INAUGURAL ARTICLE

An instrument for direct observations of seismic and normal-mode rotational oscillations of the Earth

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The rotations around the vertical axis associated with the normal mode oscillations of the Earth and those induced by the seismic and other disturbances have been very difficult to observe directly. Such observations will provide additional information for 3D modeling of the Earth and for understanding earthquakes and other underground explosions. In this paper, we describe the design of an instrument capable of measuring the rotational motions associated with the seismic oscillations of the Earth, including the lowest frequency normal mode at $\nu \approx 3.7 \times 10^{-4}$ Hz. The instrument consists of a torsion balance with a natural frequency of $\nu_0 \approx 1.6 \times 10^{-4}$ Hz, which is observed by an autocollimating optical lever of high angular resolution and dynamic range. Thermal noise limits the sensitivity of the apparatus to amplitudes of $\approx 1.5 \times 10^{-9}$ rad at the lowest frequency normal mode and the sensitivity improves as $\nu^{-3/2}$ with increasing frequency. Further improvements in sensitivity by about two orders of magnitude may be achieved by operating the balance at cryogenic temperatures. Alternatively, the instrument can be made more robust with a reduced sensitivity by increasing ν_0 to $\approx 10^{-2}$ Hz. This instrument thus complements the ongoing effort by Igel and others to study rotational motions using ring laser gyroscopes and constitutes a positive response to the clarion call for developments in rotation seismology by Igel, Lee, and Todorovska [H. Igel, W.H.K. Lee and M.I. Todorovska, AGU Fall Meeting 2006, Rotational Seismology Sessions: S22A,S23B, *Inauguration of the International Working Group on Rotational Seismology (IWGoRS)*].

Torsional oscillations of the Earth, sometimes called torroidal oscillations were predicted in the 19th century and have been calculated with increasing precision during the recent decades. The lowest of these modes involve the oscillations of the entire mantle and have periods of ≈3,000 s. Careful measurement of these oscillations is of considerable importance in understanding the Earth and the mechanism of earthquakes. The rotations associated with the normal modes of the Earth have not been directly observed with the present-day instruments. Seismological studies have been mostly responsible for our understanding of the nature and dynamics of the interior of the Earth, with additional information coming from other channels such as paleomagnetism, VLBI, and GPS. At present, the seismological studies are essentially limited to monitoring global and local wave fields at various frequencies, measuring only the three components of the translational velocity of ground displacement and in some cases directly the strains. The Fourier transform of the horizontal translations yields the frequency spectrum of these oscillations, and their correspondence with theoretical calculations carried out with the PREM model of Dziewonski and Anderson allows us to identify the frequencies and Q values associated with the rotational oscillations of the Earth's mantle. In a unique effort, Igel *et al.* (1) have been able to detect the rotational motions induced by the M8.1 Tokachi–Oki earthquake at frequencies of the order of 30–60 mHz, using ring laser gyroscopes.

Theoretical studies, for example, by Cochard *et al.* (2), indicate that the observations of seismic rotational motions will provide important new information pertaining to the Earth sciences, complementary to those obtained from the observations of the translational motions of the Earth surface using conventional seismometers. They note that combining the observation of rotational motion with the concomitant translational motion yields wave-field properties such as phase velocities and the direction of propagation. Even though it is possible to derive some of this information from the studies with arrays of seismographs (for which they provide the requisite analysis procedures), the advantages of direct observations of the rotational motions will be of great value.

Whereas Castellini and Zembaty (3) have pointed out that, generally, not only the amplitude of the rotational motions but also the damages caused by them is underestimated. Accordingly, Cochard *et al.* (2) have emphasized that recording even small, rotational motions with low potential for causing damage would be useful, especially because of possible cross-couplings between rotational motions and tilt motions (Todorovska and Trifunac, ref. 4). The amplitudes of these rotational motions were estimated to be as high as 100 μrad by Bouchon and Aki (5). Deusss and Woodhouse (6) have theoretically calculated the free-oscillation spectra and discussed the effect of the coupling of large groups of normal modes. They have particularly emphasized the cross-coupling between spheroidal modes and torroidal modes, and have shown that a detailed study of the spectra would be useful in developing a 3D structural model of the Earth. In this context, we note that the pioneering observations of Suda *et al.* (7) and their interpretation by Rhie and Romanowicz (8) have indicated the presence of a continuous "hum" in the spheroidal mode oscillations of the Earth in the frequency range 2–7 mHz, which is probably generated by the interactions amongst the ocean floor, oceans, and atmosphere. Thus, we may expect a hum in the torroidal modes as well, either generated by cross-couplings with other modes, or, more interestingly, generated directly. At present, we have several theoretical models with explicit predictions for rotational seismic motions but do not have observationally confirmed answers to several basic questions specific to rotational motions: What are the angular amplitudes of the torsional normal modes of the Earth in its quiescent state? What is the spectrum of excitations triggered by an earthquake? Are all the earthquakes the same or are there differences that signify the different underlying causes? How much energy of an earthquake is channeled into these modes? How is this energy dissipated, by cross-coupling to S modes or by transfers to high or low frequencies before thermalization?

Recently, Aki and Richards (9) have bemoaned that seismology still awaited suitable instruments for making measurements of the rotational motions. The current situation may be summa-

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rized by noting that, by and large, seismic observations have been confined to the measurement of the three translational motions along the $\hat{x}$, $\hat{y}$, $\hat{z}$ axes, and the measurements of the rotations about these three axes are either nonexistent or limited to occasional measurements of strong fields carried out in close proximity to earthquakes or volcanoes (10). Recently, a special session was organized by Igel, Lee, and Todorovska, at the fall meeting of the American Geophysical Union to stimulate studies in Rotational Seismology.†

In this paper, we describe the design and performance capabilities of an instrument is capable of measuring the rotational seismic motions of the earth. The essential idea behind the design may be briefly stated as follows: A mechanical torsional oscillator with a natural frequency significantly below the lowest normal mode frequencies will couple negligibly to the rotational motions of the Earth, even though the housing of the oscillator is firmly fixed to the Earth. An optical lever fixed to the Earth observing such an oscillator will record faithfully its own angular motion as it follows faithfully the rotational seismic motions. Below, we provide a detailed physical description of the instrument and analyze its response to rotational seismic signals. The conceptual aspects of this instrument are different from that of the ring laser gyroscope developed by Schreiber *et al.* (11) and adopted by Igel *et al.* (1) in their pioneering efforts to observe the rotations induced by the Tokachi–Oki earthquake, and the instrument described here will complement their efforts and will be capable of making highly sensitive new observations, especially at low frequencies. We finish with general remarks about the instrument, its performance, and possible avenues for development in the future.

Brief Description of Rotational Normal Modes

Following closely Stein and Wysession (12), the displacement vector corresponding to pure rotations around the local zenith $\hat{z}$ axis, with no radial displacements of the Earth, is written in the spherical polar coordinate system, with the origin at the center of the Earth and the polar and azimuthal angles defined according to the specific seismic event under consideration

$$u^T(r,\theta,\phi) = \sum_n \sum_l \sum_m {}_nA_l^m W_l(r) T_l^m(\theta,\phi) e^{i({}_n\omega_l^m)t}, \qquad [1]$$

where ${}_nA_l^m$ are the amplitude coefficients corresponding to the eigenfrequencies ${}_n\omega_l^m$, which become independent of m for a laterally homogeneous (spherically symmetric) Earth. The radial dependence of the rotational amplitudes are represented by $W(r)$, and the surface eigenfunctions T_l^m are given in terms of the vector spherical harmonics, with the $(r,\ \theta,\ \phi)$ components

$$T_l^m = \left(0, \frac{1}{\sin\theta}\frac{\partial Y_l^m(\theta,\ \phi)}{\partial\phi}, -\frac{\partial Y_l^m(\theta,\ \phi)}{\partial\theta}\right). \qquad [2]$$

The torsional modes have only horizontal displacements, and for a spherically symmetric non-rotating Earth, all the "singlet" frequencies ${}_n\omega_l^m$ become degenerate and may be specified by ${}_n\omega_l$. In a real Earth, in general, there will be a $(2l + 1)$ splitting of the frequencies.

The rotations around the $\hat{z}$ axis in which we are interested are given by the curl of the displacement field

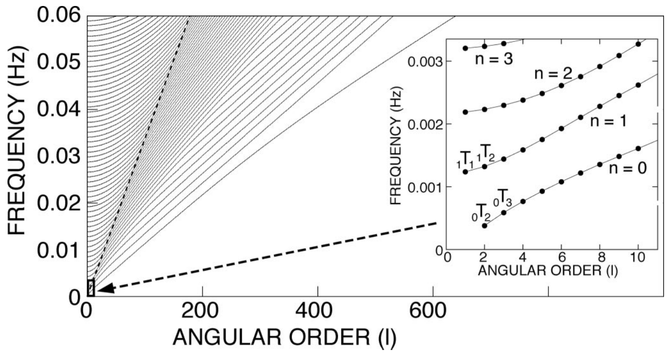

Fig. 1. Frequency of torsional modes as a function of angular order computed from the PREM model of Dziewonski and Anderson and/or estimated from the study of translational motions (13). (*Inset*) The region of low frequency and low angular order (M. Wysession, personal communication).

$$\begin{aligned} R(r,\theta,\phi) &= (\nabla \times u^T)\cdot\hat{r} \\ &= \sum_n \sum_l \sum_m {}_nA_l^m W_l(r) e^{i({}_n\omega_l^m)t} \times \frac{1}{r\sin\theta} \\ &\quad \times \left[-\frac{1}{\sin\theta}\frac{\partial^2 Y_l^m}{\partial\phi^2} + \cos\theta\frac{\partial Y_l^m}{\partial\theta} + \sin\theta\frac{\partial^2 Y_l^m}{\partial\theta^2}\right] \\ &= \sum_n \sum_l \sum_m R(\omega)(\cos\omega t + i\sin\omega t). \end{aligned} \qquad [3]$$

In the final part of this equation, we have written for convenience ${}_n\omega m_l{}^m = \omega$. In later discussions, we will focus attention on the real part of $R(r,\theta,\phi)$.

The eigenfrequencies computed by using the "Preliminary Reference Earth-Model" of Dziewonski and Anderson (13) or obtained form the Fourier spectra of lateral displacements are shown in Fig. 1 (from Stein and Wysession). These modes have Q values generally in excess of 200, and some may have Q values $>$1,000. Because the magnitude of torsional angular motions with respect to the local vertical axis depends on the curl of the horizontal displacement field $u^T(r,\theta,\phi)$, it is likely that the torsional angular amplitudes of the higher harmonics may be augmented. Accordingly, it would be interesting to see whether the signals in the instrument presented here are dominated by the Love waves. Notice from Fig. 1 that, at high frequencies corresponding to high l-orders, the eigenfrequencies of the modes get very close to one another, and their constructive interference may generate large amplitudes without any discernible peaks. On the other hand, for low l-orders, the eigenfrequencies change slowly, with large fractional change, so that the oscillation spectra will be resolved into separate peaks. For example, the frequencies of ${}_0T_2$ and ${}_0T_3$ are estimated to be $\nu({}_0T_2) = 0.371$ and $\nu({}_0T_3) = 0.586$ mHz, respectively; an estimate of the eigenfrequencies of the lowest five modes and their corresponding periods is given in Table 1.

Table 1. The lowest eigenfrequencies and their corresponding periods

Mode	Frequency, H_z	Period, min
${}_0T_2$	3.71×10^{-4}	44.9
${}_0T_3$	5.86×10^{-4}	28.5
${}_0T_4$	7.70×10^{-4}	21.6
${}_0T_5$	1.08×10^{-3}	15.4
${}_0T_6$	1.23×10^{-3}	12.5

†Igel, H., Lee, W. H. K., Todorovska, M. I., AGU Fall Meeting 2006, Rotational Seismology Sessions S22A and S23B, *Inauguration of the International Working Group on Rotational Seismology (IWGoRS).*

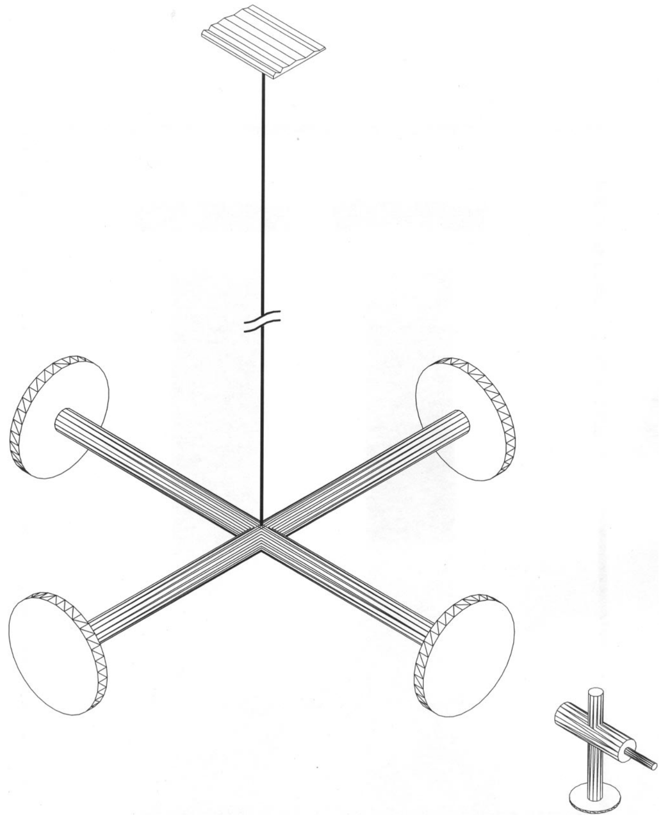

Fig. 2. Schematic of the Torsion Balance: the point of suspension and the autocollimator are fixed on to the ground and follow its motion faithfully.

The rotations associated with these modes have not yet been observed with the existing seismometers, sensitive primarily to linear displacements; one would have to correlate the signals from seismometers located at different locations to be able to observationally estimate the torsional motion of the Earth's surface. We describe below the design and estimated performance characteristics of an instrument that is capable of observing the rotations of some of the key torsional modes from a single geographic location, by directly measuring the angular amplitude of the torsional oscillations.

Physical Description of the Instrument

In essence, the instrument consists of a torsion balance having a natural period of oscillation significantly longer than the lowest frequency normal modes of the Earth. The angular position of the Earth is recorded at regular intervals by means of a high-resolution optical lever of large dynamic range. As noted in the Introduction, such a torsion balance will couple negligibly to the rotational oscillations of the Earth and an optical lever that is firmly coupled to the Earth will faithfully record the seismic rotational oscillations. Here, the challenge faced by the experimentalist is twofold: the development of a mechanical torsional oscillator with a low enough natural frequency $\approx 10^{-4}$ Hz and the fabrication of an optical lever with an angular resolution $\Delta\psi$ better than $\approx 10^{-9}$ rad·Hz$^{-1/2}$. These two basic subsystems of the instrument, which meet these design requirements, are described below.

Description of the Torsion Balance. The balance is designed to have its natural frequency of oscillations substantially below that of the lowest normal mode of torsional oscillation of the Earth, i.e., below $\approx$0.37 mHz. Furthermore, it is to be made insensitive to ground tilts and several other unwanted background torques. These requirements are met in the following way: the balance bob consists of a cross with all four arms equally long, $\approx$16 cm, suspended from its center such that the arms lie in the horizontal plane as shown in Fig. 2. The tips of the prongs carry masses, all equal and $\approx$10 g each; two of these act also as mirrors, and either may be viewed by the optical lever. Such a symmetric configuration is insensitive to gradients in the local gravitational field. The moment of inertia of the balance about the suspension axis is, $I = 10{,}240$ g·cm^2. The suspension fiber is made of SS-304 alloy, which has a cross section $(x \times y) = 7 \times 110$ μm and length, $l = 100$ cm, and its rigidity modulus is $n = 7.8 \times 10^{11}$ dyne·cm^{-2}. Following Champion and Davy (14), the torsional rigidity of the fiber is given by

$$k_f = \frac{x^3 yn}{3l} = 10^{-2}\ \text{dyne·cm·rad}^{-1}. \qquad [4]$$

The angular frequency of natural oscillations is given by

$$\omega_0 = \left(\frac{k_f}{I}\right)^{1/2} \approx 10^{-3}\ \text{rad·s}^{-1}, \qquad [5]$$

which corresponds to a period of 105 min or a frequency of $\approx 1.6 \times 10^{-4}$ Hz, i.e., significantly smaller than the frequencies even of the lowest torsional modes of the Earth.

The oscillations of the balance as a simple pendulum are damped by attaching the top of the fiber of the torsion balance to a copper disk suspended by a torsionally stiff wire of circular cross-section, thus constituting a small simple pendulum. A set of permanent magnets generate fields that thread through the disk. Eddy currents generated by pendular motions are dissipated in the disk, thereby damping the pendular motions. It is important to isolate the balance from ground tilts, because a suspension fiber of rectangular cross-section will, in principle, generate a finite coupling between tilt and torsion of the fiber. However, because the torsionally stiff wire above it has a circular cross-section, the balance will be isolated from tilts.

For torsion balances to work at such low frequencies as 10^{-3} rad·s^{-1}, the balance will have to be placed inside an ultra-high vacuum chamber. An ion pump with a pumping speed of $\approx$ 30 liter·s^{-1} maintains a vacuum of $\approx 10^{-9}$ Torr inside the chamber, without generating any vibrations whatsoever. The chamber is surrounded by magnetic shielding materials like mumetal, conetic and netic alloys that shield it from the fluctuations in the Earth's magnetic field. The whole apparatus, including the vacuum chamber, is surrounded by thermal insulators that shield it from temperature variations and gradients. One of the view ports of the vacuum chamber is made of optically flat glass, and the optical lever, which is described below, views one of the mirrors at the ends of the arms of the balance, and records the angular position of the balance.

Design and Performance of the Optical Lever. The optical lever consists of a well illuminated array of 110 slits, each having a width of 90 μm and a pitch of 182 μm, thus covering a total length of $\approx$22 mm. This is located in the focal plane of a field lens of focal length $f \approx 1{,}000$ mm. The light rays from the array of slits are collimated into parallel beams as they pass through the lens, and these beams are incident on the mirror of the torsion balance, which, after reflection, pass through the lens again, generating a well focused image of the slits on the focal plane. This optical design is generally referred to as the autocollimating configuration, and it ensures that the image quality and the angular displacement of the image due to motions of the mirror are sensibly independent of changes in the temperature of the surroundings. The optical image is sampled by a linear CCD camera of 2,048 pixels, each of dimension 14×14 μm. Thus, the

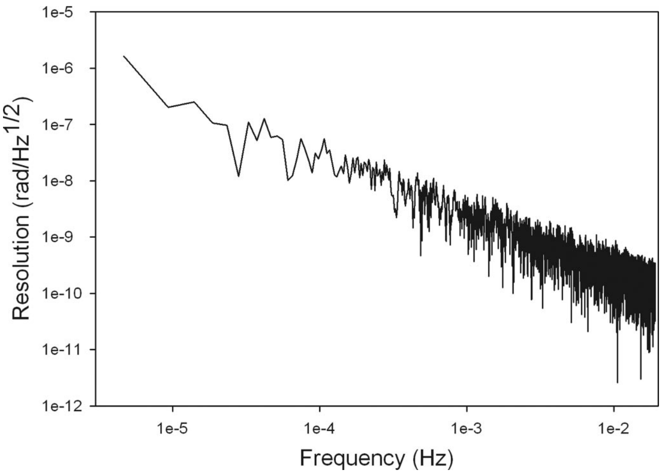

Fig. 3. A typical signal from the autocollimator.

diffraction broadened image of each slit is sampled by 13 pixels, thereby ensuring that spatial digitization effects are negligible. The well depth is ≈70,000 electrons, which is digitized by an 8-bit ADC, and the image is stored in a buffer. The centroid of the image x_c is calculated, and the ratio $(x_c/2f)$ is a measure of the angle between the optic axis of the autocollimating optical lever and the normal to the mirror on the torsion balance. In this way, typically ≈300 images are recorded per second, and the angles are calculated and stored as a time sequence. To test the performance characteristics of this system, we have observed a mirror held fixed in the laboratory. The Fourier transform of this signal is shown in Fig. 3. The angular resolution $\Delta\psi$ of the lever at frequencies below $\approx 10^{-2}$ Hz may be represented as

$$\Delta\psi = 2 \times 10^{-10}(0.01/\nu)\ \text{rad}\cdot\text{Hz}^{-1/2} \text{ for } \nu < 10^{-2} \text{ Hz}. \quad [6]$$

On the basis of a second data set (not shown) acquired with shorter integration period, we note that the resolution becomes nearly constant at higher frequencies

$$\Delta\psi \approx 2 \times 10^{-10}\ \text{rad}\cdot\text{Hz}^{-1/2} \text{ for } \nu > 10^{-2} \text{ Hz}. \quad [7]$$

Because the Q factor of the seismic rotational motions is large, ≥200, which restricts the bandwidth of the seismic signals to $\Delta\nu < \nu/Q \approx 10^{-5}$ Hz, the autocollimator may be used to measure angles of

$$\Delta\theta \approx \Delta\psi \times (\Delta\nu)^{1/2} \approx 10^{-10} \text{ rad} \quad [8]$$

at all frequencies of relevance to seismic signals.

Response Function and Sensitivity of the Balance

We begin this section with an analysis of the response of the instrument to torsional seismic oscillations and show that the signal put out by it has a unity gain at frequencies substantially above the natural frequency of the torsion balance, but has a phase lag of 180°. We then discuss the rms amplitudes induced by the thermal fluctuations and show that these essentially define the detection threshold.

Response Function of the Balance. In deriving the response function, we first note that the point of suspension of the balance and the autocollimating optical lever will faithfully follow the seismic angular oscillations defined in Eq. **3** by $R(\text{r}, \theta, \phi)$. Considering one of the Fourier components $R(w)$, the torque acting on the balance due to the angular twist of the fiber is given by

$$S(\omega) = k_f R(\omega)\cos\omega t = I\omega_0^2 R(\omega)\cos\,\omega t. \quad [9]$$

The response of the balance to this component may be written as (see Marion, ref. 15)

$$\ddot{\psi} + 2\beta\dot{\psi} + \omega_0^2\psi = a(\omega)\cos(\omega t), \quad [10]$$

where the angular displacement of the balance is represented by ψ, the damping parameter by β, the resonant frequency by ω_0, and the driving term by $a(\omega) = \omega_0{}^2 R(\omega)$. The particular solution to this equation is

$$\psi = \frac{a(\omega)}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}}\cos(\omega t - \delta) \quad [11]$$

with

$$\sin(\delta) = \frac{2\omega\beta}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}}\cos(\delta)$$
$$= \frac{\omega_0^2 - \omega^2}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}}\tan(\delta)$$
$$= \frac{2\omega\beta}{\omega_0^2 - \omega^2}. \quad [12]$$

The quality factor Q of the balance is given by

$$Q = \frac{(\omega_0^2 - 2\beta^2)^{1/2}}{2\beta} \approx \frac{\omega_0}{2\beta}. \quad [13]$$

Now the axis of the optical lever is coupled to the earth and follows its motion faithfully

$$\psi_L = \frac{1}{\omega_0^2}a(\omega)\cos(\omega t). \quad [14]$$

Accordingly, the angle α measured by the optical lever is given by

$$\alpha = \psi - \psi_L$$
$$= \frac{a(\omega)}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}}\cos(\omega t - \delta) - \frac{a(\omega)}{\omega_0^2}\cos(\omega t)$$
$$= a(\omega)\left\{\frac{\cos(\delta)}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}} - \frac{1}{\omega_0^2}\right\}\cos(\omega t) + a(\omega)$$
$$\cdot\left\{\frac{\sin(\delta)}{[(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2]^{1/2}}\right\}\sin(\omega t)$$
$$= -\frac{a(\omega)}{\omega_0^2}\cos(\omega t) = -R(\omega)\cos(\omega t), \quad \text{for } \omega >> \omega_0. \quad [15]$$

We show in Fig. 4 the coefficients of the $\cos(\omega t)$ and the $\sin(\omega t)$ terms for various values of (ω/ω_0) in units of $a(\omega)$, for $\beta = \omega_0/200$ corresponding to $Q \approx 100$.

Several interesting features emerge from inspection of Eq. **15** and Fig. 4. The response of the instrument at frequencies above the natural frequency ω_0 rapidly reaches unity, even though it is exactly π out of phase with the seismic oscillations. At frequencies below ω_0, the response declines, initially rapidly close to ω_0 and subsequently as $(\omega/\omega_0)^2$. In the quadrature phase, the response is weaker and decreases as $(\omega_0/\omega)^2$ at higher frequencies. Because ω_0 is below the lowest frequencies of the normal modes, the instrument will be capable of recording the seismic motions with unit response.

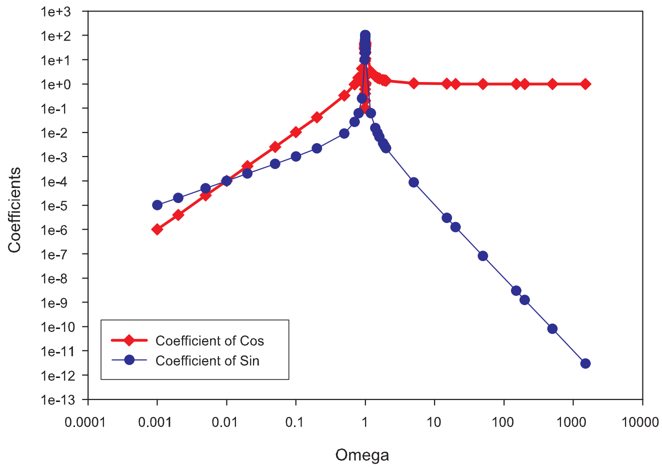

Fig. 4. Coefficients of the cos(ωt) and sin(ωt) terms in Eq. **15** in units of $a(\omega)$. Note that the x axis shows the angular frequencies in units of the natural frequency ω_0 of the balance. The coefficient of the cosine term (solid line) indicates that the instrument attains unit response quickly as the signal frequency exceeds ω_0 and the phase reaches a constant value equal to π.

Sensitivity of the Instrument. The thermal fluctuations of the torsion balance determine the threshold sensitivity of the balance. These are easily calculated by noting the interconnection that the fluctuating thermal torques bear with the damping term β or the quality factor of the balance, as first explained by Einstein (16) and worked out with progressively increasing detail by others (17, 18), who emphasized that the autocorrelation function of the thermal torques is a Dirac-delta function. Thus, the thermal torques are represented by

$$\langle S_T(t)S_T(0)\rangle = 4\beta IkT\delta(t) = \frac{2\beta IkT}{\pi}\int_{-\infty}^{\infty} e^{-i\omega t}d\omega = \frac{IkT\omega_0}{Q\pi}\int_{-\infty}^{\infty} e^{-i\omega t}d\omega. \quad [16]$$

Accordingly, the rms thermal amplitudes of the balance per square root Hertz, α_{Th}, are described by an equation similar to Eq. **10**, but with a driving term $a_{Th}(\omega)\cos\omega t$. The rms value of a_{Th} per $Hz^{-1/2}$ is given by

$$\langle a_T^2(\omega)\rangle^{1/2} = \frac{\langle S^2\rangle^{1/2}}{I} = \left(\frac{2\beta kT\omega_0}{\pi I}\right)^{1/2} = \left(\frac{kT\omega_0}{\pi IQ}\right)^{1/2}. \quad [17]$$

Thus

$$\langle \alpha_{Th}^2(\omega)\rangle^{1/2} = \frac{\langle a_T^2\rangle^{1/2}}{[(\omega^2-\omega_0^2)^2+4\beta^2\omega^2]^{1/2}} \approx \langle a_T^2\rangle^{1/2}/\omega^2 \text{ for } \omega >> \omega_0. \quad [18]$$

Now consider a seismic signal that peaks at some frequency ω_s and has a quality factor Q_s. The thermal amplitude within the bandwidth sets the typical threshold for the observations

$$\alpha_{sensitivity}(\omega_s) = \frac{\langle a_{Th}^2(\omega_s)\rangle^{1/2}}{\omega_s^2}\cdot\left(\frac{\omega_s}{Q_s}\right)^{1/2} = \left(\frac{kT\omega_0}{\pi I\omega_s^3 QQ_s}\right)^{1/2}. \quad [19]$$

Inserting typical values T = 300 K, $\omega = 10^{-3}$ rad·s^{-1}, $I = 10^4$ g·cm^2, $\omega_s \approx 2.5\times10^{-3}$ rad·s^{-1}, corresponding to the lowest torsional mode, Q = 100 and Q_s = 300, we find $\alpha_{sensitivity} \approx 3\times10^{-9}$ rad. Or, conveniently,

$$\alpha_{sensitivity}(\omega_s) = 1.2\times10^{-8}\left(\frac{10^{-3}}{\omega_s}\right)^{3/2} \text{rad}. \quad [20]$$

For example, the thermally limited sensitivity is $\approx 1.2\times10^{-10}$ rad at a period of $\approx$60 s ($\omega_s \approx 10^{-1}$ rad·s^{-1}), where Igel *et al.* (1) have observed the Tokachi–Oki earthquake. The autocollimating optical lever we have constructed and the rotational seismometer we have presented here have the requisite sensitivity to observe these seismic motions with very high fidelity (see Fig. 3).

Insensitivity of the Instrument to Tilts. Let us consider a small tilt of the ground

$$\delta = \delta_0\cos\omega_h t \quad [21]$$

about the same axis in the horizontal plane. The frequency of the tilt oscillations ω_h is expected to be much smaller than that of the pendular oscillations, ω_p of the torsion balance as a whole, which is

$$\omega_p = \sqrt{\frac{g}{l}} \approx 3 \text{ rad}\cdot s^{-1}. \quad [22]$$

Accordingly, the suspension fiber of the balance will continue to be aligned with the local gravity field. The body of the apparatus would, however, always be aligned with the ground. Because the cross-section of the fiber of the eddy-current damper, which attaches the balance to the vacuum chamber, has a circular cross-section (as described earlier), the bending of the fiber due to tilts does not cause any twists. Thus, we may conclude that the normal to the mirror attached to the pendulum bob will point always in the horizontal plane, i.e., perpendicular to local gravity.

In contrast, the axis of the autocollimator will follow the tilt faithfully. Accordingly, the measured rotation angle will have a change in the absolute normalization by a factor $\cos\delta \approx 1 - \delta^2/2$. Because δ is small, this normalization change may be neglected.

General Remarks and Conclusions

In this paper, we have described an instrument highly sensitive to seismic rotations. The sensitivity of the instrument may be improved dramatically by operating the torsion balance at cryogenic temperatures. Referring to Eq. **19**, two factors will contribute to this improvement: (*i*) decrease in the value of T, which reduces the thermal torques, and (*ii*) substantial increase of the quality factor Q of the fiber for suitably selected materials, which in turn reduces the coupling of the balances to the thermal fluctuations. These two factors will increase the sensitivity by a factor of $\approx10^2$ for the measurement of amplitude of the seismic signal or a factor of $\approx10^4$ in the observation threshold energy of the seismic events. Alternatively, it is straightforward to develop a robust instrument that is less sensitive, and having unit response only above $\approx$10 mHz. Such an instrument would be of great value in the study of seismic rotations in the near-field zone of earthquakes.

Substantial progress has been achieved in the fabrication of the instrument and we should be able to respond positively to the clarion call of Igel and others for new initiatives in the field of "Rotational Seismology." The high-resolution autocollimating optical lever described here briefly, may be adopted for the measurement of tilts as well, thereby improving the sensitivity with respect to conventional tilt meters.

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