variables do exist, in principle, to NLO, these are not in a form that is easily amenable to the analysis we have done. This is something we are endeavouring to incorporate at present.

Table 3. Difference between choosing massless and massive quarks above the top quark threshold.

Q (GeV)	α_s	$\langle 1-T \rangle$ (Massless)	$\langle 1-T \rangle$ (Massive)
360	0.0995	0.0323	0.1605
500	0.0957	0.0311	0.0994
1000	0.0888	0.0289	0.0435

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