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Sporadic acceleration of cosmic rays

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Summary. The spectra of primary and secondary cosmic rays generated by the Fermi-acceleration process is derived analytically, exact to all orders. The observed L/M ratios in galatic cosmic rays constrain the frequency of encounters and the amount of acceleration by this process so severely as to imply that cosmic rays at high energies suffer negligible reacceleration in the interstellar medium.

Key words: acceleration mechanisms – cosmic rays

1. Introduction

In recent years the high efficiency with which shocks can accelerate nuclei to relativistic energies is becoming clear [Axford et al., 1977; Axford, 1981; Bell, 1978; Blandford and Ostriker, 1978; Krimsky et al., 1979; Cowsik and Lee, 1981, 1982; L.O'C Drury, 1982; Toptygin, 1980; Lee and Fisk, 1982; Volk et al., 1979] and specific models for the origin of cosmic rays are being proposed (Blandford and Ostriker, 1980; Silberberg et al., 1983). One has to combine the ideas of acceleration with the models of propagation to derive the spectra of primaries like H, C, O, etc. and those of secondaries like L, Be and B. The exact sources of cosmic rays where such acceleration might be taking place is by no means known but recently it has been argued that it could be the interstellar medium itself where the cosmic rays are stored for ~10 Myr before leaking into the intergalactic space. In this model the high temperature tenuous phase of the interstellar medium is invoked to allow the supernova blast waves to propagate to distances of ~100 pc where they accelerate cosmic rays to a spectral form $\sim E^{-2.2}$ at least up to 10^4–10^5 GeV, and also re-energise the cosmic rays in the medium. One has to assume that the residence time of cosmic rays in the Galaxy decreases with energy as $\sim E^{-0.5}$ to reproduce the spectra (Ormes and Frier, 1978; Orth et al., 1978; Cowsik et al., 1967) and the ratios secondary to primary cosmic rays. Even though our views on this are quite different (Cowsik and Lee, 1979, 1981, 1982; Cowsik and Wilson, 1973) it is this model on which we focus our attention here.

When in a single encounter with a shock the energy of the cosmic ray increases by a large factor then we cannot treat the transport under the diffusion-approximation as it is done for the cannonical Fermi-process (Cowsik 1979, 1980), but we must take into account the higher order terms in the expansion of the collision integral. The fluctuations in the number of encounters become critically important in determining the spectra of primaries and secondaries, and we study the effect of these fluctuations which has received very little attention in the earlier models of cosmic ray transport (Eichler, 1980). In order that the model be mathematically simple we neglect the wide distribution of energies expected after an encounter with a shock. The rather severe constraint on acceleration parameter derived in this paper justifies this assumption, a posteriori.

2. Steady state spectra with sporadic acceleration

In an encounter with a shock we assume that the energy of the particle E is multiplied by a factor α. The probability of such an encounter per unit time is given by A, the probability of leakage from the volume is represented by $B(E)$ and s represents the spallation rate. If the injection with the spectral form $I(E)$ started at time $t=0$, then the spectrum of particles at time t which have suffered exclusively n encounters is given by

$$\mathcal{M}(n,E,t)=\int_0^\infty\int_0^t \mathcal{M}(n-1,E,t)A\delta(E-\alpha E)\exp\left[-\{A+B(E)+s\}(t-t')\right]dt'\,dE' \tag{1}$$

with

$$\mathcal{M}(O,E,t)=\int_0^t I(E,t')\exp\left[-\{A+s+B(E)\}(t-t')\right]dt' \tag{2}$$

Assuming that the cosmic rays are in steady state the spectrum is given by

$$M(E)=\lim_{t\to\infty}\sum_{n=0}^{\infty}\mathcal{M}(n,E,t) \tag{3}$$

It is straight-forward to derive the result

$$M(E)=\sum_{n=0}^{\infty} I(E/\alpha^n)\frac{\alpha}{A}\cdot\prod_{k=0}^{n}\left[\frac{A}{\alpha[A+s+B(E/\alpha^k)]}\right] \tag{4}$$

where we have assumed that the injection of cosmic rays is sensibly constant. The spectra of secondaries $L(E)$ is obtained by simply substituting $sM(E)$ for $I(E)$ in the above expression. Thus with obvious notation

$$L(E)=\sum_{n=0}^{\infty} sM(E/\alpha^n)\frac{\alpha}{A}P_n(E) \tag{5}$$

Even here it is to be noted that Eqs. (4), (5) constitute exact solutions to the Fermi-problem without any approximation of the effect of the collisions simply as convective and diffusive transport in energy space. Also, it is easily generalised when A, B and s are different for primaries and secondaries.

Now the various cosmic-ray models can be investigated using Eqs. 4, 5 by specifying I, α, A and B. For example, with $I = I_0 E^{-\beta}$, and $B = B_0$ independent of energy the sum in Eq. 4 converges for

$$\frac{A\alpha^{\beta-1}}{(A+B_0+s)} < 1 \tag{6}$$

In applying the formalism developed here we make use of the excellent review of cosmic-ray phenomena and astrophysical parameters relevant to acceleration by interstellar shocks given by Blandford and Ostriker (1980). An estimate of the probability of encounter, A is obtained by noting that it is bracketed between the two rates, one calculated assuming free streaming and the other assuming rapid pitch angle scattering:

$$\left(\frac{\xi_{SN} R_s}{V_G v_{SN}}\right) \pi R_s^2 c \geq A \geq \frac{4\pi}{3} R_s^3 \xi_{SN}/V_G\,;\; 10^{-6}\,\mathrm{yr}^{-1} \geq A \geq 10^{-7}\,\mathrm{yr}^{-1} \tag{7}$$

where c = velocity of a cosmic ray particle, $\xi_{SN}\, 10^{-2}\,\mathrm{yr}^{-1}$ the supernova rate in the galaxy, $V_G \approx 3.10^{67}\,\mathrm{cm}^3$ galactic volume in which the cosmic rays are stored and v_{SN} the mean speed with which the blast wave propagates in the hot phase of the interstellar medium and $R_s \approx 3\,10^{20}\,\mathrm{cm}$, the radius of the shock at which it is most effective in energising cosmic rays. The value of α is somewhere in the range 1–10, representing the range of no acceleration to rather intense acceleration.

It is convenient to choose

$$\begin{aligned} I &= I_0/(E_0+E)^{\beta-\delta} \\ B &= B_0 E^{\delta} + h \\ E_0 &= 1.0\,\mathrm{GeV} \\ h &= 10^{-7}\,\mathrm{yr}^{-1} \\ \beta &\approx 2.7 \\ \delta &= 0.5\,. \end{aligned} \tag{8}$$

Note that h represents the minimum rate of leakage of cosmic rays from the Galaxy at low energies. With such a choice the observed spectra of primaries with index β, the decrease of L/M with energy, and the convergence of I and $B^{-1} = \tau_{CR}$ at low energies is ensured in the absence of acceleration by shocks. The energy E is expressed in GeV/nucleon hereafter.

The nature of the product $P_n(E)$ for various values of $(\alpha, A/B_0)$ expresses the relative probabilities of the occurrence of 'n' collisions leading to a final energy E. The function $P_n(E)$ decreases exponentially with n, with the exponent increasing with increasing energy and decreasing acceleration factor. In other words, with increasing energy the rate of escape increases and consequently the average number of encounters decreases, leading to much smaller effects of reacceleration. With increasing acceleration parameter the decrease in the average number of encounters at high energies becomes more gentle. Also for the range of accelerations considered the contribution of multiple collisions tends to zero at very high energies, so that in the absence of a strong divergence of I at small E, the spectrum $M(E)$ asymptotically tends to the power-law $E^{-\beta}$.

In Fig. 1 the $L(E)/M(E)$ ratio is shown for the two values of α 1.26, 1.6 and for $A/B_0 = 1$, 3 and 6 respectively. In the same figure we also show the ratio of (boron/carbon) as measured in cosmic rays (Engelmann et al., 1981; Bouffard et al., 1982; Ormes and Protheroe, 1983]. For approximate results corresponding to different choices for the interstellar gas density the theoretical curves can be scaled up or down. We can draw several interesting conclusions:

(a) All curve tends towards $E^{-\delta}$ at high energies as expected.

(b) As the acceleration increases the L/M ratios progressively increase in magnitude (Curve 1 is essentially identical to the one with no acceleration).

(c) Also, as the acceleration increases the shape of the theoretical curves of L/M becomes flat or even increases with energy before dropping towards the curve $\sim E^{-\delta}$ at high energies. The shapes of the curves calculated for acceleration rates exceeding $\alpha = 1.6$ and $A/B_0 = 3$ do not fit the observations even with rescaling.

In the same Fig. 1, we also show the L/M expected when the acceleration process is as described by second order Fermi-theory stochastic but continuous (Cowsik, 1979; Fransson and Epstein, 1980). The injection for this is taken to be $\delta(E - E_i)$ and $B = B_0$, a constant, not dependent on energy. The expected dependence is logarithmic. Thus it appears that any substantial reacceleration is ruled out in the interstellar medium. At low energies below 1 GeV/nucleon the typical energy gain can be as much as $(1.6)^3 \sim 4$ which is just enough to accommodate some recently discovered anomalies in the composition of cosmic rays at low energies (Silberberg et al., 1983). At energies $\gtrsim 10$ GeV/nucleon the product $P_n(E)$ is so small that the typical increase in energies is less than 30% and the reacceleration becomes rapidly negligible at higher energies. We have also studied other forms of the injection function such as Gaussian $\sim \exp -(E - E_0)^2$, for which the L/M ratio increases even more pathologically with energy.

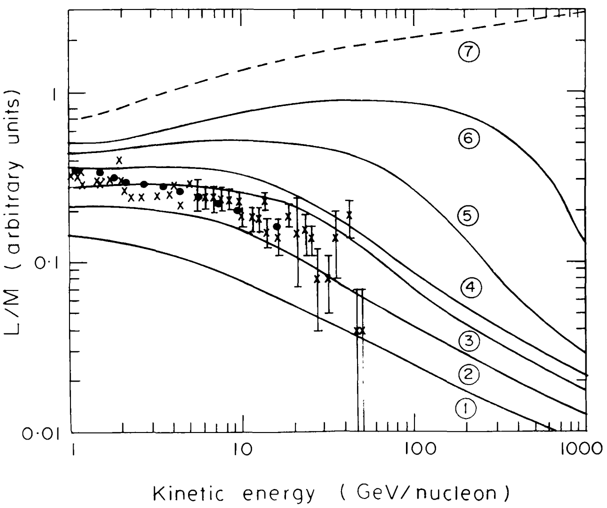

Fig. 1. The theoretical secondary to primary ratios for $(\alpha, A/B_0) = (1.26, 1)$, $(1.26, 3)$, $(1.26, 6)$, $(1.6, 1)$, $(1.6, 3)$ and $(1.6, 6)$ are labelled 1–6 and the standard Fermi process is labelled 7. The experimental data on B/C ratios are adopted from a compilation of cosmic-ray data by Engelman et al (filled circles) and by Ormes and Protheroe (crosses)

3. Discussion

At this point it is worthwhile to compare and contrast the results of our calculations with those of other workers. Eichler (1980) has an explicit solution for the spectra of primaries and secondaries when the scattering process instantaneously generates a power law $E^{-\beta+\delta}$. It is straight forward to show that the (L/M) dependence in his model is

$$(L/M)_{\text{Eichler}} \sim \left\{A + B\left(\frac{E}{E_t}\right)^{-\delta}\right\}^{-1} + \text{constant} \tag{9}$$

Thus for energies much greater than the threshold energy E_t the ratio becomes a constant. Lerche and Schlickeiser (1985) make an important point that the logarithmic behaviour of (L/M) derived by Cowsik (1980) is specific to the assumption that the leakage probability of primaries, B_p (say), is identical to that of secondaries, B_s. This is indeed true and following the prescription of Cowsik (1980), the (L/M) ratio with Fermi-acceleration for $B_s > B_p$, both independent of energy, is given by

$$(L/M)_{\text{Fermi-}B_s>B_p} \sim 1 + \frac{\zeta_s + \zeta_p}{\zeta_s - \zeta_p}\left\{1 - \left(\frac{E_t}{E}\right)^{\zeta_s - \zeta_p}\right\} \tag{10}$$

Here $\zeta_{p,s} = \left[\frac{1}{k}\left\{\frac{(k+a)^2}{4k} + B_{p,s}\right\}\right]^{1/2}$ with a and k being the first and second order Fermi-coefficients. Note that the (L/M) ratio increases monotonically from its value at the threshold and asymptotically becomes constant. But, typical values of $\zeta_p = 3.2$, $\zeta_s = 3.5$ are obtained for $B_s = 1.3B_p$, so that the high energy limit of (L/M) is 22 times the value at $E = E_t$. The high energy limit is reached to within 25% only at energies of $\sim 100E_t = 10$ GeV/nucleon. Thus neither the results of the Fermi theory with $B_s > B_p$, as given in Eq. 10, nor the dependence shown in Eq. 9, when the redistribution function is a power-law (Eichler, 1980) fits the observed dependence in cosmic rays shown in Fig. 1. Of course, allowing B_p and B_s to increase with E will change these results. But the important point to note is that the models are restricted by the requirement that the spectrum of primaries be a power-law of the form $\sim E^{-2.7}$. But Lerche and Schlickeiser (1985) solve directly for the ratio (L/M) without explicitly deriving either of their spectra. In general, when the leakage probability increases with energy, Fermi-theory predicts spectra which are cut-off exponentially. Thus, even though one has to await further work assessing the full import of their suggestion to cosmic rays in the galaxy, it seems safe to conclude that, as yet, no viable theories allowing for substantial reacceleration either stochastically or sporadically in the interstellar medium have been constructed.

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