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Transport of neutrinos, radiation and energetic particles in accretion flows

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The equations describing the transport of suprathermal charged particles, electromagnetic radiation and neutrinos across accretion flows onto compact objects are solved analytically, the effects of shocks in the flow being included. These solutions are used in discussing three illustrative astrophysical examples: acceleration of cosmic rays, generation of spectral continua in quasars and the effect of neutrinos during the collapse of supernova precursors. The main results are:

(*a*) Accretion flows with shocks accelerate cosmic rays very efficiently up to the highest energies.

(*b*) The emergent spectra of electromagnetic radiation from such flows reproduce the observed spectra of quasars from infrared to the hard X-ray region.

(*c*) The neutrinos in the collapsing cores of red giants develop a very hard non-thermal tail in their distribution facilitating the rebound of the gravitational collapse leading to the supernovae.

1. Introduction

Most of the powerful astronomical objects discovered over the past two decades appear to be energized by the accretion of matter on to condensed massive bodies residing at their centres. Quasars, X-ray binaries, cores of double radio sources showing relativistic jets and other such interesting phenomena are some examples for which an important part of the gravitational energy associated with the inflow of matter is radiated away in the form of non-thermal particles and quanta. Pulsars, which are believed to be neutron stars and derive their energy from rapid rotation, seem to be the only exception. But, even here, it is the gravitational energy, which is made available during the collapse of the core of the red-giant progenitor, that is converted to kinetic energy of rotation owing to the conservation of angular momentum. Indeed, it is also the gravitational energy of the collapse of the core into a neutron star that is responsible for the supernova explosion when the surrounding stellar mantle is thrown out by the energetic neutrinos emanating from the central core.

In this paper, we present a study of the transport of photons, neutrinos and energetic particles (hereafter referred to jointly as particles) emitted in the central regions of these objects across the inflow of matter. We formulate the transport equations in the diffusion approximation and derive several analytical solutions relevant to the astrophysical context. The classes of solutions derived in this paper exhibit very many fascinating features that are well suited to building models for quasars, supernova explosions and the sources of high energy cosmic rays. Our main finding is that particles are accelerated very effectively to high energies and achieve spectra with power-law behaviour. The gravitational energy of the systematic inflow of matter is transferred very efficiently to high energy particles. This process might then provide the basis for understanding the non-thermal phenomena associated with accreting white dwarfs, neutron stars and black holes and for making neutrinos emitted by the collapsing cores of red giants more energetic and consequently more effective in blowing off the stellar mantle in supernovae.

Qualitatively there are two processes that are operative in energizing the particles in accretion flows. First, the divergence of the velocity field of the fluid being accreted is in general negative and any particle that is being convected in such a flow experiences a continuous gain in energy because of adiabatic compression. Second, the surface gravity on the condensed astronomical objects is so high that the flow of the accreted material, be it from a companion star or from a surrounding cloud of gas, becomes supersonic and often passes through a shock transition before finally being absorbed: particles that traverse the shock suffer first-order Fermi acceleration as they are repeatedly scattered back and forth across the large discontinuity in the velocity of the flow on either side of the shock.

That shocks are extremely effective in energizing the charged particles in the astrophysical context has been recognized for a long time (Hoyle 1960; Parker 1963; Schatzmann 1963; Axford 1980; Toptyghin 1980; Fisk & Lee 1980). Clear discussions of how cosmic rays gain energy in multiple encounters between approaching shocks can be found in the works of Parker (1958), Wentzel (1963) and Axford & Reid (1962, 1963), Jokipii (1966), and Levy *et al.* (1976). With the realization that magnetic inhomogeneities exist or could easily be excited in virtually all astrophysical plasmas and that one does not need a second shock nor any special geometry for the multiple scattering of charged particles at the shock, many papers have appeared clarifying the subtleties of cosmic-ray and solar-particle acceleration, notably Fisk (1971), Scholer & Morfill (1975), Axford *et al.* (1977), Bell (1978), Blandford & Ostriker (1978), Eichler (1980 personal communication) and Decker *et al.* (1980). Invariably all these papers discuss the transport of charged particles in either expanding or planar shocks. The shocks in accreting flows that are discussed in this paper exhibit some special features exclusive to this geometry. The divergence of the flow is always negative, leading to continuous adiabatic compression; also, the particles downstream of the shock find no path other than diffusion across the convective inflow before escaping to large distances. Both these features make the process of acceleration very effective and it drains very effectively the energy made available by the gravitational infall of matter. Further, the converging flow concentrates matter to high densities near the object so that the transport of photons in several of the energetic astrophysical objects and indeed even that of the neutrinos in the collapsing cores of supernova progenitors can very well be approximated by diffusion, and these particles are energized as effectively as the cosmic rays.

Fortunately, extensive studies of the accretion phenomena in the energetic astrophysical objects are available: studies directed at building specific models for these objects include Zeldovich (1964), Davidson (1973), Maraschi *et al.* (1974, 1978*a*, *b*), Shakura (1974), Illyarionov & Sunyaev (1975), Meszaros (1975), Shapiro & Salpeter (1975), Meszaros & Silk (1977), Rees (1978, 1980), McCray (1979). These studies have concentrated on the effects of electromagnetic radiation on the inflow and on the nature of the emergent spectrum. With only a few exceptions, the treatment of radiation is like that of a black body (Zeldovich & Raizer 1966; Pai 1966; Weinberg 1971). There have been extensive discussions as to how the radiation might develop a non-thermal character owing to repeated Thompson scattering, with a high temperature electron gas expected to be present at the shock in the accreting flow (Shapiro *et al.* 1976; Katz 1976; Sunyaev & Titarchuk 1980; Payne 1980). What we study in this paper is the complementary effect where it is the velocity of the bulk flow that energizes the photons, neutrinos and cosmic rays. In view of the fact that the flow is in general highly supersonic and the bulk flow induces a systematic acceleration rather than a stochastic one, the effect under study here could become very important and indeed even dominant under the astrophysical conditions expected to prevail in these objects.

Since the scattering cross sections for the energetic particles, photons and neutrinos are expected to have different dependences on particle energy, our discussion includes suitably parametrized dependences of the diffusion constant on particle momentum. Some of the aspects of the transport of energetic particles, photons and neutrinos (Cowsik & Lee 1979*a*, *b*, 1980*a*, *b*) have been discussed by the authors earlier. Recently, there has been an excellent discussion of the physics of the convection of radiation and behaviour in planar shocks by Blandford & Payne (1980). Finally there have been attempts to study the effects of the inflow on the spectrum of neutrinos in supernovae by using Monte-Carlo techniques (Kazanas 1980).

In §2 of this paper we derive the basic transport equations and in §3 we solve analytically several illustrative examples. We discuss specific applications in §4 and we conclude the paper in §5 by discussing future prospects for this exciting field.

2. Basic transport equation

We consider energetic particle (energetic charged particle, photon, or neutrino) transport in a plasma accretion flow, $V(x,t)$, in which the particles are scattered (via magnetic irregularities, electron collisions, or nuclear interactions, respectively). Let λ be the scattering mean free path and v the particle speed. If the characteristic length-scale and time-scale for variation of the distribution function of the energetic particles are large compared with λ and λ/v, respectively, then the random walk of the scattered particles can be described by spatial diffusion. Under this restriction the distribution function is nearly isotropic in the local plasma rest frame.

We also make the assumption that the energetic particles have no influence on the evolution of the accretion flow; both the flow and the scattering mean free path are assumed given. The energetic particles are then treated as test particles satisfying a linear kinetic equation decoupled from the dynamics of the flow. There exist specialized one-dimensional calculations of coupled systems (Axford *et al.* 1977; Bell 1978; Eichler 1979; Drury & Volk 1980 personal communication; Blandford 1980 personal communication; Kazanas 1980) that indicate that the coupling can be important. Axford *et al.* (1977) have shown that acceleration in a convergent flow can be very efficient in the sense that a large fraction (about 90 %) of the energy can be channelled into accelerating energetic particles with the remainder being expended in plasma heating. We present the test particle formalism here to illustrate energetic particle behaviour in convergent accretion flows in an analytically tractable fashion rather than from a deep conviction that the coupling is weak in most astrophysical applications. Indeed we shall present an illustrative example for neutrinos for which the dependence of λ on energy ensures that the coupling is essential. In any application, it must be substantiated that particle intensity is sufficiently small compared with the energy density associated with the flow and thus that the approximation is justified.

The basic energetic particle transport equation expressing conservation of particles under the restrictions just outlined, and the additional assumption that V^2/v^2 is negligible (implying also relativistic flows), were originally derived by Parker (1965) and Gleeson & Axford (1967) to treat the transport of energetic particles in the solar wind. In terms of the omnidirectional distribution function, $f(p, x, t)$, and the magnitude of the particle momentum, p, that equation takes the form

$$\frac{\partial f}{\partial t}+\nabla\cdot S+\frac{1}{3p^2}\frac{\partial}{\partial p}(V\cdot\nabla p^3 f) = Q, \tag{1a}$$

$$S = -\kappa\cdot\nabla f+Vf-\tfrac{1}{3}Vp^{-2}\frac{\partial}{\partial P}(p^3 f), \tag{1b}$$

where $4\pi p^2 S$ is the differential current density or streaming, Q is a source (injection) term, and κ is the spatial diffusion tensor (for isotropic scattering $\boldsymbol{\kappa} = \kappa\delta_{ij}$ and $\kappa = \frac{1}{3}\lambda v$). The last term on the left-hand side of equation (1a) describes the differential power expended by the background flow on the energetic particles. The first term in expression ($1b$) is the diffusive streaming of a distribution that is isotropic in a frame moving with velocity $\boldsymbol{V}$. Equations ($1a$) and ($1b$) simplify to

$$\frac{\partial f}{\partial t} - \nabla\cdot(\kappa\cdot\nabla f) + \boldsymbol{V}\cdot\nabla f - \tfrac{1}{3}\nabla\cdot\boldsymbol{V}p\frac{\partial f}{\partial p} = Q. \tag{2}$$

The distribution changes as a function of p if the last term on the left-hand side of equation (2) is non-zero: $\nabla\cdot\boldsymbol{V} \neq 0$. This term describes, for example, adiabatic deceleration in the divergent solar wind where $\nabla\cdot\boldsymbol{V} > 0$. A convergent accretion flow leads to adiabatic acceleration.

This term also describes acceleration at the shock transition at the base of the accretion flow due to plasma compression ($\nabla\cdot\boldsymbol{V} < 0$) at the shock. Even though $\nabla\cdot\boldsymbol{V}$ is localized (and singular) at the shock, particles are coupled to the compression by scattering upstream and downstream of the shock front. The effect of the shock on the energetic particles may then be included as boundary conditions at the shock front: (i) the distribution, f, must be continuous in order that the streaming, S, be finite; (ii) integration of equation ($1a$) normal to the shock front should yield continuity of the component of S in that direction unless Q is singular, implying particle injection or loss at the shock front.

Acceleration in a convergent supersonic accretion flow should be particularly effective since acceleration can occur throughout the flow *and* at the shock front. A problem similar, but complementary, to particle acceleration in a divergent flow has been treated by Fisk & Lee (1980) to explain particle enhancements in association with corotating interaction regions in the solar wind. Here acceleration via compression at the shock is counteracted by adiabatic *deceleration* in the divergent solar wind flow.

We consider an idealized spherically symmetric stationary supersonic accretion flow with a shock transition at $r = r_s$ and with $V = -V_0(r/r_s)^{-\alpha}$ and $\kappa_{rr} = \kappa_0(r/r_s)^{\beta} \times (p/p_0)^{\gamma}$ for $r > r_s$. Clearly an accretion flow cannot be rigorously stationary: we assume that the characteristic time-scale for change for the flow pattern is large compared with characteristic times for energetic particle transport. We assume that monoenergetic particles of momentum p_0 are injected continuously at the shock front in order to model injection near the front via, for example for charged particles, pre-acceleration in downstream plasma turbulence. The solution should not be sensitive to the exact spatial location of the injection, which is chosen to be at r_s, for simple incorporation into the shock boundary conditions. For $r > r_s$ the energetic particle distribution then satisfies (cf. equation (2))

$$-\kappa_0\left(\frac{p}{p_0}\right)^{\gamma}\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{r}{r_s}\right)^{\beta}\frac{\partial f}{\partial r}\right] - V_0\left(\frac{r}{r_s}\right)^{-\alpha}\frac{\partial f}{\partial r} + \tfrac{1}{3}V_0\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\frac{r}{r_s}\right)^{-\alpha}\right]p\frac{\partial f}{\partial p} = 0. \tag{3}$$

We let the compression at the shock be described by the parameter σ defined through the relation between the post-shock velocity V and the velocity of infall at the shock V_0: $V(\text{at } r = r_s - \epsilon) = V_0(1-3\sigma^{-1})$ as $\epsilon \to 0$. For a 'strong' shock $\sigma = 4$. The boundary conditions on f at $r = r_s$ may be written as

$$-\kappa_0 \left(\frac{p}{p_0}\right)^{\gamma} \frac{\partial f}{\partial r} + \kappa_-(r_s, p)\frac{\partial f_-}{\partial r} + \frac{V_0}{\sigma} p \frac{\partial f}{\partial p} = N(4\pi p_0 r_s)^{-2}\delta(p-p_0), \tag{4a}$$

$$f(r_s, p) = f_-(r_s, p), \tag{4b}$$

where N is the total number of particles injected per second, and $\kappa_-(r,p)$ and $f_-(r,p)$ are the downstream diffusion coefficient and distribution function, respectively.

3. Three illustrative examples

The variety of problems that can be addressed within the formalism presented in §2 is extensive even within the restriction outlined there. To illustrate as simply and clearly as possible the behaviour of energetic particles in convergent supersonic accretion flows we shall present three calculations. Each illustrates primarily a different one of the three particular species: charged particles, photons and neutrinos. They are not comprehensive. Nevertheless, each isolates and illustrates a different facet of the problem depending primarily on the structure of $\kappa_{rr}(r,p)$, which depends on the scattering of the species under consideration. Taken together the examples create a picture of the variety of energetic particle spectra expected to be produced in supersonic accretion flows by acceleration via compression in the converging flow and shock.

3.1. *Charged particle*

In this case the spatial dependence of κ_{rr} depends on the properties of the magnetic field irregularities and need not be related to the spatial dependence of $V(r)$. We assume that turbulence downstream of the shock ($r < r_s$) is sufficiently strong that $\kappa_-(r,p) \approx 0$. Now we consider the special case $\alpha + \beta = 1$. In terms of the variable x ($\equiv \ln r/r_s$) equations (3) and (4) may be rewritten as

$$-\left(\frac{p}{p_0}\right)^{\gamma}\left[\frac{\partial^2 f}{\partial x^2} + (1+\beta)\frac{\partial f}{\partial x}\right] - \eta\frac{\partial f}{\partial x} + \tfrac{1}{3}\eta(1+\beta)p\frac{\partial f}{\partial p} = 0, \tag{5}$$

$$-\left(\frac{p}{p_0}\right)^{\gamma}\frac{\partial f}{\partial x} + \sigma^{-1}\eta p\frac{\partial f}{\partial p} = r_s N \kappa_0^{-1}(4\pi p_0 r_s)^{-2}\delta(p-p_0) \quad \text{at} \quad x = 0, \tag{6}$$

where $n \equiv r_s V_0/\kappa_0$.

3.1.1. *The case $\gamma = 0$*

We consider first the case in which κ_{rr} is independent of p: $\gamma = 0$. Fourier transforming in the variable t ($\equiv \ln p/p_0$) and requiring that $f \to 0$ as $x \to \infty$, we obtain from equations (5) and (6).

$$f(x,t) = -A\int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\{2\sigma^{-1}\eta i\omega - \xi - [\xi^2 - \tfrac{4}{3}\eta(1+\beta)i\omega]^{\frac{1}{2}}\}^{-1}$$
$$\times \exp[\![-\tfrac{1}{2}x\{\xi + [\xi^2 - \tfrac{4}{3}\eta(1+\beta)i\omega]^{\frac{1}{2}}\}]\!], \tag{7}$$

where $\qquad \xi = 1+\beta+\eta \quad \text{and} \quad A = Nr_\mathrm{s}\,(\pi\kappa_0\,p_0)^{-1}\,(4\pi p_0 r_\mathrm{s})^{-2}.$

Expression (7) exhibits a branch point at

$$\omega_\mathrm{b} - -\tfrac{3}{4}\mathrm{i}\xi^2(1+\beta)^{-1}\,\eta^{-1}$$

and, if $\eta > (1+\beta)\,(\frac{2}{3}\sigma-1) \equiv \eta_\mathrm{c}$, a pole at

$$\omega_\mathrm{p} = -\mathrm{i}\sigma\eta^{-1}\,[\eta+(1+\beta)\,(1-\tfrac{1}{3}\sigma)].$$

Both pole and branch point are in the lower-half ω-plane with $|\omega_\mathrm{b}| \geqslant |\omega_\mathrm{p}|$. Equality holds when $\eta = \eta_\mathrm{c}$. For $p < p_0$, $f = 0$, indicating that particles do not lose energy since the flow is everywhere compressive. For $p > p_0$ and $\eta > \eta_\mathrm{c}$, the pole contribution, which is dominant for large p, yields

$$f_\mathrm{p} = \pi A\sigma\eta^{-2}(\eta-\eta_\mathrm{c})\,\delta^{-1}(r/r_\mathrm{s})^{-\eta\delta}\,(p/p_0)^{-\sigma\delta}, \tag{8}$$

where $\qquad \delta = 1-\eta^{-1}(1+\beta)\,(\frac{1}{3}\sigma-1).$

The branch contribution yields

$$\begin{aligned} f_\mathrm{b} = {} & 2A\exp\left(-\mathrm{i}t\omega_\mathrm{b}-\tfrac{1}{2}x\xi\right) \\ & \times\int_0^\infty \mathrm{d}z\,z\mathrm{e}^{-tz^2}\,[(\eta\sigma^{-1}\,\mathrm{i}\omega_\mathrm{b}+\eta\sigma^{-1}z^2-\tfrac{1}{2}\xi)^2+\phi^2z^2]^{-1} \\ & \qquad \times[\phi z\cos(x\phi z)-(\eta\sigma^{-1}\,\mathrm{i}\omega_\mathrm{b}+\eta\sigma^{-1}z^2-\tfrac{1}{2}\xi)\sin(x\phi z)] \end{aligned} \tag{9}$$

where $\phi^2 = \frac{1}{3}\eta(1+\beta)$. In the special case in which $\eta = \eta_\mathrm{c}$, and the pole and branch point concide, expression (9) can be evaluated exactly and yields

$$f(x,t) = A\pi\sigma\eta_\mathrm{c}^{-1}\,\mathrm{erfc}\,(t^{\frac{1}{2}}\sigma\phi\eta_\mathrm{c}^{-1}+\tfrac{1}{2}t^{-\frac{1}{2}}x\phi), \tag{10}$$

which for large $p \gg p_0$ has the asymptotic behaviour

$$f(x,p) \sim A\pi^{\frac{1}{2}}\,t^{-\frac{1}{2}}\,\phi^{-1}\,(p/p_0)^{-\frac{1}{3}\sigma^2(\frac{2}{3}\sigma-1)^{-1}}. \tag{11}$$

For large $p \gg p_0$ the branch-cut contribution (expression (9)) can be approximated to yield

$$\begin{aligned} f_\mathrm{b} \sim {} & \tfrac{8}{9}A\pi^{\frac{1}{2}}\sigma^2(1+\beta)^2\xi^{-2}\,\phi(\eta-\eta_\mathrm{c})^{-2}\,[1-\tfrac{3}{4}x\sigma^{-1}\xi(1+\beta)^{-1}\,(\eta-\eta_\mathrm{c})] \\ & \qquad \times\mathrm{e}^{-\frac{1}{2}\xi x}\,xt^{-\frac{3}{2}}\,(p/p_0)^{-\frac{3}{4}\xi^2\eta^{-1}(1+\beta)^{-1}} \end{aligned} \tag{12}$$

Expressions (8), (11) and (12) describe for $p \gg p_0$ the spectra resulting from acceleration at the shock in the convergent accretion flow in the three cases $\eta > \eta_\mathrm{c}$, $\eta = \eta_\mathrm{c}$ and $\eta < \eta_\mathrm{c}$ respectively. Clearly the spectra are predominantly power laws: the inverse dependence of expressions (11) and (12) on p through t ($\equiv \ln p/p_0$) is weak. For $\eta < \eta_\mathrm{c}$ (κ_{rr} large) particles spend negligible time near the shock. They are accelerated in the accretion flow, exhibiting a spectral index, Γ, which is seen from expression (12) to be independent of shock compression, σ. In the limit in which shock compression approaches zero ($\sigma^{-1}\to 0$), $\eta_\mathrm{c}\to\infty$, so that this form of Γ holds for all η.

For $\eta > \eta_\mathrm{c}$ (κ_{rr} small) particles are confined primarily near the shock before being convected downstream. Acceleration, and therefore the spectral index, Γ, depend primarily on shock compression. In the limit $\eta \rightarrow \infty$ ($\kappa_{rr} \rightarrow 0$), the shock front appears planar to the particles and we recover $\Gamma = \sigma$, the index obtained in one-dimensional (1-D) calculations of shock acceleration (see for example Axford *et al.* 1977; Blandford & Ostriker 1978). If we had omitted the last term in equation (5) describing acceleration in the accretion flow we would have obtained

$$\Gamma = \sigma[1+(1+\beta)\eta^{-1}],$$

a softer spectrum than obtained in 1-D simply owing to the 3-D geometry. However, the combination of 3-D geometry and acceleration in the convergent accretion flow yields the spectral index of expression (8), which is smaller (harder) than that obtained in 1-D. The index in expression (8), of course, increases (softens) with increasing σ (i.e. decreasing shock compression).

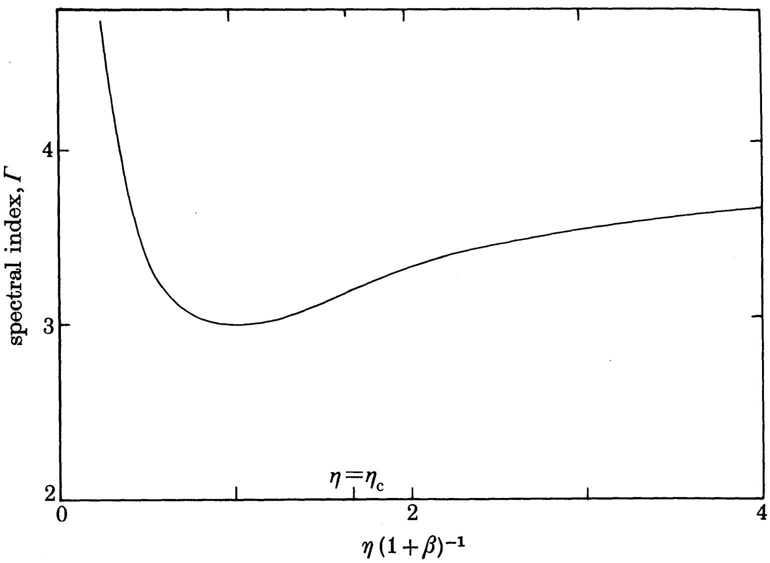

FIGURE 1. Spectral index in phase space for charged particles, Γ, as a function of $\eta(1+\beta)^{-1}$.

In figure 1 we present the power-law spectral index Γ for $p \gg p_0$ with $\sigma = 4$ ('strong' shock) as a function of $\eta(1+\beta)^{-1}$. The hardest spectra occur when the acceleration is dominated by the accretion flow ($\eta < \eta_c$) even for a 'strong' shock. Nevertheless, either the shock or the flow can be dominant and yield very hard non-thermal spectra.

It should be noted that the chosen r-dependence of $V(r)$ and $\kappa_{rr}(r)$, leading to the condition $\alpha+\beta = 1$ used in this section restricts particles from escaping to $r \rightarrow \infty$. All particles are eventually convected downstream towards $r = 0$. For example, the total differential flux across a sphere of radius $r(4\pi r^2 4\pi p^2 s)$ for $p \gg p_0$ and $\eta > \eta_c$ may be calculated from expression (8):

$$(4\pi pr)^2 S = -4\pi V_0 r_s^2 \delta(\tfrac{1}{3}\sigma - 1)\exp\{-\sigma\delta t - x[\eta - \tfrac{1}{3}\sigma(1+\beta)]\}, \tag{13}$$

which is negative and approaches zero as $x \to \infty$. This feature should not affect the interplay illustrated here between acceleration at the shock and in the accretion flow, which determines the energetic particle spectrum. Indeed, insertion of a 'free-escape' boundary at $r = L$ should not substantially affect the spectrum if $\lambda \ll L$. In §3.2 we consider an example that allows the accelerated particles to escape, and the solutions in that case do yield power-law spectra.

3.1.2. *The case* $\gamma > 0$

We now consider the more general case in which κ_{rr} increases with increasing momentum. With $\theta = (p/p_0)^\gamma$ equations (5) and (6) may be rewritten as

$$\frac{\partial^2 f}{\partial x^2} + (1+\beta)\frac{\partial f}{\partial x} + \frac{\eta}{\theta}\frac{\partial f}{\partial x} - \tfrac{1}{3}\eta\gamma(1+\beta)\frac{\partial f}{\partial \theta} = 0, \tag{14}$$

$$\frac{\partial f}{\partial x} - \eta\sigma^{-1}\gamma\frac{\partial f}{\partial \theta} = -r_s N \kappa_0^{-1}(4\pi p_0 r_s)^2 \gamma_0^{-1}\delta(\theta - 1) \quad \text{at} \quad x = 0. \tag{15}$$

The convective term $(\eta\theta^{-1}\partial f/\partial x)$ in equation (14) defies a complete analytical solution to the system (14) and (15), but an asymptotic solution for large θ can be obtained by assuming that f may be expanded in W.K.B. form

$$f = \exp[a(x)\theta + b(x)\ln\theta + c(x) + d(x)\theta^{-1} + e(x)\theta^{-2} + \ldots].$$

At each order in θ, f must be bounded as $x \to \infty$. Substituting the assumed form into equations (14) and (15), and noting that the source term in equation (13) does not contribute at high momenta, yields ordinary differential equations in x for each order (i.e. $\theta^\alpha(\ln\theta)^\beta$). Order θ^2 yields $(a')^2 = 0$ implying $a(x) = a_0$, where a prime indicates differentiation with respect to x. Order $(\ln\theta)^2$ yields $(b')^2 = 0$ implying $b(x) = b_0$.

The coefficient a_0 must be determined separately by solving the system (14) and (15) with the convective term neglected and finding the dominant behaviour (the value of a_0). The solution is similar to that presented in §3.1, with the exception that the integrand has no poles. The dominant behaviour is determined by the location of the branch point: there obtains $a_0 = -\frac{3}{4}(1+\beta)(\eta = \gamma)^{-1}$. Order θ^0 the yields equations

$$\left.\begin{aligned} c'' + c'^2 + (1+\beta)c' + \tfrac{1}{4}(1+\beta)^2 = 0,\\ c'(0) = -\tfrac{3}{4}(1+\beta)\sigma^{-1},\end{aligned}\right\} \tag{16}$$

with solution

$$c = -\tfrac{1}{2}(1+\beta)x + \ln[1 + \tfrac{3}{4}\times(1+\beta)\sigma^{-1}(\tfrac{2}{3}\sigma - 1)] + c_0,$$

where c_0 is an overall multiplicative constant that cannot be determined by asymptotic analysis.

Order θ^{-1} yields an equation and boundary condition with the solution

$$\begin{aligned} d = D + \tfrac{1}{2}(1+\beta)\eta\xi^{-2}(1+\zeta x)^2 - \tfrac{1}{2}\eta x + \tfrac{1}{18}\eta\gamma(1+\alpha\beta)b_0\zeta^{-2}(1+\zeta x)^2 \\ - \zeta^{-1}(1+\zeta x)^{-1}[\eta\sigma^{-1}\gamma b_0 - \tfrac{1}{6}(1+\beta)\eta\zeta^{-1} + \tfrac{1}{2}\eta - \tfrac{1}{9}\eta\gamma(1+\beta)\zeta^{-1}b_0],\end{aligned} \tag{17}$$

where D is a constant and $\zeta = \frac{3}{4}\sigma^{-1}(1+\beta)(\frac{2}{3}\sigma - 1)$. The term in d involving $(1+\zeta x)^2$ gives anomalously sensitive (if convergent) dependence of f on x for large x. To eliminate it we demand that $d_0 = -\frac{3}{2}\gamma^{-1}$ whence

$$d = -\tfrac{1}{2}\eta x - \tfrac{1}{2}\eta\zeta^{-1}(1+\zeta x)^{-1}(1-3\sigma^{-1}) + D.$$

At order θ^{-2} it is no longer possible to eliminate the leading anomalously sensitive dependence of x by judicious choice of constants. We stop the expansion at this level.

Including the first three orders we have

$$f(x,\theta) \approx (1+\zeta x)\,\theta^{-\frac{3}{2}\gamma^{-1}} \exp\left[-\tfrac{3}{4}(1+\beta)(\eta\gamma)^{-1}\theta - \tfrac{1}{2}(1+\beta)x\right]. \tag{18}$$

This method was used by Fisk & Lee (1980) and gave excellent agreement with numerical analysis to the same order considered here.

In contrast with the power law obtained for a diffusion coefficient independent of p in §3.1.1, this spectrum is dominated by an exponential of the form $\exp(-cp^{\gamma})$, where c is constant, and thus exhibits the momentum dependence of κ_{rr}. Because of the increase of κ_{rr} with energy, high energy particles are less likely to encounter and shock, and also favour the outer regions of the accretion flow where compression is small. A softer spectrum results. Exponential spectra are characteristic of 3-D geometries when the diffusion coefficient depends on p (see for example Fisk & Lee 1980). In contrast, simple 1-D shock acceleration yields a power law independent of the momentum dependence of the diffusion coefficient. If energetic nuclei accelerated in accretion flows are the source of cosmic rays, then, with $\kappa = \frac{1}{3}\lambda v$ and $v \approx c$, λ must be independent of particle energy in order to yield the power laws observed up to the highest energies.

3.2. *Photons*

In this case, κ_{rr} is not independent of the accretion flow since its spatial dependence is inversely proportional to electron density, $n_e(r)$. In a stationary flow, $n_e(r)\,V(r)\,r^2$ is constant, thus requiring $\alpha+\beta = 2$. Furthermore, $\kappa_{rr}^{-}(r,p)$ cannot be neglected as in §3.1. Payne & Blandford (1980) have considered radiative transfer in such an accretion flow for the same power-law dependence of V and κ_{rr} on r with κ independent of p, but have neglected the shock transition at the base of the accretion flow. In this section we extend their results to include the shock transition (§3.2.1) and also to consider transport at energies greater than $\frac{1}{2}$ MeV for which κ_{rr} increases with p with an index $\gamma \approx 1$ (§3.2.2). The same calculation may of course apply to energetic charged particles. In contrast with the case considered in the last section, the more rapid increase of κ_{rr} with r accommodated in this case $(\alpha+\beta = 2)$ allows particles to escape to $r \to \infty$.

3.2.1. *The case $\gamma = 0$*

We consider first the case in which κ_{rr} is independent of p, appropriate for photons with energies well below $\frac{1}{2}$ MeV. For illustration and to compare our results with those of Payne & Blandford (1980), we assume that $V(r)$ and $\kappa_{rr}(r)$ exhibit the same power-law radial dependence for $r < r_s$ as for $r > r_s$, but with discontinuous jumps at the shock, $r = r_s$. We note that η must be the same in both $r < r_s$ and $r > r_s$, that $f_-(r,p)$ also satisfies equation (3) for $r < r_s$. With $\alpha+\beta = 2$, $z \equiv \eta r_s r^{-1}$, $t \equiv \ln p/p_0$ and $g = fz^{-(1+\beta)}$, equations (3) and (4) may be rewritten as

$$z\frac{\partial^2 g}{\partial z^2} + (2+\beta-z)\frac{\partial g}{\partial z} - (1+\beta)g - \tfrac{1}{3}\beta\frac{\partial g}{\partial t} = 0, \tag{19}$$

$$\left.\frac{\partial g}{\partial t}\right|_{\eta-} - (1-3\sigma^{-1})\left.\frac{\partial g}{\partial z}\right|_{\eta+} + 3(1+\beta)(\eta\sigma)^{-1}g + \sigma^{-1}\frac{\partial g}{\partial t} = c\delta(t) \quad \text{at} \quad z = \eta, \tag{20}$$

$$g(z = \eta_-) = g(z = \eta_+),$$

where $c \equiv N(\kappa_0^+ p_0)^{-1}\eta^{-(2+\beta)}(4\pi p_0 r_s)^{-2} r_s$. The Fourier transform of g with respect to t satisfies the confluent hypergeometric equation with standard solutions $M(a,b,z)$ and $U(a,b,z)$ (see for example Abramotwitz & Stegun 1970), where $a = 1+\beta-\frac{1}{3}\mathrm{i}\beta\omega$ and $b = 2+\beta$. Using boundary conditions (20) and the condition that $f \to 0$ as $r \to \infty$, we obtain the solution for $r > r_s$ ($z < \eta$) as

$$g = (2\pi)^{-1} c \int_{-\infty}^{\infty} \mathrm{d}\omega\, \mathrm{e}^{-\mathrm{i}\omega t}\, \Gamma(a)\, M(z)\, U(\eta)$$
$$\times [\![\Gamma(b)\,\eta^{-b}\,\mathrm{e}^{\eta} + \Gamma(a)\, M(\eta)\,\sigma^{-1}\, 3U'(\eta) + U(\eta)\,[3\eta^{-1}(1+\beta) - \mathrm{i}\omega]\}]\!]^{-1}, \tag{21}$$

where the indices a and b in M and U have been suppressed, $U' = \mathrm{d}U(a,b,z)/\mathrm{d}z$ and $\Gamma(x)$ is the gamma function. The functions $M(a,b,z)$ and $U(a,b,z)$ are entire functions of ω. As $\sigma \to \infty$, the discontinuities at the shock are reduced to zero. The analytic structure of the integrand is then determined by simple poles of $\Gamma(a)$ at $a = -r$ (integer) or $\mathrm{i}\omega_r = 3\beta^{-1}(1+\beta+\eta)$. These are simply the eigenvalues obtained by Payne & Blandford (1980) for the case in which the shock transition is not considered. For large momentum the spectrum is then dominated by the first eigenvalue: $f = p^{-3\beta^{-1}(1+\beta)}$.

The structure of the integrand is more complicated for finite σ. The integrand is in general analytic at $a = -n$. The only singularities are poles arising from zeros of the denominator in equation (21). From the spectra obtained in §3.3.3, we expect the effect of the shock to be most important for $\eta \gg 1$. In that case we may use asymptotic expansions of the hypergeometric functions to calculate the dominant zero of the denominator, which results in the spectrum

$$f \sim (p/p_0)^{-\sigma[1-2\beta\eta^{-1}(\frac{1}{3}\sigma-1)]}. \tag{22}$$

The spectrum is very similar to that obtained previously in equation (8) for $\eta > \eta_c$ and $\alpha+\beta = 1$. Again, the leading term in the exponent is $-\sigma$, the spectral index obtained for a 1-D shock, but the convergent accretion flow hardens the spectrum further.

It is instructive to compare the spectral index, Γ, in equation (22) with that obtained by Payne & Blandford (1980): $-3\beta^{-1}(1+\beta)$. Payne & Blandford obtain the hardest spectrum for $\beta = 2$ ($\Gamma = \frac{9}{2}$); for a free-fall accretion flow ($\alpha = \frac{1}{2}$), $\beta = \frac{3}{2}$

they obtain $\Gamma = 5$. Inclusion of the shock transition yields a substantially harder spectrum; for a 'strong' shock $\Gamma < 4$.

The total differential flux $\mathscr{F}$ of particles that escape is $(4\pi pr)^2 S|_{r\to\infty}$, which may be evaluated to yield

$$\mathscr{F} = (4\pi)^2 r_s \kappa_0 \eta^{1+\beta}(1+\beta)p^2 f \quad (z = 0, p). \tag{23}$$

The spectrum of escaping particles is modified only by the phase-space factor p^2.

3.2.2. *The case $\gamma \neq 0$*

We now consider the modification of the photon spectrum that occurs by virtue of the increase in $\kappa(p)$ at high frequencies where the scattering cross sections decrease as given by the Klein–Nishina formula ($\gamma \approx 1$). With $y = (p/p_0)^\gamma$ and $x = r/r_s$, equations (3) and (4) may be rewritten for $\alpha + \beta = 2$ as

$$\frac{\partial}{\partial x}\left(x^{2+\beta}\frac{\partial f}{\partial x}\right) + x^\beta \frac{\eta}{y}\frac{\partial f}{\partial x} - \tfrac{1}{3}\eta^\gamma \beta x^{\beta-1}\frac{\partial f}{\partial y} = 0, \tag{24}$$

$$\left.\frac{\partial f}{\partial x}\right|_{x=1^+} - (1-3\sigma^{-1})\left.\frac{\partial f}{\partial x}\right|_{x=1^-} - \eta\sigma^{-1}\gamma\frac{\partial f}{\partial y} = -B\delta(y-1) \quad \text{at} \quad x = 1, \tag{25}$$

where $B \equiv N\gamma(\kappa_0 p_0)^{-1}(4\pi p_0 r_s)^{-2}$. Following the assumption made in the last subsection, we take the accretion flow and diffusion coefficient to exhibit the same spatial dependence in $x < 1$ as in $x > 1$ so that equation (24) must be satisfied in either domain with the same value of η.

We restrict our considerations to the behaviour of the spectrum for large p. We therefore neglect the convective term proportional to y^{-1} in equation (24). Having eliminated inward convection, however, the divergent accretion flow at small r yields artificially divergent spectra since convection no longer inhibits the escape of photons adiabatically accelerated deep in the accretion flow. Accordingly we assume the presence of an absorption boundary at $x = \epsilon < 1$.

Fourier transforming equations (24) and (25) and using the boundary conditions that $f(\epsilon, y) = 0$ and $f(x,y) \to 0$ as $x \to \infty$, we obtain the solution in $r > r_s$ as

$$f(x,y) = (2\pi)^{-1} B x^{-\frac{1}{2}(1+\beta)} \int_{-\infty}^{\infty} d\omega\, e^{-i\omega(y-1)} J_{1+\beta}(\Omega x^{-\frac{1}{2}})\, D^{-1}$$
$$\times [J_{1+\beta}(\Omega)\, J_{-(1+\beta)}(\Omega\epsilon^{-\frac{1}{2}}) - J_{-(1+\beta)}(\Omega)\, J_{1+\beta}(\Omega\epsilon^{-\frac{1}{2}})], \tag{26}$$

$$\begin{aligned} D \equiv\; & -\pi^{-1} J_{1+\beta}(\Omega\epsilon^{-\frac{1}{2}}) \sin(1+\beta)\pi \\ & + 3\sigma^{-1} J_{1+\beta}(\Omega)\,\{\tfrac{1}{2}\Omega[J'_{1+\beta}(\Omega)\, J_{-(1+\beta)}(\Omega\epsilon^{-\frac{1}{2}}) - J'_{-(1+\beta)}(\Omega)\, J_{1+\beta}(\Omega^{-\frac{1}{2}})] \\ & + \tfrac{1}{2}[(1+\beta) - \tfrac{1}{3}\eta i\omega\gamma]\,[J_{1+\beta}(\Omega)\, J_{-(1+\beta)}(\Omega\epsilon^{-\frac{1}{2}}) - J_{-(1+\beta)}(\Omega)\, J_{1+\beta}(\Omega\epsilon^{-\frac{1}{2}})]\}, \end{aligned}$$

where $\Omega^2 = \frac{4}{3}\eta\gamma\beta i\omega$ and $J_\beta(\Omega)$ is the standard Bessel function of index β. The integrand is analytic at $\omega = 0$; singularities arise only from zeros of D. The dominant behaviour for large y arises from the uppermost zero in ω-space, which gives

$$f(x,y) \sim x^{-\frac{1}{2}(1+\beta)} J_{1+\beta}(x^{-\frac{1}{2}}\Omega_0) \exp[-\tfrac{3}{4}\Omega_0^2(\eta\gamma\beta)^{-1} y], \tag{27}$$

where $\Omega_0 > 0$ and is evaluated at the uppermost zero $\omega = \omega_0$, which must lie on the negative imaginary axis.

In the absence of a shock transition ($\sigma \rightarrow \infty$), $D \propto J_{1+\beta}(\Omega\epsilon^{-\frac{1}{2}})$ and the spectrum is determined entirely by adiabatic compression in the accretion flow. The first zero of D then occurs at $\Omega_0 = \epsilon^{\frac{1}{2}}\xi_{1+\beta}$, where $\xi_{1+\beta}$ is the first zero of $J_{1+\beta}(\xi)$. We note that the spectrum in equation (28) then becomes flat as $\epsilon \rightarrow 0$, an artificiality introduced by the divergence of the accretion flow as $r \rightarrow 0$ and by the neglect of the accompanying inward convection. For $\epsilon^{\frac{1}{2}} \lesssim 1$, however, we obtain the spectrum resulting from acceleration in the flow at radii greater than that of injection. If $\beta = \frac{3}{2}$, for example, $\Omega < 5.8$.

In general, if $\epsilon = 1$ the numerator in expression (26) is zero, indicating that injected particles are immediately absorbed. As $\epsilon \rightarrow 1$ D is well behaved, however, so the effect of the shock transition on the spectrum can be assessed when $\epsilon < 1$. For $\epsilon = 1$,

$$D = -\pi^{-1}\sin(1+\beta)\pi(1-3\sigma^{-1})J_{1+\beta}(\Omega)$$

indicating that the shock has no effect on the spectrum in the limit $\epsilon \rightarrow 1$, an obvious result since there can be no acceleration via compression across the shock if absorption occurs immediately downstream. Expanding equation (27) in powers of $1-\epsilon$, the uppermost zero of D, gives

$$\Omega_0 \approx \xi_{1+\beta}[1-\tfrac{1}{2}(1-\epsilon)(1-3\sigma^{-1})^{-1}],$$

indicating a harder spectrum with decreasing σ (stronger shock). If $\beta = \frac{3}{2}$ and $\epsilon = \frac{3}{4}$ for example, $\Omega_0^2 \approx 25$ if $\sigma \rightarrow \infty$, and $\Omega_0^2 \approx 8$ if $\sigma = 4$.

The total differential flux of particles that escape, $[(4\pi rp)^2 S]_{r\rightarrow\infty}$, is finite and has the same dominant behaviour as the distribution function.

As expected, more rapid escape of higher energy particles from the flow due to the increase of κ_{rr} with p results in an exponential spectrum. This contrasts with the power law obtained for κ_{rr} independent of p. The e-folding value of y $(=(p/p_0)^\gamma)$ increases, as expected, with V_0/κ_0: larger V_0 and smaller κ_0 keep particles confined to the flow near the shock.

3.3. *Neutrinos*

We finally consider an example that illustrates the transport of neutrinos in spherically symmetric accretion flows. The new feature here is a cross section of nuclear scattering of neutrinos that increases with momentum as p^2, yielding a diffusion coefficient that decreases as p^{-2}. The asymptotic analysis used in the previous two examples is ineffective since neglecting terms in the transport equation with inverse dependence on p eliminates diffusion. All particles would convect into the accreting object with no acceleration at, or upstream of, the shock.

We might expect that neutrinos exhibit singular behaviour. As they are accelerated their scattering mean free path decreases making it less likely that they escape and more likely that they remain confined near the shock where the convergence of the flow is greater and the probability of shock traversals is higher, both of which lead to an increased acceleration rate. To illustrate this singular feature we consider the conservative case in which $V = V_0(r/r_s)^{-2}$ for which the divergence of the accretion flow is identically zero and acceleration occurs only via compression at the shock. In order that the neutrinos are not simply convected downstream, artificially releasing the singular increase in neutrino energy density, we incorporate the deceleration and convergence of the accretion flow downstream of the shock into the shock boundary condition by assuming that the flow is decelerated to zero at an effective 'shock front' with no flow within.

As in the previous two examples, monoenergetic particles of momentum p_0 are injected at the shock continuously and diffuse according to $\kappa_{rr} = \kappa_0(p/p_0)^\gamma (r/r_s)^\beta$. The solution of equation (3) for $r < r_s$ is a function $f(r,p)$ that is independent of r. For $r > r_s$ the solution of equation (3) and boundary condition (4) that vanishes as $r \to \infty$ is

$$f(r,p) = 3N(V_0 p_0)^{-1}(4\pi p_0 r_s)^{-2}[\exp(\zeta y^{-1}) - 1]^{-1}[\exp(\zeta y^{-1} x^{-(1+\beta)}) - 1] \times \exp\left\{-3\int_{p_0}^{p} \mathrm{d}p' p'^{-1}[1-\exp(\zeta y'^{-1})]^{-1}\right\}, \tag{28}$$

where $\zeta = \eta(1+\beta)^{-1}$, $\eta = V_0 r_s \kappa_0^{-1}$, $y \equiv (p/p_0)^\gamma$, $x = r/r_s$ and N is the particle injection rate. The total differential flux, $(4\pi pr)^2 S$, as $r \to \infty$ is then

$$(4\pi pr)^2 S = 3N(r_s p_0)^{-1}(p/p_0)^2[\exp(\zeta t^{-1}) - 1]^{-1} \times \exp\left\{-3\int_{p_0}^{p} \mathrm{d}p' p'^{-1}[1-\exp(-\zeta y'^{-1})]^{-1}\right\}. \tag{29}$$

For $r < r_s$ the distribution function is given by the final exponential factor in equation (28), which exhibits the following asymptotic behaviour for large p:

$$f(r < r_s, p) \sim \exp\left[-\frac{3(1+\beta)}{\gamma r_s V_0}\kappa_0(p/p_0)^\gamma\right], \quad \gamma > 0, \tag{30a}$$

$$f(r < r_s, p) \sim (p/p_0)^{-3}, \quad \gamma < 0, \tag{30b}$$

$$f(r < r_s, p) \sim (p/p_0)^{-3[1-\exp(-\zeta)]^{-1}}, \quad \gamma = 0. \tag{30c}$$

Equation (29) exhibits the following asymptotic behaviour:

$$(4\pi rp)^2 S \sim (p/p_0)^\gamma \exp\left[-\frac{3(1+\beta)}{\gamma r_s V_0}\kappa_0(p/p_0)^\gamma\right], \quad \gamma > 0 \tag{31a}$$

$$(4\pi rp)^2 S \sim (p/p_0)^{-3}\exp\left[-\frac{r_s V_0}{(1+\beta)\kappa_0}(p/p_0)^{|\gamma|}\right], \quad \gamma < 0, \tag{31b}$$

$$(4\pi rp)^2 S \sim (p/p_0)^{-3}[1-\exp(-\zeta)]^{-1}, \quad \gamma = 0. \tag{31c}$$

The behaviour of a charged particle distribution ($\gamma > 0$) under the assumptions outlined is straightforward. If $\gamma > 0$, higher energy particles are more likely to escape than those at lower energies, resulting in a soft exponential spectrum of the form obtained in the previous two illustrative cases, §§3.1.1 and 3.2.1. The special case, $\gamma = 0$, yields power law spectra as obtained in §3.1.1, with an index that increases as κ increases, indicating that acceleration is more efficient if particles are confined near the shock.

The case $\gamma < 0$, appropriate for neutrinos if $\gamma = -2$, however, is dramatically different, and indeed singular. The distribution function for $r < r_s$ is extremely hard, indeed divergent in both particle number and energy densities. The spectrum of escaping particles, however, is exponential. The high energy particles simply cannot escape the accretion flow, and continually accelerate and accumulate within the accreting object. The calculation breaks down, of course. A divergent neutrino number and energy density repudiate the test particle formalism and assumed time independence. A build-up in neutrino energy density would disrupt the accretion flow. Nevertheless, the lack of a stationary non-divergent spectrum is indicative of possible catastrophic behaviour. A more general dependence of V on r would only exacerbate the divergence owing to added acceleration in the accretion flow.

4. Some applications to astrophysical problems

As already mentioned in the Introduction, the primary source of energy that powers the exciting new objects discovered during the last two decades in several branches of modern astronomy is accretion. It is not the purpose of this section to develope detailed models for these objects, quasars, γ-ray sources, etc. but rather to indicate qualitatively the applicability of the discussions of the last two sections to these objects and show that several essential observed features of these objects could be understood in terms of the ideas developed so far. We start this section by reviewing briefly the topic of accretion by compact objects and we refer readers to several detailed papers that have appeared on the subject for a fuller discussion. We then proceed to discuss specific applications to the acceleration of charged particles in these objects, including special applications to the galactic cosmic rays. This is followed by a discussion of the spectra of the continuum in quasars. We end by showing how the neutrino transport theory in the shocked flow prior to the supernova outburst might help in ejecting the debris and leaving an object of mass *ca.* $M_\odot$ behind as a neutron star. In this section the reader should bear in mind that our purpose is neither to be exhaustive as to the coverage of the earlier work on these topics, nor to explain in fine detail every observation, but to indicate a new avenue of thought whose significance is broader than the specific details of any particular model.

We consider four idealized situations as examples for further discussion and quote order-of-magnitude numerical results. A neutron star of $r = 10^6$ cm and mass $M \approx M_\odot$ is placed inside a typical interstellar cloud of molecular hydrogen with number density in the range 10^3–10^6 atoms cm^{-3}. Though the kinetic temperatures in the cloud could be as low as *ca.* 100 K, temperatures ten times as large could be produced by turbulence etc. We also consider white dwarfs with $r \approx 10^{8.5}$ cm and $M \approx M_\odot$ inside similar clouds. As a model for quasars and nuclei of Seyfert galaxies one may consider a supermassive black hole of mass *ca.* $10^9\,M_\odot$. The surrounding medium may have densities in the range 1–1000 atoms cm^{-3} and temperatures in the range 10^3–10^4 K. Finally we consider the core of a red giant undergoing gravitational collapse. The core mass is typically in the range 1–2 $M_\odot$ and during the collapse densities in excess of $10^{11}\,g\,cm^{-3}$ are achieved. The hydrodynamic flow of material here is much more complicated than the simple flow discussed in this paper, yet the qualitative behaviour derived in § 3.4 should indicate the general aspects of the neutrino transport in the collapse.

4.1. *Qualitative features of accretion on to compact objects*

Though the early studies of accretion by Bondi (1952), Mestel (1954) and others were directed towards finding possible changes in the luminosity of ordinary stars, it is very easy to extend their pioneering work to more compact objects like white dwarfs, neutron stars and black holes. Excellent and exhaustive reviews by Lightman *et al.* (1979) and Sunyaev (1979) provide good coverage of the field showing how the earlier considerations were extended in an attempt to understand the X-ray binaries, quasars, etc.

The gravitational potential $U \sim GM/rc^2$ indicates roughly the energy that can be released by infalling matter, in units of its rest energy mc^2. This ranges from $U \sim 10^{-3}$ for a white dwarf to $U \sim 10^{-1}$ for neutron stars and a similar effective value for black holes even though the theoretical limit is *ca.* 0.5. When one considers the simple case of accretion in which the gravitating body is submerged in a gas of temperature T, we can define a characteristic length, called the accretion radius r_a, signifying the distance at which the escape velocity becomes equal to the speed of sound in the gas:

$$r_a \approx \frac{GM}{C_S^2} \sim 5\times 10^{13}\frac{M}{M_\odot}\left(\frac{T}{10^4\,\mathrm{K}}\right)^{-1}\mathrm{cm}. \tag{32}$$

The significance of r_a is that beyond this distance the surrounding gas is not seriously affected by the presence of the massive body; following the review of Lightman *et al.* we can write

$$\left.\begin{aligned} n = n_\infty,\quad & V \sim r^{-2}, \quad r \gg r_a, \\ & V \sim V_\infty, \quad r \approx r_a, \\ n \sim r^{-\frac{3}{2}},\quad & V \sim r^{-\frac{1}{2}}, \quad r \ll r_a, \quad \text{for matter in free fall.} \end{aligned}\right\} \tag{33}$$

The behaviour described in equation (33) is only qualitative and the actual flow pattern particularly at $r \ll r_a$ will depend critically on several details of the problem such as the exact ratios of the specific heats of the gas and the degree to which the momentum of the infall is counteracted by gradients in the pressure, especially due to the radiation and energetic particles in the system. From the discussion in § 3 it is amply clear that radiation is energized very effectively by the inflow, and the pressure due to the radiation can, in the limit, become equal to the momentum flux due to the accreted gas. Under such circumstances V will increase much less rapidly than $r^{-\frac{1}{2}}$ with decreasing radius. Though a self-consistent treatment may be necessary we might hazard a guess that the parameter α defined in the §3 through the relation $V = V_0(r/r_s)^{-\alpha}$ will be smaller than 0.5 and may be about 0.45. When copious amounts of energetic charged particles are present α could become much smaller at short distances.

Since the mean free path of radiation increases rapidly as we move away from the star the typical state of accretion may be expected to be only weakly sensitive to the behaviour at short distances, and the classical formula given below may be applicable:

$$\dot{M} = 2\times 10^{10}\left(\frac{M}{M_\odot}\right)^2\left(\frac{T}{10^4\,\mathrm{K}}\right)^{-\frac{3}{2}} n_\infty\,\mathrm{g\,s^{-1}}. \tag{34}$$

When the massive object and the surrounding gas are in relative motion, kT is replaced by the effective kinetic energy per particle as seen from the object. Combining the expression (34) for the rate of accretion with the expression for the gravitational potential we can write the power released as

$$\dot{E} = \frac{GM\dot{M}}{R} \sim \frac{4\times 10^{36}}{r_0}\left(\frac{M}{M_\odot}\right)^3\left(\frac{T}{10^4\,\mathrm{K}}\right)^{-\frac{3}{2}} n_\infty\,\mathrm{erg\,s^{-1}},\dagger \tag{35}$$

where r_0 is the 'size' of the object. For example a neutron star of radius *ca* 10^6 cm, when placed at the centre of a typical molecular cloud in the interstellar medium, with $n_\infty \sim 10^3$ and $T \sim 10^2$ K, can radiate *ca.* $4\times 10^{36}\,\mathrm{erg\,s^{-1}}$.

When the luminosity of the object in electromagnetic radiation becomes very large, the radiation pressure can balance the gravitational attraction and can in principle switch off a spherically symmetric accretion. This limiting luminosity is called the Eddington luminosity and is given by

$$L_{\mathrm{Edd}} = \frac{4\pi GM\,m_p c}{\sigma_T} \sim 1.3\times 10^{38}\left(\frac{M}{M_\odot}\right)\mathrm{erg\,s^{-1}}. \tag{36}$$

The Eddington limit is to be considered as qualitative and there are a variety of ways in which this limit can be exceeded (see for example the review of Lightman *et al.* (1979)).

When the γ of the infalling gas becomes smaller than $\frac{5}{3}$ the flow has a tendency to become supersonic and the flow passes through a shock transition before finally being absorbed by the central object. Depending on the specific conditions that obtain at the shock one may expect different processes like the pre-acceleration of charged particles by turbulent magnetized plasma or emission of radiation through bremsstrahlung in the hot plasma to take place.

4.2. *Acceleration of charged particles*

There exists ample evidence that relativistic charged particles are copiously produced in quasars and in the nuclei of Seyfert galaxies. Indeed our discussion in the previous sections implies that the conditions that obtain in accretion flows

† erg = 10^{-7} J = 10^{-1} μJ.

around compact objects are highly conducive to the acceleration of the particles. Though § 3.2 referred to the transport of photons, the solution given is as applicable as that discussed in § 3.1 for the acceleration of particles. These two cases with $\alpha+\beta = 2$ and $\alpha+\beta = 1$ span a large region of the relevant parameter space for the variation of the flow velocity and the diffusion coefficient with radial distance and indicate that invariably particles are accelerated to spectra having rather flat indices. The reason for this is that the time in which the energy of any particle increases by a factor e is extremely small. Noting that the energy of a particle increases by a factor of $[V_0 - V(r_s - \epsilon)]/c$ every time it crosses the shock and that the typical time for crossing is λ/c where λ is the scattering mean free path, we can write

$$t_c = \lambda[V_0 - V(r_s - \epsilon)]^{-1} = \frac{\sigma}{c}\frac{\kappa_0}{V_0}. \tag{37}$$

This time is quite small under most conditions discussed here as the shock occurs in the relatively high density turbulent plasma close to the condensed object where the velocity of flow is large In fact the net acceleration time is so short that one may expect even electrons to be accelerated efficiently without substantial radiation losses up to rather high energies Assuming the conservation of magnetic flux in the accretion flow we can write

$$\tau_{\text{sync}} \sim \tau_0[B_\infty^2 (r_a/r_s)^{\frac{4}{3}(2+\alpha)} E]^{-1} \tag{38}$$

with $\tau_0 \approx 2.5 \times 10^5$ when B_∞ is expressed in Gauss and E in GeV and r_a is the accretion radius. Equating τ_{sync} and t_c of equation (37), we can obtain the energy up to which the electrons are accelerated efficiently Since the radiation length of electrons in a hydrogenic plasma is *ca.* 100 g cm^{-2} the bremsstrahlung losses can be neglected, in anticipation of our discussion in § 4.3.2 that the particles traverse only about 0.1 g cm^{-2} during acceleration.

Another important consequence of this high acceleration rate is that it eases considerably the problem of injecting particles with energies sufficiently high as to make the rate of energy losses due to coulombian interactions with electrons in the plasma small compared with the acceleration rate. In the present picture a considerable number of such particles may be expected to be produced due to acceleration in the post-shock turbulent plasma (Tsytovich 1977).

The scattering of the charged particles, which we have represented by the diffusion coefficient κ, is expected to be sensibly independent of particle rigidities up to a certain large energy where the gyro-radius of the particle becomes equal to the length scale of the magnetic irregularities in the system. This sets the limit up to which we can expect the power-law spectra derived in § 3.1.1 to be valid. Beyond this energy we expect the spectra to steepen into the exponential form derived in § 3.1.1 for κ dependent on p. Assuming that the accretion flow conserves magnetic flux, we have

$$R_{\max} \sim R_\infty(r_a/r_s)^{\frac{1}{3}(1-2\alpha)} \sim 3\times 10^{-7}\, r_a B_\infty (r_a/r_s)^{\frac{1}{3}(1-2\alpha)}\,\text{GV}. \tag{39}$$

R_{max} is *ca.* 10^3, 10^5 and $> 10^{12}$ GV for white dwarfs, neutron stars and black holes of $10^8 M_{\odot}$ respectively placed under typical conditions. Thus it appears that whereas the cosmic rays of energy below 10^5 GeV could be generated by neutron stars, the cosmic rays of highest energy are probably accelerated in the nuclei of galaxies and quasars and fill the intergalactic medium.

4.3. *Galactic cosmic rays*

We now consider the question as to how far these general considerations are applicable to the galactic cosmic rays. From typical rates of occurrence of supernovae and the birth of pulsars in the galaxy we might expect that there are 10^9–10^{10} neutron stars distributed over the galactic disc. Most of these are more than 10^9 years old and their magnetic field has decayed away to rather small values. Similarly their spin has also decayed away and presently they would have periods longer than about 10 s. They are essentially dead but for the intense gravitational fields close to their surface. Now, the interstellar space is interspersed with clouds of molecular hydrogen, with number densities often exceeding $10^3\,\mathrm{cm}^{-3}$ and with radii of the order of several tens of parsecs. These clouds of gas are concentrated at the galactic plane and occupy a net volume roughly 10^{-3} of the galactic disc. The neutron stars are expected to have a scale height several times larger than the clouds but even so the probability that a neutron star is surrounded by a thick cloud of gas is not negligible. Let us then entertain the possibility that such systems might provide the basis for building a model for the sources of cosmic rays in the galaxy. One might expect 10^4–10^5 such systems in the galactic disc so that each of them need only accelerate 10^{35}–10^{36} erg in cosmic rays to generate the requisite luminosity. (For typical conditions the luminosity of a neutron star will be *ca.* $10^{36.5}\,\mathrm{erg\,s^{-1}}$; see equation (35).) But the maximum energy up to which κ may be taken to be constant is 10^5 GV so the power-law form of the spectrum can extend only up to this energy. The cosmic rays of higher energy must therefore originate elsewhere and are probably extragalactic in origin. It is quite easy to generate $f(p) \sim p^{-4.7}$ corresponding to $\mathrm{d}N/\mathrm{d}E \sim E^{-2.7}$, required for the galactic cosmic rays, by a suitable choice of η. But other detailed knowledge of the galactic cosmic rays and the astrophysical conditions in the galaxy places many critical constraints on the model and we shall proceed to investigate these in turn.

4.3.1. *X-ray astronomy*

Intense X-ray emission has been discovered from many neutron stars in binary systems. The number of such systems over the whole galaxy is expected to be only about 100. But for generating the cosmic rays we need about 10^4 neutron stars accreting matter at about 10^{-2} of the Eddington limit. Thus if all accreting neutron stars are necessarily intense sources of X-rays then they would not be responsible for generating the galactic cosmic rays. But should all accreting neutron stars necessarily release all their gravitational energy as X-rays? The efficiency of emitting X-rays depends on the ratio of the cooling time of the infalling gas to the infall time, and indeed one can define a critical infall rate $\dot{M}_c$, for spherical accretion below which cooling is negligible:

$$\dot{M}_c \sim 7\times 10^{17}\,(M/M_\odot)\,(T_\infty/10^4\,\mathrm{K})^{\frac{1}{2}}\,\mathrm{g\,s^{-1}}. \tag{40}$$

With this we can define the maximum power in cosmic rays that we can extract from a neutron star without many X-rays being emitted:

$$P_c = GM\dot{M}_c/r \sim 10^{36}\,\mathrm{erg\,s^{-1}}. \tag{41}$$

P_c is fortunately at the upper end of the band of luminosities required for generating cosmic rays with 10^4–10^5 sources (Cowsik & Lee 1979*b*). Besides, the critical accretion rate given in equation (40) has been derived with $\gamma = \frac{5}{3}$ and for matter in free fall. In the presence of cosmic rays the infall is slowed down progressively over the whole region of flow from r_a to r_s so we might expect P_c to be considerably larger. Also, since the neutron star is located inside a thick cloud of gas, which could absorb the emitted X-rays efficiently, particularly at large wavelengths, X-ray astronomical results do not unduly constrain the model. The exact treatment of this problem is beyond the scope of this paper as it requires a solution of a self-consistent set of equations including the hydrodynamics and the heat balance in the flow along with the transport of the particles. We suspect that the problem may have to be treated by numerical methods.

4.3.2. *Amount of matter traversed during acceleration*

Observations of heavy nuclei in cosmic rays and the spectrum of their secondaries, which have a considerably steeper spectrum in the energy range below about 100 GeV, demand that the amount of matter traversed during acceleration be smaller than the amount subsequent to acceleration (Cowsik 1980*a*). Otherwise, since the time for acceleration is logarithmically dependent on particle energy, the secondary to primary ratio will increase logarithmically with energy. The density of matter at r_s is given by

$$n(r_s) = n(r_a)\,(r_a/r_s)^{2-\alpha}, \tag{42}$$

which yields $n(r_s) \approx 10^{17}$ atoms $\mathrm{cm^{-3}}$ for a neutron star accreting matter under typical conditions. Since it takes only a few milliseconds for a relativistic particle to traverse *ca.* 1 g $\mathrm{cm^{-2}}$ of matter at such densities, t_c should be much smaller than this. Equation (37) then implies that $\kappa_0 \ll 10^{17}\,\mathrm{cm^2\,sec^{-1}}$ at the shock and this restriction again could also be easily accommodated.

4.3.3. *Composition of cosmic-ray nuclei*

It is well known that cosmic rays are richer in heavy nuclei than is the general interstellar medium. To generate this excess of heavy nuclei we can rely on two possible mechanisms: (i) the injection process at the shock selects preferentially the heavier nuclei, through a variety of mechanisms discussed in cosmic-ray literature such as dependence on first ionization potential (Meyer *et al.* 1979); (ii) in the accretion process the solid grains in the interstellar gas fall inward with nearly the free-fall velocity, much faster than the general gas in the cloud, which is slowed down owing to gradients in pressure of gas and radiation (Cowsik 1980*b*, Cesarsky & Bibring 1980, Tarafdar & Apparao 1981). Upon reaching the shock the grain is evaporated and sputtered, a population of ions rich in heavy nuclei thus being injected. This mechanism might be an attractive way to understand the rather large abundance of ^{22}Ne seen in cosmic rays: as the radiogenic component of ^{22}Na trapped in the grain. This scheme for cosmic-ray origin encounters no serious problems that are not present in the other theses, but it might have some advantages.

The cosmic ray emerging at a distance r_a from the central object have to diffuse over the whole size of the cloud before escaping into the general interstellar medium. During this transport some of the nuclei will be spalled generating secondaries such as Li, Be and B; this aspect is adequately described in terms of the nested leaky box model (Cowsik & Wilson 1973, 1975; Cowsik & Gaisser 1981), which provides good agreement with observational details.

4.3.4. *The galactic γ-ray sources*

During their traversal of the enshrouding cloud, the cosmic rays would generate γ-rays through π^0 production in nuclear interactions and bremsstrahlung. The brightest of the cosmic-ray sources, even when shrouded in a particularly dense cloud of gas, would have sufficient γ-ray luminosity to be visible against the diffuse background generated by cosmic-ray interactions in the weaker sources and in the general interstellar medium. (See Cowsik (1980) and Cowsik & Gaisser (1981) for details.) Of course one does not expect that the luminosity would increase indefinitely when the neutron star is surrounded by clouds of increasing density. One would expect that conditions of self-limiting flow would yield a sharp cut-off on the maximum luminosity.

Summarizing the discussion in this section, one would say that acceleration of cosmic rays by defunct neutron stars embedded in interstellar clouds provides an attractive scenario for understanding several aspects of the problem and might also provide a model for the galactice γ-ray sources. Though it appears that one can understand details such as enhancement of heavy nuclei, much further work is necessary for a fuller understanding of the problem. Cosmic rays of energy exceeding 10^6 GeV cannot be produced by the accretion of neutron stars and black holes present at the nuclei of active galaxies; quasars are perhaps their sources. Under such conditions the cosmic rays with $E > 10^6$ GeV are essentially extragalactic.

4.4. *Spectrum of continuum radiation from quasars*

4.4.1. *Brief overview of earlier work*

An interesting observed feature of quasistellar objects and the nuclei of active galaxies is that their spectra are approximate power laws extending from the near-infrared to the hard X-ray regions, ranging over more than six decades in the energy of the protons (Hermsen *et al.* 1977; Worrall *et al.* 1980). There have been many attempts to generate these spectra through a process of 'comptonization' of the photons in the tenuous high temperature plasma; Thompson collisions with the electrons having random thermal motions lead to a stochastic acceleration of the photons, in a manner akin to the well known Fermi process (Kompaneets 1957; Illyarionov & Sunyaev 1975; Pozdnyaleov, Sobol & Sunyaev 1979; Katz 1976; Eardley & Lightman 1976). Even though the models for the quasars (as for several galactic X-ray sources) involve a rapid accretion flow that often becomes even supersonic, the studies listed here assume a static background plasma through which the transport of radiation takes place. Because of this the non-thermal power-law nature of the photon spectrum is limited to at most two decades in frequency, beyond which the spectrum is cut off exponentially. Only recently, has the importance of the bulk motion of the plasma in controlling the spectral shape of the emergent radiation been recognized (Cowsik & Lee 1980*a*; Blandford & Payne 1980, 1981). In these recent studies it was hoped that the compression of the photon gas in the convergent flow would bring the theoretical spectra into conformity with the observed spectra of the continua of the compact sources.

Even within the context of converging flows two distinct situations obtain: (*a*) without any shocks in the flow (Blandford & Payne 1980; §3.2 of this paper); and (*b*) with a shock transition before the final infall (Cowsik & Lee 1980, §3.2 of this paper). When the flow is continuous without any shocks then the energizing of the photons is not very efficient with a typical gain of only about 50% in the energy of the individual photons. The resulting photon spectrum is too steep to be applicable to the quasar spectra. On the other hand, when the accretion flow passes through a shock, the general compression is augmented, as the photon gains energy by repeated reflexion across the shock, analogous to the first-order Fermi-process. The spectra thus obtained are much flatter and fit the observed spectra of quasars rather well.

4.4.2. *Comparison with the observed spectra of quasars*

A plausible model for the quasistellar objects is that there is a massive black hole accreting mass from the surrounding medium at a rate sufficient to energize the quasar. Presumably, thermal and non-thermal processes generate an intense photon field with a relatively narrow frequency distribution near the shock transition in the accretion flow. These photons have now to diffuse outwards through the infalling hot plasma and the colder gas in the outer regions. Attempts to understand the generation of the line spectra as being due to excitation by the continuum have been remarkably successful. Thus attention needs to be focused here on the underlying continuum; the local variations in the spectrum would then be attributed to modifications caused by the surrounding gas. We also neglect the effect of the energy-spread introduced by comptonization by the thermal motions of the electrons in comparison with the effect of their systematic inflow and the shock. The reasons for this are twofold: first, the flow being generally supersonic, the random motions are slower than the systematic speed of infall; secondly, and more importantly, the systematic motions induce a continuous monotonic increase in the energy of the photon, in contrast to the much slower stochastic increase due to small differences between the rates of head-on and overtaking collisions.

Thus for simplicity we may assume that the injection of photons takes place at a single frequency somewhere in the infrared or microwave bands. At optical frequencies and beyond we may use the asymptotic expression (22), at least up to $\frac{1}{2}$ MeV. (Beyond this energy we should use expression (27), which is relevant to the Klein–Nishina region.) For equation (22) to be applicable, the convection near the shock should dominate over the diffusive transport, which is expressed through the condition

$$\eta = r_{\mathrm{s}} V_0/\kappa_0 \gg 1. \tag{43}$$

To estimate the value of η for a typical quasar, note that if the efficiency of conversion of the gravitational energy made availaable by the accretion flow into radiation is ϕ, then the power emitted by a quasar P is given by

$$P = 2\pi\phi r_{\mathrm{s}}^2 \rho_0 V_0^3. \tag{44}$$

Since $\kappa_0 = \frac{1}{3}c(\sigma_{\mathrm{T}}\rho_0 A)^{-1}$ and $V_0 = (2GM/r_{\mathrm{s}})^{\frac{1}{2}}$,

$$\eta = 3A\sigma_{\mathrm{T}} P/4\pi\phi c\, GM. \tag{45}$$

Typically a quasar emits *ca.* 10^{46} erg s^{-1} and models suggest a $10^8\, M_{\odot}$ black hole radiating the gravitational energy of accretion with about 10% efficiency; with these values $\eta \approx 20$ so we may assume the asymptotic form for the spectrum given in equation (22) to be applicable to most quasars. The spectral index, Γ, of the differential power spectrum of the radiation is obtained simply by adding 3 to the index of the phase space distribution:

$$\Gamma = \sigma[1 - 2\beta\eta^{-1}(\tfrac{1}{3}\sigma - 1)] - 3. \tag{46}$$

In figure 2 we show Γ as a function of the compression factor $\sigma = 3V_0/(V_0 - V_1)$, where V_1 is the post-shock velocity. If the post-shock velocity is taken (unphysically) to be zero we get the minimum value of $\sigma = 3$; for $\sigma = \infty$ there is no shock and $\sigma = 4$ is the most probable value for all strong shocks in a gas with $\gamma = \frac{5}{3}$. We choose $\beta = 1.5$ corresponding to free fall.

Notice that the spectral index of the emergent radiation is very weakly dependent on the choice of parameters describing the shock and that the spectral indices are in the right range for fitting the spectra of quasars and of the active galactic nuclei. Further, since the flow is highly supersonic one expects that a strong shock obtains, yielding a value of σ in the range 4–5, corresponding to a velocity discontinuity $4 \lesssim V_0/V_1 \lesssim 2.5$; when one restricts considerations to these expected values of the parameters the quality of the fit with the quasar spectra becomes even more remarkable. In figure 3, we show the fit to the spectra of 3C 273 and 3C 120. The spectra from the infrared up to *ca.* 1 MeV fit excellently. The spectral intensities below the infrared frequencies could be due to other processes. We have implicitly assumed that the photons are injected with a δ-function spectrum at the frequencies

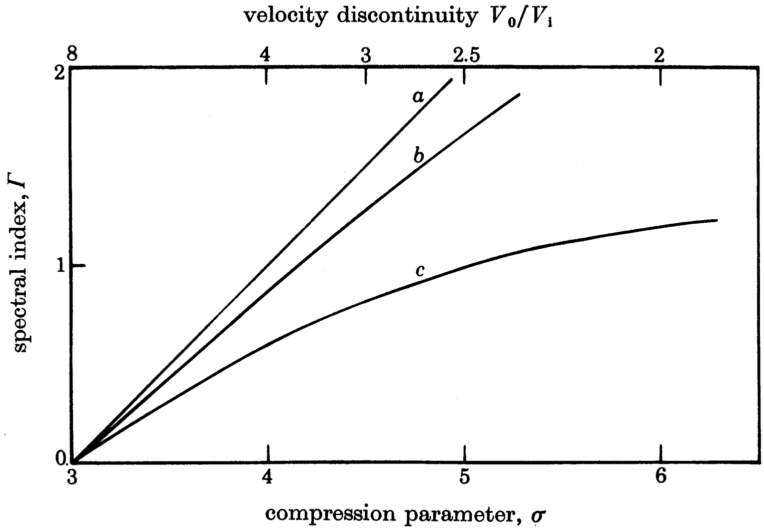

FIGURE 2. Spectral index of radiated power in the energy band from infrared to hard X-rays; typical values of η are in the range 20–100 and of σ in the range 4–5. Curve a, $\eta = \infty$; curve b, $\eta = 30$; curve c, $\eta = 10$.

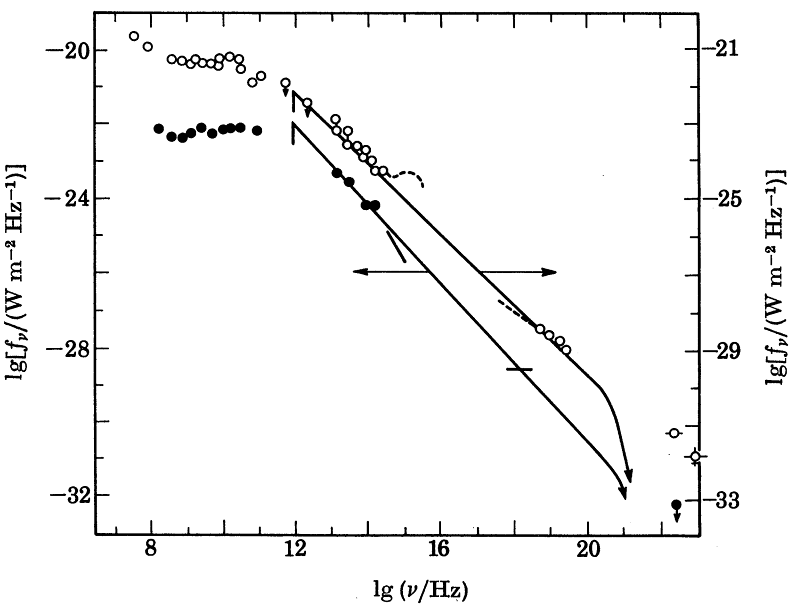

FIGURE 3. Power-law fits to the spectra of two quasars ($\Gamma_1 = 0.97$, $\eta = 30$, $\sigma = 4.05$; $\Gamma_2 = 1.07$, $\eta = 30$, $\sigma = 4.22$); ●, 3C 120; ○, 3C 273. The γ-rays in 3C 273 are probably produced by high energy nuclear interactions of charged particles also accelerated in the accretion flow.

indicated in the figure. A small spread, perhaps thermal, could explain the rounding-off of the spectrum seen at these frequencies.

At energies beyond about 1 MeV the photons cannot be energized directly by the transport and the spectrum must cut off exponentially as the scattering cross section decreases according to the Klein–Nishina formula. Consequently all the photons seen at higher energies, as for 3C 273, must arise as secondaries of the charged particles, which are also accelerated in the flow. The secondary photon generation at high energies can occur efficiently mainly through bremsstrahlung, inverse compton-scattering by relativistic electrons and the decay of neutral pions generated in the nuclear interactions of high energy protons. From looking at the spectral details of 3C 273 we infer that it is the last two of the aforementioned processes that are likely to be important.

In the radiofrequency domain the most likely process of emission is synchrotron radiation by relativistic electrons in the magnetic fields at large distances from the central black hole. As the discussion in §§3.1.2 and 4.2.3 has made it clear that acceleration power-law spectra are very efficient and ubiquitous, the radio-spectra of quasars having a simple $v^{-\alpha}$ form can be rationalized easily.

In concluding this section it is worth while to note that radiation transport in an accretion flow yields spectra that are power laws with indices that match remarkably well with the observations of quasistellar objects, when the effect of the shock transition before the final infall is explicitly included. This mechanism of generating the non-thermal spectrum does not encounter the difficulty inherent in the synchrotron models, namely the 'Compton-paradox' in which the synchrotron photons rescatter on the relativistic electrons very effectively, causing severe loss of energy, thus vitiating the construction of self-consistent models.

4.5. *Neutrino-driven supernovae*

The standard supernova model originally proposed by Colgate & White (1966) envisages a highly evolved star, usually a red giant, that has developed an iron core of about $1.5\,M_{\odot}$. This core becomes dynamically unstable leading to gravitational collapse where, progressively, densities exceeding $10^{14}\,\mathrm{g\,cm^{-3}}$ are achieved. A variety of processes, including capture of the electrons, generate neutrinos copiously. The pressure due to the neutrinos, stiffening of the equation of state and other complex circumstances presumably generate an outward-moving shock wave and possibly bounce the infalling stellar mantle, which results in the supernova explosion. Despite the tremendous amount of attention this idea has received in the literature (Bruenn *et al.* 1978; Arnett 1977; Smarr *et al.* 1981; Bethe *et al.* 1979; Tubbs 1978; Tubbs & Schramm 1975) the efficacy of neutrinos in inducing the bounce is still not well understood. Complications, such as the effects of electron degeneracy, the precise equation of state, the dissociation of nuclei and the effect of the neutrinos in preventing such a dissociation, the neutral currents in weak interactions and the effect of the hydrodynamic flow of matter on the distribution of neutrinos in phase space, have all conspired to prevent a definitive resolution of the problem.

Our study in § 3.3 focuses attention on the last of the complications mentioned. Of course we have neglected most of the details essential for complete understanding of the problem. Yet we feel that the analysis points out the remarkable fact that since the scattering cross sections for neutrinos increase with energy, indeed as E^2, once the densities become large enough for the neutrinos to be trapped, their spectrum hardens, because of the inward flow of matter during the collapse. This enhances further their affinity with the matter. Such an iterative increase in the coupling between the neutrinos and matter was shown to generate an extremely flat spectrum and could in principle lead to a reversal of the inward flow of matter in the mantles. Several points regarding such a scheme are worth noting.

We have neglected the transfer of energy from the neutrinos to the surrounding matter during the scattering process and have assumed that only the momentum is transferred. Under the conditions of the collapse of the core most of the scattering of the neutrinos takes place on the nuclei rather than on the electrons, especially because of the coherent scattering induced by the neutral currents (Freedmann 1974; Bethe 1980). This assumption is therefore valid up to large neutrino momenta, $p = Am_{\mathrm{p}}c$, so we may consider the assumption well justified at the typical neutrino momenta of 10–100 meV/c.

The assumption of the stationary shock used in the calculations is harder to justify since the piling-up of matter at the centre causes the shock to migrate outwards. But since there is a substantial contrast in the density between core and outer regions the outward velocity of the shock is not too high and our results may be expected to indicate the trends qualitatively. The more exact problem is not tractable analytically and there have been attempts at solving the problem numerically (Kazanas 1980). Of course the definitive answers can be obtained only by a self-consistent treatment of the flow as modified by the neutrinos with an ever-changing phase-space distribution. Yet the calculations presented here do provide an initial look at how the neutrinos, energized by the inflow of matter and the shock, can attain increased affinity with the matter and cause a neutrino-driven supernova.

5. Epilogue

In this paper we have studied the transport of particles and radiation across converging flows, including the effects of stationary shocks that might be present in the system. These calculations were applied to a few astrophysical systems to illustrate the role they might play in a variety of situations where the emergent spectra are non-thermal in character. The role of such energizing processes in increasing the affinity between neutrinos and stellar matter during gravitational collapse was also briefly pointed out. The fit to the continuum spectra of quasi-stellar objects turned out to be remarkably good over the wide range of frequencies from infrared to hard X-rays.

In considering the possible avenues for further development of these ideas, self-consistent studies of the transport and the flow should enjoy priority. The acceleration process discussed here is so effective that it would drain the energy associated with the flow very efficiently. Thus it becomes imperative that the equations governing the flow contain the forces exerted on them by the particles and radiation.

Finally, even though this study has assumed spherical symmetry throughout, the results, we feel, have a broader applicability. The qualitative aspects of our calculations, such as the generation of power-law and exponential spectra, would survive even when there is some departure from exact spherical symmetry. Also, in the context of the transport of charged particles in flows where the dominant ambient magnetic field is along the flow our results would be applicable since the diffusion constant transverse to the field is much smaller than that in the direction of the field. Such conditions might obtain near the poles of neutron stars and near rotating black holes where relativistic particle jets could be generated.

In concluding, we remark that in accretion flows there is a self-contained energy source and a very efficient process of transferring this energy to energetic particles and radiation. Thus massive objects such as neutron stars and black holes accreting matter from the surrounding medium could form the essence of models for many exciting astrophysical objects displaying non-thermal spectra.

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