EVALUATION OF HIGH ENERGY NATURAL NEUTRINO EXPERIMENTS

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Abstract

The directional intensities of neutrinos of various types produced in the decay of cosmic ray secondaries in the earth's atmosphere have been estimated. The calculated energy spectra are believed to be uncertain by $\lesssim 20\%$ for neutrino energies up to 100 GeV.

Using these fluxes and making various assumptions about the behaviour of neutrino cross-section with energy, the energy spectra of neutrino-induced muons at large depths underground have been calculated. It is shown that a cross-section increasing only linearly with energy up to ~ 100 GeV. would account only for about one-fifth of the preliminary counting rate observed in a recent underground experiment. A more rapid increase in cross-section is indicated somewhere between 10 GeV. and 100 GeV. and this is shown to be quantitatively consistent with the existence of a charged intermediate boson of mass 2 GeV. $\lesssim M_w < 5$ GeV. (However see the note added in proof.)

The question of detecting high energy neutrino signals from extra-terrestrial sources is briefly discussed.

I. Introduction

Interactions of high energy cosmic rays in the atmosphere lead to the production of unstable particles which, through their decay, result in production of neutrinos whose energy extends into regions which will not be attained by the laboratory machines for a long time to come. Several experiments to detect high energy cosmic ray neutrinos have already been initiated.[1, 2] Amongst the important aims of these experiments would be the study of the cross-section and the nature of neutrino interactions at very high energies. For a proper analysis of such experiments, which essentially detect the total number of secondary particles (for the present muons, without charge determination) emerging from thick slabs of matter, it is necessary to have a detailed knowledge of the energy spectra of all types of neutrinos produced in the earth's atmosphere. Besides this aspect involving the study of weak interactions at ultra-high energies, if one is to undertake more ambitious experiments to detect the cosmic fluxes of high energy neutrinos, originating either in point sources or in the interstellar medium. it is essential that one knows the background of the atmospheric neutrino flux in various directions.

The early detailed calculations[3] of the overall fluxes of atmospheric neutrinos were reported by Greisen and Zatsepin and Kuzmin. These did not take into account, quantitatively, the contribution of strange particle decays to different types of neutrinos; as remarked by these authors, this contribution is significant and, as will be shown below, dominant under certain conditions. The present calculation has been made to determine the spectra of various types of neutrinos produced in the atmosphere, using the best known present-day parameters about the primary cosmic radiation, high energy interactions and decay of unstable particles. In particular, use has been made of a detailed calculation of the propagation of cosmic rays through the atmosphere.[4] Since neutrinos have a generic relationship with muons, the fluxes of muons in different directions and their charge ratio as a function of energy must be represented accurately by the choice of parameters used for the neutrino flux calculations. This has been ensured.

Section II deals with the details of flux calculations while the results are discussed in Section III. Section IV includes a discussion of the expected energy spectrum of neutrino-induced muons at large depths underground, under various assumptions about the behaviour of neutrino interaction cross-section with energy; in particular the contribution due to production of the hypothetical charged intermediate bosons has been estimated in detail. In Section V the results of the calculation are reviewed and the possible existence of the intermediate boson is discussed in the light of a recent deep mine experiment. Possibility of detecting neutrino signals from extraterrestrial sources is briefly discussed in Appendix A.

II. CALCULATION OF THE NEUTRINO FLUXES

A. *Nature of the Atmosphere*

A spherical atmosphere whose density falls exponentially with increasing radius has been assumed; the scale height h_0 of the atmosphere has been taken as 7 km. Referring to Fig. 1, the amount of matter traversed by a particle arriving at a zenith angle θ up to a distance t (measured in the direction θ) from sea-level is given by

$$x(t, \theta) = \frac{X_0}{h_0} \int_t^{\infty} e^{-(t^2/2R + t\cos\theta)/h_0} \, . \, dt \qquad \text{II (1)}$$

where R is the radius of the earth and X_0 (= 1030 gm./cm.^{2}) is the total atmospheric thickness in the vertical direction. For θ up to ~70° one can use the approximation

$$x(t, \theta) = \frac{X_0}{\cos\theta} e^{-t\cos\theta/h_0}. \qquad \text{II (2)}$$

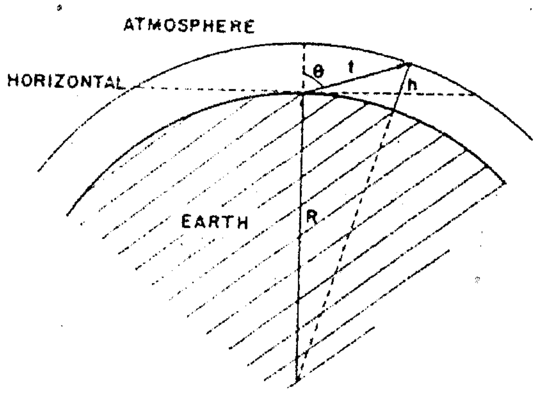

Fig. 1. Definition of symbols used in relation II·1.

B. In this section expressions have been given for the fluxes of the unstable particles which, through their decay, give rise to neutrinos. Since the branching ratios for leptonic decays of hyperons are small we neglect the contribution of these to the neutrino flux.

Production Spectra and Fluxes of Pions and Kaons: In the part of the atmosphere in which the density distribution along the trajectory of the incident particle can be approximated by an exponential, as is the case for all depths in the vertical direction and for first 1000 gm./cm.^{2} in the horizontal direction, the expressions for these fluxes have been given by Pal and Peters.[4] As discussed there, most of the flux of pions and kaons at depths less than ~ 500 gm./cm.^{2} is comprised of particles produced directly by the nucleons (first generation particles). Pion production by pions occurs deep in the atmosphere in a restricted energy interval and its contribution to the neutrino flux is small. Therefore here we give only the simple expressions for the first generation pion and kaon fluxes; small corrections have been applied during numerical evaluations, using the more complicated expressions given in reference 4.

The differential flux of particles of type j at depth x is given by

$$F_j(x, E) = \langle B_j \rangle \frac{x}{\lambda} e^{-x/\Lambda} \frac{S_0}{E^{\gamma+1}(1+u_j)}. \qquad \text{II (3)}$$

Here

$\frac{S_0}{E^{\gamma+1}}$ is the cosmic ray nucleon spectrum at the top of the atmosphere, where

$S_0 = 2 \cdot 35 \times 10^4$ if the flux is measured in ($m.^{-2}$ sec.$^{-1}$ ster.$^{-1}$ GeV.$^{-1}$) and $\gamma = 1 \cdot 67$.

λ : interaction mean free path of nucleons $\approx$ 75 gm./cm.^{2};

Λ: attenuation mean free path of nucleons $\approx$ 120 gm./cm.^{2};

[In deriving II (3) it is assumed that interaction mean free path of pions and kaons is equal to Λ].

U_j: $\frac{hm_j}{c\tau_j E}$, m_j and τ_j being the mass and proper life time of particle j, and h the scale height of the atmosphere in the direction in which the flux is measured;

$\langle B_j \rangle = \int_0^1 n_j(r_j) \, . \, r_j^{\gamma} \, . \, dr_j$, where $n_j(r_j)$ is the number of particles of type j which receive a fraction r_j of the incident nucleon energy in an interaction.

Values of $\langle B_{\pi\pm} \rangle$ and $\langle B_{K\pm} \rangle$ have been fixed in reference 4 by comparison of the primary cosmic ray spectrum and the observed spectrum of muons of both charges as a function of energy. Value of $\langle B_{K\pm} \rangle / \langle B_{\pi\pm} \rangle$ estimated from other data is consistent with the values adopted in reference 4; this ratio represents the ratio of production spectra of charged kaons and charged pions and its value is taken as 15%.

As discussed in reference 4 the ratio of production spectra of K^+ and K^- is expected to be very large. We have taken a value of ~ 20 for this ratio. This means, for example, that charged kaons will contribute mostly to the production of ν_μ, via the decay mode $K^+ \rightarrow \mu^+ + \nu_\mu$, and very little to $\bar{\nu}_\mu$. Further it is assumed that the K°/K^+ production ratio is unity. The K_1°

particles decay mostly by 2 pion mode and hence are relatively unimportant. On the other hand the contribution of K_2°, because of its three body leptonic decay modes, would be significant for the production of electron-neutrinos. These are therefore included.

Flux of Muons

In all directions the first 1000 gm./cm.^{2} of the atmosphere is well approximated by taking an exponential variation of density with distance. Hence in all directions the muon flux up to a depth of 1000 gm./cm.^{2} can be calculated using standard expressions of the type II–17 given in reference 4, replacing h_0 by the appropriate scale height h for the direction concerned. Further, it may be assumed without any loss of accuracy that the *production* of muons below a depth of $\sim$ 1000 gm./cm.^{2} is negligible in all directions. The flux of muons below a depth of 1000 gm./cm.^{2} is then calculated by taking into account their ionization and decay loss and neglecting the production term. The survival probability, defined as the probability that a muon of energy E survives decay till a *further* depth of x [arriving at x with energy $(E - bx)$, where b is the energy loss in traversing unit thickness], is given by

$$W(x, E) = \exp. - \left[\int_0^x \frac{m_\mu \cdot dz}{c \cdot \tau_\mu \cdot \rho(z) \cdot (E - bz)} \right] \qquad \text{II (6)}$$

where $\rho(z)$ is the local density of the atmosphere in gm./cm.^{3} at depth z.

C. *Flux of Neutrinos*

Table I lists the various decay modes of unstable particles which have been taken into account for calculating the fluxes of neutrinos of various types. The relative flux $(\nu_\mu/\bar{\nu}_\mu)_\pi$ arising from pion decay must conform to the observed ratio $\mu^+/\mu^- = 1{\cdot}25$ at sea-level at moderate energies ($\lesssim$ 100 GeV.). On the other hand the ratios $(\nu_\mu/\bar{\nu}_\mu)_\mu$ and $(\bar{\nu}_e/\nu_e)_\mu$ from muon decay would be $\approx 1/1{\cdot}25$. Charged kaons contribute significantly only to ν_μ and ν_e, negative kaon contribution being very small. K_2° contribution to neutrinos and antineutrinos would be equal.

The production spectrum of neutrinos of type i from parents of type j (pions, muons or kaons) emitting neutrinos in direction ϕ in their rest system is given by

$$P_{\nu_i}^{\,j}(x, E; \phi) = \frac{m_j}{c \cdot \tau_j \cdot \rho(x)} \cdot \frac{b_{ij}}{E} \cdot F_j\left[x, \frac{E}{r_{j,\phi}} \right]. \qquad \text{II (7)}$$

Here b_{ij} is the relevant branching ratio, $r_{j,\phi} = r_j\,(1 + \cos\phi)$ is the fractional energy* received by the neutrino when it is emitted in the direction ϕ, and $\rho\,(x)$ is the local density of the atmosphere. Weighting each angle by $d\cos\phi/2$ (isotropic emission), and integrating over ϕ,

$$P_{\nu_i}^{\,j}\,(x, E) = \frac{m_j}{c\cdot\tau_j\cdot\rho\,(x)}\cdot b_{ij}\int_{-1}^{+1} F_j\left[x, \frac{E}{r_{j,\phi}}\right]\cdot\frac{d\cos\phi}{2E}$$

$$= \frac{m_j}{c\cdot\tau_j\cdot\rho\,(x)}\cdot\frac{b_{ij}}{2r_j}\int_{E/2r_j}^{\infty} F_j\,(x, E')\cdot\frac{dE'}{E'^2}. \qquad \text{II (8)}$$

TABLE I

Various decay modes taken into account

Particle and decay mode	Branching ratio	Life time (Sec.)
$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$	100%	$2{\cdot}2 \times 10^{-6}$
$\pi^+ \rightarrow \mu^+ + \nu_\mu$	100%	$2{\cdot}55 \times 10^{-8}$
$\pi^+ \rightarrow e^+ + \nu_e$	$1{\cdot}24 \times 10^{-4}$	$2{\cdot}55 \times 10^{-8}$
$K^+ \rightarrow \mu^+ + \nu_\mu$	63·1%	$1{\cdot}229 \times 10^{-8}$
$K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu$	3·4%	$1{\cdot}229 \times 10^{-8}$
$K^+ \rightarrow \pi^0 + e^+ + \nu_e$	4·8%	$1{\cdot}229 \times 10^{-8}$
$K_2^0 \rightarrow \pi^- + \mu^+ + \nu_\mu$	15·5%	$5{\cdot}62 \times 10^{-8}$
$K_2^0 \rightarrow \pi^- + e^+ + \nu_e$	18·5%	$5{\cdot}62 \times 10^{-8}$

(Same decay parameters have been used for antiparticles).

For pion and kaon primaries the neutrino production is essentially confined to the first 1000 gm./cm.^{2} of the atmosphere where, for all directions, the density $\rho\,(x)$ may be expressed as

$$\rho\,(x) = \frac{x}{h} \qquad \text{II (9)}$$

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[*] For three body decays r_j is not unique, so an additional integration over r_j is required.

where h is the appropriate scale height. For muon primaries such a simple formulation is not possible for zenith angles beyond 70°.

The neutrino flux at depth X_0 is then given by

$$F_{\nu_i}^{j}(X_0, E) = \int_0^{X_0} P_{\nu_i}^{j}(x, E) \cdot dx. \qquad \text{II (10)}$$

III. Results of Calculations

Calculated muon spectra at sea-level at zenith angles of 0° and at 80°, 88·75° and 90° are plotted in Figs. 2 and 3 respectively. The experimental points of Durham group[5] are also given. Fit to the vertical spectrum is excellent over the entire range (this figure is from reference 4). Also 80° and 88·75° calculations give fortuitously good representations of the measured spectra at these angles; slight deviations arising from finite aperture effects and multiple coulomb scattering would not have been surprising. In Fig. 4,

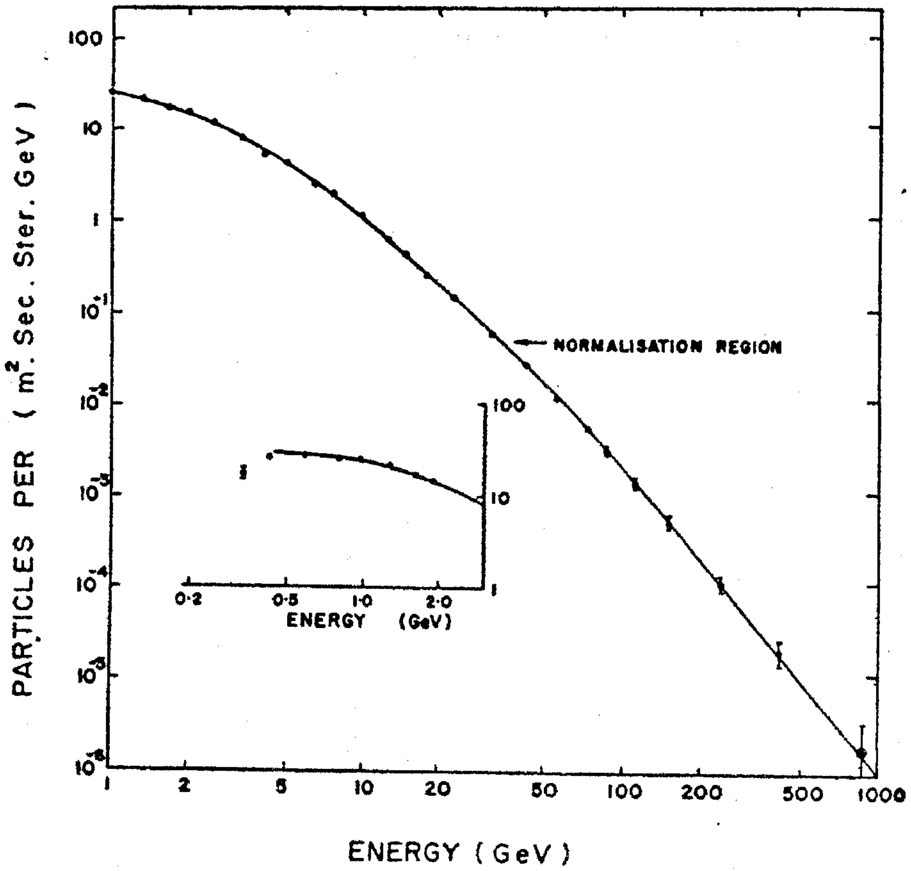

Fig. 2. Calculated vertical flux of muons (after ref. 4) with experimental points.[5a, 5]

again taken from reference 4, the fit to the (μ^+/μ^-) ratio, which would correspond to the calculated $\nu_\mu/\bar{\nu}_\mu$ ratio, has been given.

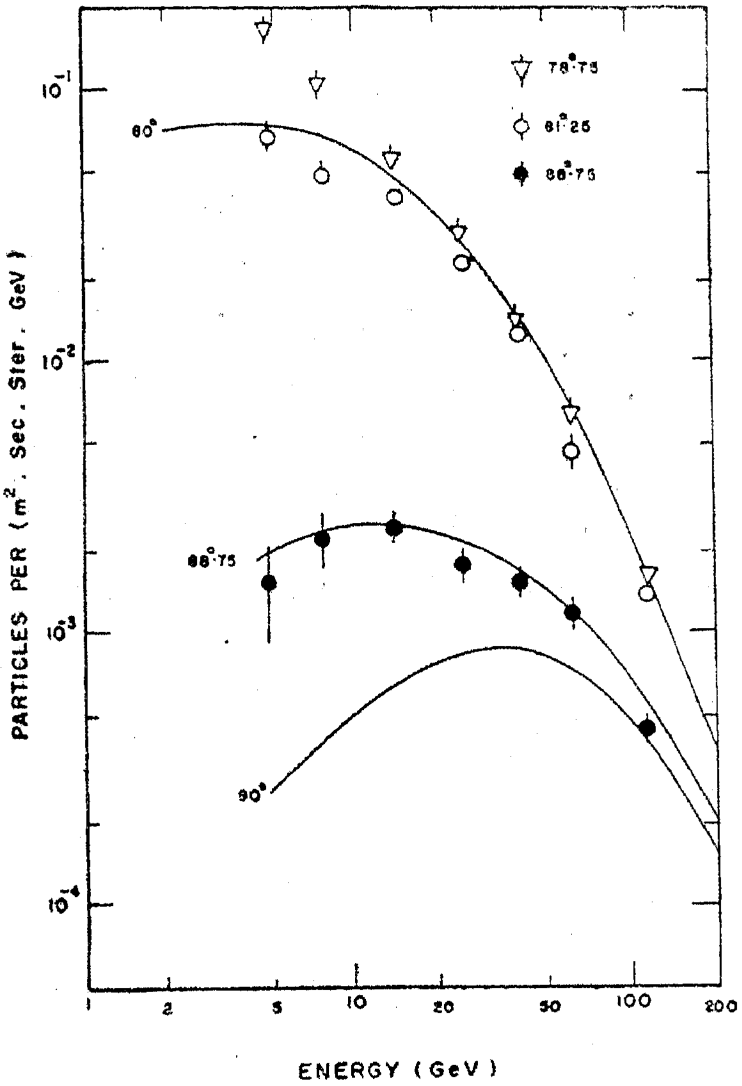

FIG. 3. Calculated flux of muons at large zenith angles with experimental points.[14]

Representative flux values for neutrinos of different types in the vertical and horizontal directions are given in Table II. The total flux of $(\nu_\mu + \bar{\nu}_\mu)$ at 0°, 80° and 90° are plotted in Fig. 5. One may also write down convenient power law expressions for the differential flux of different types of neutrinos. These expressions for the vertical and horizontal directions are given in Table III and represent the flux in the appropriate intervals within $\sim 10\%$. It is seen that:

TABLE. II *Fluxes of different kinds of neutrinos, arising from different sources, at sea-level in the vertical and horizontal directions in units of particles/(m.^{2} sec. ster. GeV.)*

Type of Neutrino		ν_μ				$\bar{\nu}_\mu$			
E(GeV.) \ Source		Pion	Muon	Kaon	Total	Pion	Muon	Kaon	Total
2	Vert.	$1\cdot1\times10^{1}$	$6\cdot0$	$7\cdot0$	$2\cdot4\times10^{1}$	$9\cdot1$	$7\cdot4$	$7\cdot3\times10^{-1}$	$1\cdot7\times10^{1}$
	Hor.	$1\cdot1\times10^{1}$	$2\cdot0\times10^{1}$	$7\cdot8$	$3\cdot8\times10^{1}$	$9\cdot1$	$2\cdot4\times10^{1}$	$7\cdot6\times10^{-1}$	$3\cdot4\times10^{1}$
5	Vert.	$6\cdot6\times10^{-1}$	$3\cdot6\times10^{-1}$	$5\cdot8\times10^{-1}$	$1\cdot6$	$5\cdot5\times10^{-1}$	$4\cdot3\times10^{-1}$	$6\cdot0\times10^{-2}$	$1\cdot0$
	Hor.	$8\cdot8\times10^{-1}$	$1\cdot6$	$6\cdot1\times10^{-1}$	$3\cdot1$	$7\cdot2\times10^{-1}$	$1\cdot8$	$6\cdot4\times10^{-2}$	$2\cdot6$
10	Vert.	$1\cdot1\times10^{-1}$	$2\cdot3\times10^{-2}$	$9\cdot0\times10^{-2}$	$2\cdot2\times10^{-1}$	$9\cdot0\times10^{-2}$	$2\cdot7\times10^{-2}$	$9\cdot3\times10^{-3}$	$1\cdot3\times10^{-1}$
	Hor.	$1\cdot3\times10^{-1}$	$1\cdot8\times10^{-1}$	$9\cdot5\times10^{-2}$	$4\cdot1\times10^{-1}$	$1\cdot1\times10^{-1}$	$2\cdot2\times10^{-1}$	$9\cdot9\times10^{-3}$	$3\cdot4\times10^{-1}$
20	Vert.	$1\cdot4\times10^{-2}$	$2\cdot0\times10^{-3}$	$1\cdot5\times10^{-2}$	$3\cdot1\times10^{-2}$	$1\cdot2\times10^{-2}$	$2\cdot4\times10^{-3}$	$1\cdot4\times10^{-3}$	$1\cdot6\times10^{-2}$
	Hor.	$2\cdot0\times10^{-2}$	$2\cdot6\times10^{-2}$	$1\cdot5\times10^{-2}$	$6\cdot1\times10^{-2}$	$1\cdot7\times10^{-2}$	$3\cdot1\times10^{-2}$	$1\cdot6\times10^{-3}$	$5\cdot0\times10^{-2}$
50	Vert.	$7\cdot4\times10^{-4}$	$5\cdot7\times10^{-5}$	$1\cdot0\times10^{-3}$	$1\cdot8\times10^{-3}$	$6\cdot1\times10^{-4}$	$7\cdot0\times10^{-5}$	$1\cdot0\times10^{-4}$	$8\cdot3\times10^{-4}$
	Hor.	$1\cdot7\times10^{-3}$	$1\cdot4\times10^{-3}$	$1\cdot3\times10^{-3}$	$4\cdot4\times10^{-3}$	$1\cdot4\times10^{-3}$	$1\cdot6\times10^{-3}$	$1\cdot3\times10^{-4}$	$3\cdot1\times10^{-3}$
100	Vert.	$7\cdot3\times10^{-5}$	$2\cdot6\times10^{-6}$	$1\cdot8\times10^{-4}$	$2\cdot6\times10^{-4}$	$6\cdot4\times10^{-5}$	$3\cdot1\times10^{-6}$	$1\cdot4\times10^{-5}$	$8\cdot1\times10^{-5}$
	Hor.	$2\cdot2\times10^{-4}$	$1\cdot5\times10^{-4}$	$1\cdot9\times10^{-4}$	$5\cdot6\times10^{-4}$	$1\cdot8\times10^{-4}$	$1\cdot8\times10^{-4}$	$2\cdot0\times10^{-5}$	$3\cdot8\times10^{-4}$
200	Vert.	$7\cdot7\times10^{-6}$	$1\cdot3\times10^{-7}$	$2\cdot2\times10^{-5}$	$3\cdot0\times10^{-5}$	$6\cdot3\times10^{-6}$	$1\cdot6\times10^{-7}$	$1\cdot6\times10^{-6}$	$8\cdot1\times10^{-6}$
	Hor.	$2\cdot9\times10^{-5}$	$1\cdot1\times10^{-5}$	$3\cdot1\times10^{-5}$	$7\cdot1\times10^{-5}$	$2\cdot4\times10^{-5}$	$1\cdot3\times10^{-5}$	$3\cdot1\times10^{-6}$	$4\cdot0\times10^{-5}$
500	Vert,	$2\cdot9\times10^{-7}$	$2\cdot2\times10^{-9}$	$1\cdot5\times10^{-6}$	$1\cdot8\times10^{-6}$	$2\cdot4\times10^{-7}$	$2\cdot6\times10^{-9}$	$9\cdot9\times10^{-8}$	$3\cdot4\times10^{-7}$
	Hor.	$1\cdot6\times10^{-6}$	$3\cdot4\times10^{-7}$	$2\cdot6\times10^{-6}$	$4\cdot6\times10^{-6}$	$1\cdot3\times10^{-6}$	$4\cdot1\times10^{-7}$	$2\cdot2\times10^{-7}$	$1\cdot9\times10^{-6}$
1,000	Vert.	$2\cdot5\times10^{-8}$	$8\cdot9\times10^{-11}$	$1\cdot6\times10^{-7}$	$1\cdot8\times10^{-7}$	$2\cdot0\times10^{-8}$	$1\cdot1\times10^{-10}$	$9\cdot8\times10^{-9}$	$3\cdot0\times10^{-8}$
	Hor.	$1\cdot8\times10^{-7}$	$2\cdot2\times10^{-8}$	$3\cdot8\times10^{-7}$	$5\cdot8\times10^{-7}$	$1\cdot5\times10^{-7}$	$2\cdot5\times10^{-8}$	$3\cdot1\times10^{-8}$	$2\cdot0\times10^{-7}$

TABLE II—*Contd.*

Type of Neutrino		ν_e				$\bar{\nu}_e$			
E(GeV.)	Source	Pion	Muon	Kaon	Total	Pion	Muon	Kaon	Total
2	Vert.	$5{\cdot}7\times10^{-3}$	$5{\cdot}8$	$8{\cdot}2\times10^{-1}$	$6{\cdot}6$	$4{\cdot}7\times10^{-3}$	$4{\cdot}7$	$5{\cdot}6\times10^{-1}$	$5{\cdot}2$
	Hor.	$6{\cdot}0\times10^{-3}$	$1{\cdot}9\times10^{1}$	$8{\cdot}0\times10^{-1}$	$2{\cdot}0\times10^{1}$	$5{\cdot}0\times10^{-3}$	$1{\cdot}5\times10^{1}$	$5{\cdot}7\times10^{-1}$	$1{\cdot}6\times10^{1}$
5	Vert.	$4{\cdot}6\times10^{-4}$	$3{\cdot}0\times10^{-1}$	$6{\cdot}9\times10^{-2}$	$3{\cdot}7\times10^{-1}$	$3{\cdot}8\times10^{-4}$	$2{\cdot}5\times10^{-1}$	$4{\cdot}5\times10^{-2}$	$3{\cdot}0\times10^{-1}$
	Hor.	$4{\cdot}7\times10^{-4}$	$1{\cdot}4$	$7{\cdot}2\times10^{-2}$	$1{\cdot}4$	$3{\cdot}9\times10^{-4}$	$1{\cdot}2$	$4{\cdot}8\times10^{-2}$	$1{\cdot}2$
10	Vert.	$7{\cdot}2\times10^{-5}$	$1{\cdot}9\times10^{-3}$	$1{\cdot}1\times10^{-2}$	$2{\cdot}9\times10^{-2}$	$6{\cdot}0\times10^{-5}$	$1{\cdot}6\times10^{-2}$	$7{\cdot}0\times10^{-3}$	$2{\cdot}3\times10^{-2}$
	Hor.	$7{\cdot}0\times10^{-5}$	$1{\cdot}7\times10^{-1}$	$1{\cdot}1\times10^{-2}$	$1{\cdot}8\times10^{-1}$	$5{\cdot}8\times10^{-5}$	$1{\cdot}4\times10^{-1}$	$7{\cdot}6\times10^{-3}$	$1{\cdot}5\times10^{-1}$
20	Vert.	$9{\cdot}5\times10^{-6}$	$1{\cdot}7\times10^{-3}$	$1{\cdot}5\times10^{-3}$	$3{\cdot}2\times10^{-3}$	$7{\cdot}8\times10^{-6}$	$1{\cdot}4\times10^{-3}$	$9{\cdot}8\times10^{-4}$	$2{\cdot}4\times10^{-3}$
	Hor.	10^{-5}	$2{\cdot}3\times10^{-2}$	$1{\cdot}8\times10^{-3}$	$2{\cdot}5\times10^{-2}$	$8{\cdot}5\times10^{-6}$	$1{\cdot}9\times10^{-2}$	$1{\cdot}2\times10^{-3}$	$2{\cdot}1\times10^{-2}$
50	Vert.	$5{\cdot}9\times10^{-7}$	$4{\cdot}6\times10^{-5}$	$1{\cdot}1\times10^{-4}$	$1{\cdot}6\times10^{-4}$	$4{\cdot}9\times10^{-7}$	$3{\cdot}7\times10^{-5}$	$6{\cdot}6\times10^{-5}$	$1{\cdot}0\times10^{-4}$
	Hor.	$8{\cdot}3\times10^{-7}$	$1{\cdot}2\times10^{-3}$	$1{\cdot}5\times10^{-4}$	$1{\cdot}4\times10^{-3}$	$6{\cdot}8\times10^{-7}$	$1{\cdot}0\times10^{-3}$	$9{\cdot}9\times10^{-5}$	$1{\cdot}1\times10^{-3}$
100	Vert.	$6{\cdot}3\times10^{-8}$	$2{\cdot}0\times10^{-6}$	$1{\cdot}4\times10^{-5}$	$1{\cdot}6\times10^{-5}$	$5{\cdot}2\times10^{-8}$	$1{\cdot}7\times10^{-6}$	$7{\cdot}6\times10^{-6}$	$9{\cdot}4\times10^{-6}$
	Hor.	$1{\cdot}3\times10^{-7}$	$1{\cdot}3\times10^{-4}$	$2{\cdot}2\times10^{-5}$	$1{\cdot}5\times10^{-4}$	10^{-7}	$1{\cdot}1\times10^{-4}$	$1{\cdot}5\times10^{-5}$	$1{\cdot}2\times10^{-4}$
200	Vert.	$6{\cdot}8\times10^{-9}$	$1{\cdot}0\times10^{-7}$	$1{\cdot}6\times10^{-6}$	$1{\cdot}7\times10^{-6}$	$5{\cdot}6\times10^{-9}$	$8{\cdot}3\times10^{-8}$	$8{\cdot}2\times10^{-7}$	$9{\cdot}1\times10^{-7}$
	Hor.	$1{\cdot}8\times10^{-8}$	$9{\cdot}3\times10^{-6}$	$3{\cdot}4\times10^{-6}$	$1{\cdot}3\times10^{-5}$	$1{\cdot}5\times10^{-8}$	$7{\cdot}7\times10^{-6}$	$2{\cdot}0\times10^{-6}$	$9{\cdot}7\times10^{-6}$
500	Vert.	$3{\cdot}1\times10^{-10}$	$1{\cdot}7\times10^{-9}$	$8{\cdot}6\times10^{-8}$	$8{\cdot}8\times10^{-8}$	$2{\cdot}6\times10^{-10}$	$1{\cdot}5\times10^{-9}$	$3{\cdot}9\times10^{-8}$	$4{\cdot}1\times10^{-8}$
	Hor.	$1{\cdot}2\times10^{-9}$	$2{\cdot}8\times10^{-7}$	$2{\cdot}5\times10^{-7}$	$5{\cdot}3\times10^{-7}$	$9{\cdot}9\times10^{-10}$	$2{\cdot}3\times10^{-7}$	$1{\cdot}4\times10^{-7}$	$3{\cdot}7\times10^{-7}$
1,000	Vert.	$2{\cdot}5\times10^{-11}$	$7{\cdot}1\times10^{-11}$	$8{\cdot}4\times10^{-9}$	$8{\cdot}5\times10^{-9}$	$2{\cdot}1\times10^{-11}$	$5{\cdot}8\times10^{-11}$	$3{\cdot}4\times10^{-9}$	$3{\cdot}5\times10^{-9}$
	Hor.	$1{\cdot}4\times10^{-10}$	$1{\cdot}6\times10^{-8}$	$2{\cdot}9\times10^{-8}$	$4{\cdot}5\times10^{-8}$	$1{\cdot}1\times10^{-10}$	$1{\cdot}4\times10^{-8}$	$1{\cdot}6\times10^{-8}$	$3{\cdot}0\times10^{-8}$

(*a*) In the vertical direction kaon decay dominates the flux of ν_μ, ν_e and $\bar{\nu}_e$ at energies above 100 GeV. and most of the $\bar{\nu}_e$ type of neutrinos come from the decay of K_2°.

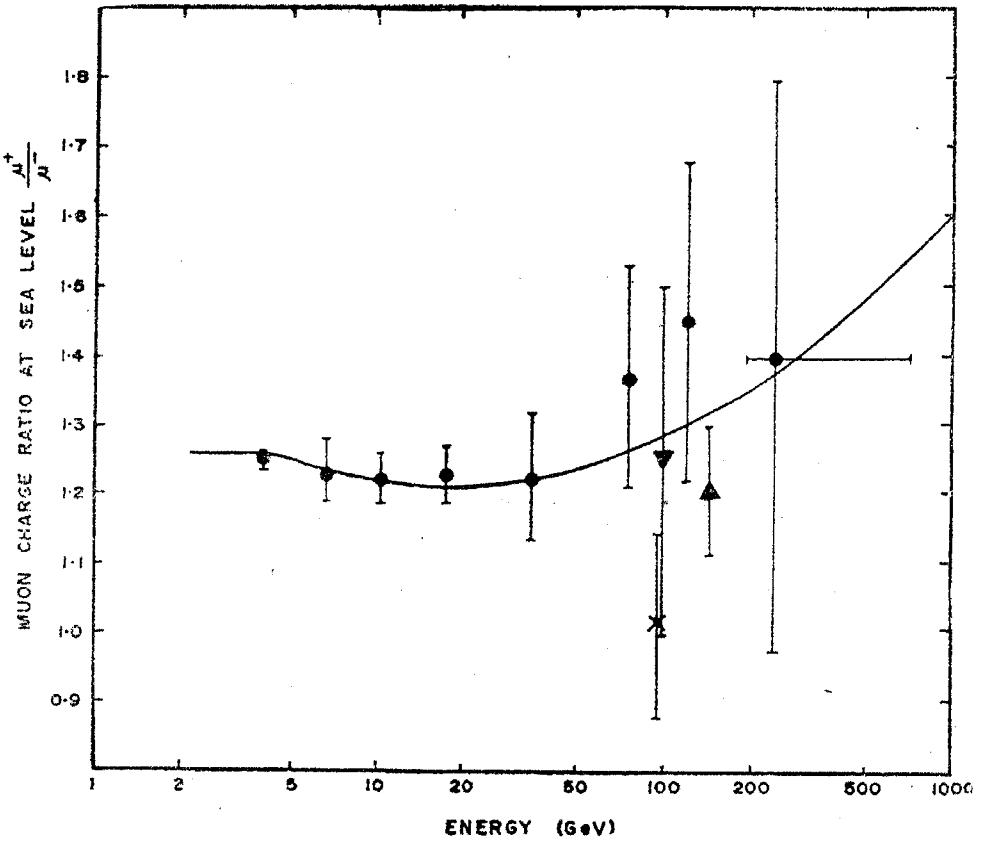

FIG. 4. Calculated charge ratio of muons at sea-level with experimental points. The slight dip at ~ 20 GeV. is due to pion production by pions. The increase at the high energy end is due to the production of charged kaons which are mostly positive. The curve is for a production ratio $(B_K \pm)/(B_\pi \pm) = 15\%$ (after ref. 4).

(*b*) In the horizontal direction pion and muon contributions are considerably enhanced and kaons are not the dominant source even at very high energies.

(*c*) The ratios of horizontal to vertical fluxes are always greater than unity; these are given in Table IV at a few typical energies.

(*d*) Ratio $\nu_\mu/\bar{\nu}_\mu$ which reflects, qualitatively, the ratio μ^+/μ^- amongst cosmic ray muons, is greater than unity. This ratio becomes very large at high energies because of the relative amplification of the K^+ contribution ($K\mu 2$ decay) with respect to the pion contribution.[4] This relative amplification is much larger for the contribution to the flux of neutrinos (because of the higher fractional energy given to them in kaon decay as compared to

TABLE III

Power law fits to the neutrino fluxes
Differential neutrino flux = $NE^{-(\alpha+1)}$ ($m.^{-2}$ $sec.^{-1}$ $ster.^{-1}$ $GeV.^{-1}$)

Type of neutrino		2 – 100 GeV.		100 – 1000 GeV.	
		N	α	N	α
ν_μ	Vertical	173	1·91	544	2·16
	Horizontal	311	1·87	511	1·98
$\bar{\nu}_\mu$	Vertical	142	2·08	587	2·43
	Horizontal	300	1·95	1380	2·28
ν_e	Vertical	83	2·36	56	2·27
	Horizontal	174	2·00	1650	2·52
$\bar{\nu}_e$	Vertical	81	2·48	72	2·43
	Horizontal	161	2·04	1910	2·6
$\nu_\mu + \bar{\nu}_\mu$	Vertical	322	1·99	910	2·21
	Horizontal	576	1·89	1360	2·08

TABLE IV

Ratios of horizontal to vertical flux of neutrinos at sea-level

Energy GeV. / Type of neutrino	2	10	100	1000
ν_μ	1·6	1·9	2·1	3·2
$\bar{\nu}_\mu$	2·0	2·6	4·7	6·7
ν_e	3·0	6·2	9·4	5·3
$\bar{\nu}_e$	3·1	6·5	13	8·6

that in pion decay) than it is for the flux of muons. As a result the high energy asymptotic value of $\nu_\mu/\bar{\nu}_\mu$ would approach 7 while the μ^+/μ^- approaches $\sim 1 \cdot 8$.

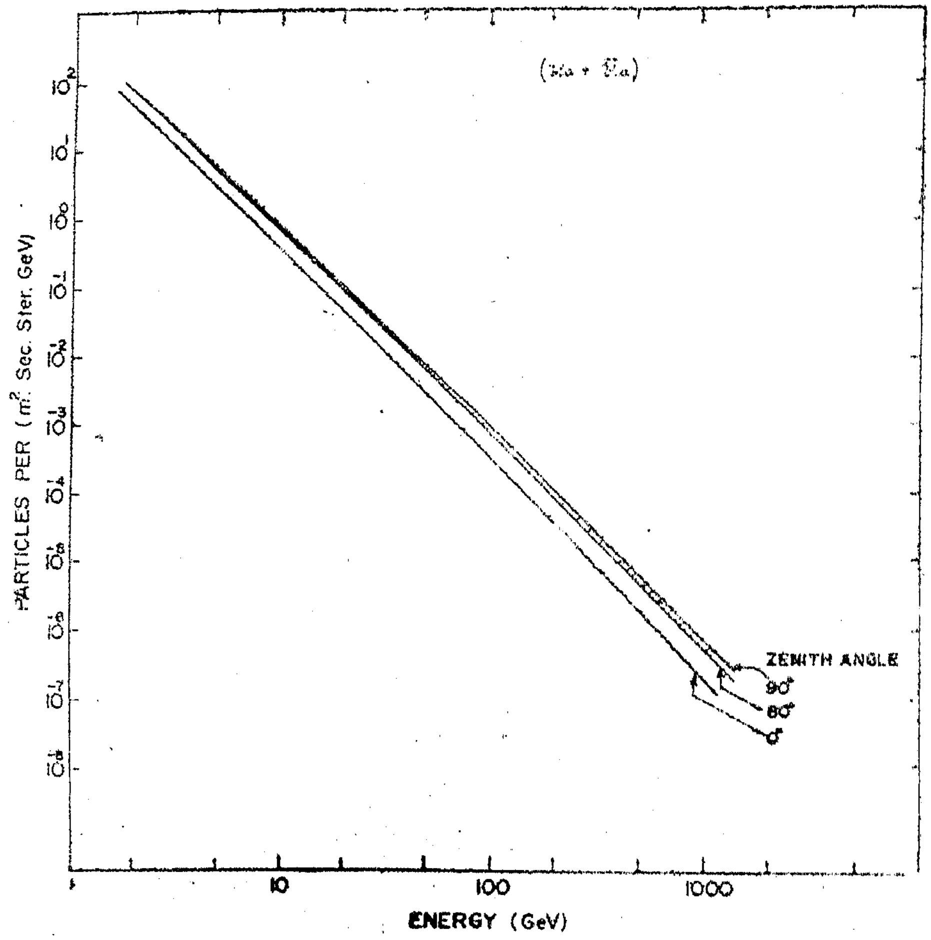

Fig. 5. **Flux of ($\nu_\mu + \bar{\nu}_\mu$) at zenith angles of 0°, 80° and 90°.**

IV. High Energy Neutrino Experiments Using Cosmic Ray Neutrinos

It is clear that we have a well-known though rather small flux of different types of neutrinos extending in energy up to thousands of GeV. and beyond. The question arises whether one may use this information to study the behaviour of neutrino interaction cross-section at high energies. One of the first discussions about the feasibility of detecting interactions of cosmic ray produced neutrinos was given by Markov and Zheleznykh.[6] They pointed out that such experiments may be done by detecting muons produced in overlaying layers of rock in a mine which is so deep that normal cosmic ray muon background is negligibly small. The pioneering experiments of

Miyake *et al.*[7] showed that such conditions are actually realised at depths $\sim$ 8400 meters water equivalent (m.w.e.). Menon *et al.*[8] analysed their experimental finding of no count in a 3 m.^{2} detector operated for 120 days at a depth of 8400 m.w.e. in terms of limits on the neutrino interaction cross-section. Since then several big experiments have been initiated to detect cosmic ray neutrions.[1, 2, 9] Most of the experiments aim at registering muons produced in earth's material by neutrino interactions. Since the penetrating power of electrons is small electron neutrinos would be detected efficiently only through the possible process $\bar{\nu}_e + e^- \rightarrow W^- \rightarrow \mu^- + \bar{\nu}_\mu$.

As discussed by Markov *et al.*,[6] Menon *et al.*[8] and Lee *et al.*,[10] a relatively high flux of neutrino induced muons could have been produced if the hypothetical intermediate boson had a mass $\lesssim$ 1 GeV. Recent GERN experiments[11] have demonstrated that if the intermediate boson exists, its mass must be greater than $\sim$ 1·8 GeV. Our preliminary estimate[12] based on the experimental results of Miyake *et al.*,[6] led us to a lower limit of $\sim$ 1·2 GeV. for the mass of the boson. Below, in subsection A, we shall calculate the integral energy spectrum of neutrino-induced muons, registered by a detector located at a large depth underground, under very general assumptions about the nature of neutrino interactions and about the behaviour of cross-section with energy. In subsection B we shall estimate the contribution to the muon flux due to production of the intermediate boson of different masses.

A. Energy Spectrum of Neutrino-induced Muons

Flux of muons of type *j*, with a total energy greater than ϵ, incident on a horizontal area A, within a cone of half angle Θ, is given by:

$$F_{\mu j}(\epsilon, k) = 4\pi A \int_{\theta=0}^{\Theta} \cos\theta \int_{E=\epsilon/k}^{\infty} \frac{F_{\nu j}(E, \theta)}{L_j(E)} \int_{r=0}^{(kE-\epsilon)/b} dr \cdot dE \cdot d\cos\theta \qquad \text{IV (1)}$$

$$= 4\pi A \int_{\theta=0}^{\Theta} \cos\theta \int_{E=\epsilon/k}^{\infty} \frac{F_{\nu j}(E, \theta)}{L_j(E)} \, \frac{kE-\epsilon}{b} \cdot dE \cdot d\cos\theta. \qquad \text{IV (2)}$$

Here

L_j is the interaction length of neutrinos of type *j* (for producing muons of type *j*),

b is the energy loss of muons per unit distance (assumed independent of energy up to E $\sim$ 100 GeV.),

k is the fraction of the neutrino energy given to the muon.

For studying the qualitative behaviour of $F_{\mu j}(\epsilon, k)$ we substitute for $F_{\nu j}(E, \theta)$ its value in the direction normal to the detector area*, in which case the integral over $\cos\theta$ gives $\sin^2\Theta/2$. Further, we take the following general form of energy dependence for the neutrino interaction length (for ν_μ and $\bar{\nu}_\mu$):

$$L(E) = \frac{\lambda_0}{E^n} \qquad E < E_c \qquad \text{IV (3)}$$

and

$$L(E) = \frac{\lambda_0}{E_c^n} \qquad E \geqslant E_c \qquad \text{IV (4)}$$

where λ_0 is the interaction length of neutrinos at 1 GeV. and $n \geqslant 0$. Dropping the subscript j and representing the differential spectrum of neutrinos by

$$\frac{N}{E^{\alpha+1}}\, dE \qquad \text{IV (5)}$$

over the entire energy range†, we have

$$F_\mu(\epsilon, k, E_c, n) = \frac{2\pi A \sin^2\Theta N k}{b\lambda_0 \alpha(\alpha - 1)} \cdot \phi\left(\frac{\epsilon}{k}, E_c, n\right) \qquad \text{IV (6)}$$

where

$$\phi\left(\frac{\epsilon}{k}, E_c, n\right) = \left[\frac{\alpha}{n - \alpha + 1}\left\{nE_c^{n-\alpha+1} - (\alpha - 1)\left(\frac{\epsilon}{k}\right)^{n-\alpha+1}\right\} - \frac{(\alpha - 1)}{(n - \alpha)} \cdot \frac{\epsilon}{k}\left\{nE_c^{n-\alpha} - \alpha\left(\frac{\epsilon}{k}\right)^{n-\alpha}\right\}\right]$$

$$\text{for } \frac{\epsilon}{k} < E_c \qquad \text{IV (7)}$$

and

$$= \left(\frac{E_c}{\frac{\epsilon}{k}}\right)^n \cdot \left(\frac{\epsilon}{k}\right)^{n-\alpha+1} \qquad \text{for } \frac{\epsilon}{k} \geqslant E_c. \qquad \text{IV (8)}$$

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* Equation IV.2 is strictly valid for flux on a horizontal area, because the neutrino flux is taken to be independent of the azimuthal angle. For a vertical area, the neutrino flux will depend on the zenith angle (measured with respect to the normal to the detector surface) as also on azimuthal angle.

† A single power law representation has been used for this general argument; it is easy enough to generalise this to the situation when two power laws need to be used, though for neutrinos of energy below 100 GeV. one power law representation is quite adequate. Note that the flux of neutrino-induced muons is approximately proportional to k.

For the total muon-neutrino spectrum up to ~ 100 GeV. in the horizontal direction (Fig. 5) we have (*see* Table III) $a = 1{\cdot}89$ and $N = 576$ ($m.^{-2}$ sec.$^{-1}$ ster.$^{-1}$ GeV.$^{-1}$). Using these values we have plotted $\phi\,(\epsilon/k, E_c, n)$ as a function of ϵ/k for three values of n ($n = 0$, 1 and 2) and different values of E_c (Fig. 6). This gives the expected energy spectrum of muons under different situations, in units of $\bar{A} \equiv [2\pi A \sin^2\theta N k/b\lambda_0 a\,(a-1)]$. It is clear that for most situations it is essential to measure the actual energy spectrum of muons and not only the flux of muons above a certain threshold of energy, in order to make any reasonable deductions about the behaviour of neutrino cross-section with energy. Up to muon energy ≈ 10 GeV. this may be done using a solid detector system of the type envisaged by Keuffel.[9]

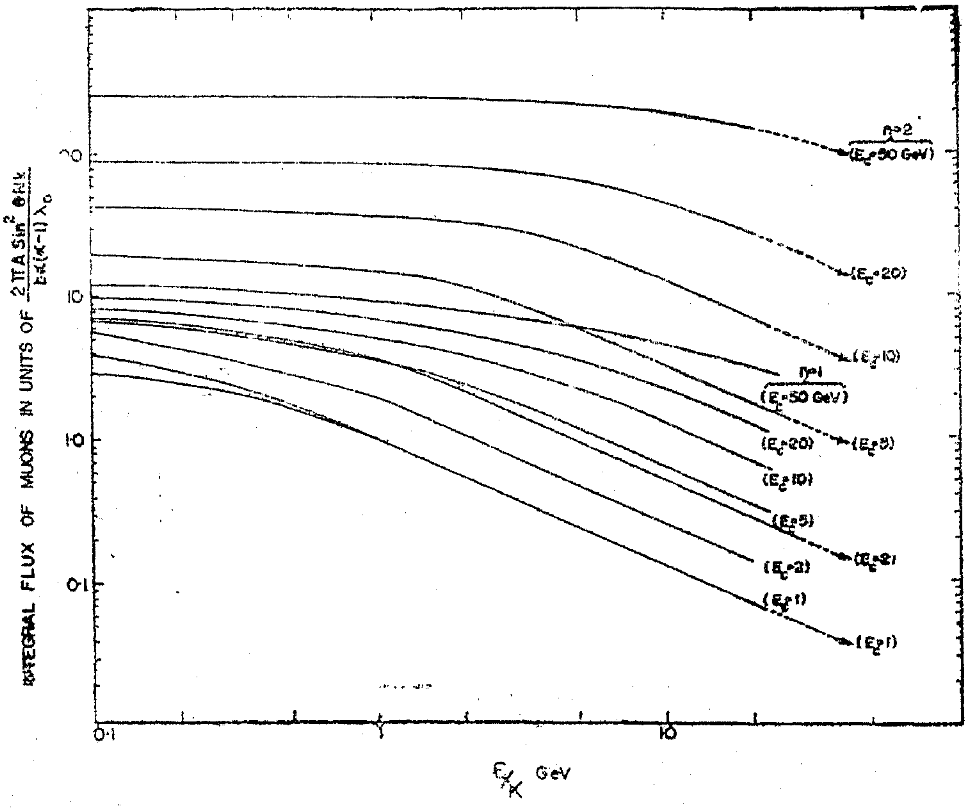

FIG. 6. Integral energy spectrum of neutrino-induced muons for inelastic neutrino cross-section increasing as $\sigma_0.E^n$ to saturation at E_s, where $\sigma_0 = 0{\cdot}34\times10^{-38}$ cm.^{2}/nucleon is the cross-section at 1 GeV. The ordinate is in units of $\bar{A} \equiv 2\pi A \sin^2\theta\, Nk/b\lambda_0 a\,(a-1)$, (*see* text).

If one uses the latest experimental results from CERN,[11] one can limit drastically the number of situations to be considered. For example it appears that

(i) the cross-section for elastic production of muons saturates around 1 GeV. at a value of $0 \cdot 5 \times 10^{-38}$ cm.^{2}/nucleon;

(ii) that the inelastic cross-section from 1 GeV. to 10 GeV. is given by $0 \cdot 34 \times 10^{-38}$ E cm.^{2}/nucleon, where E is in Gev.

Therefore for the inelastic cross-section one may now assume $n = 1$ and study the correlation of E_c, the saturation energy, and k, the fraction of the neutrino energy given to the muon, with the expected muon energy spectrum. This is shown in Fig. 7.

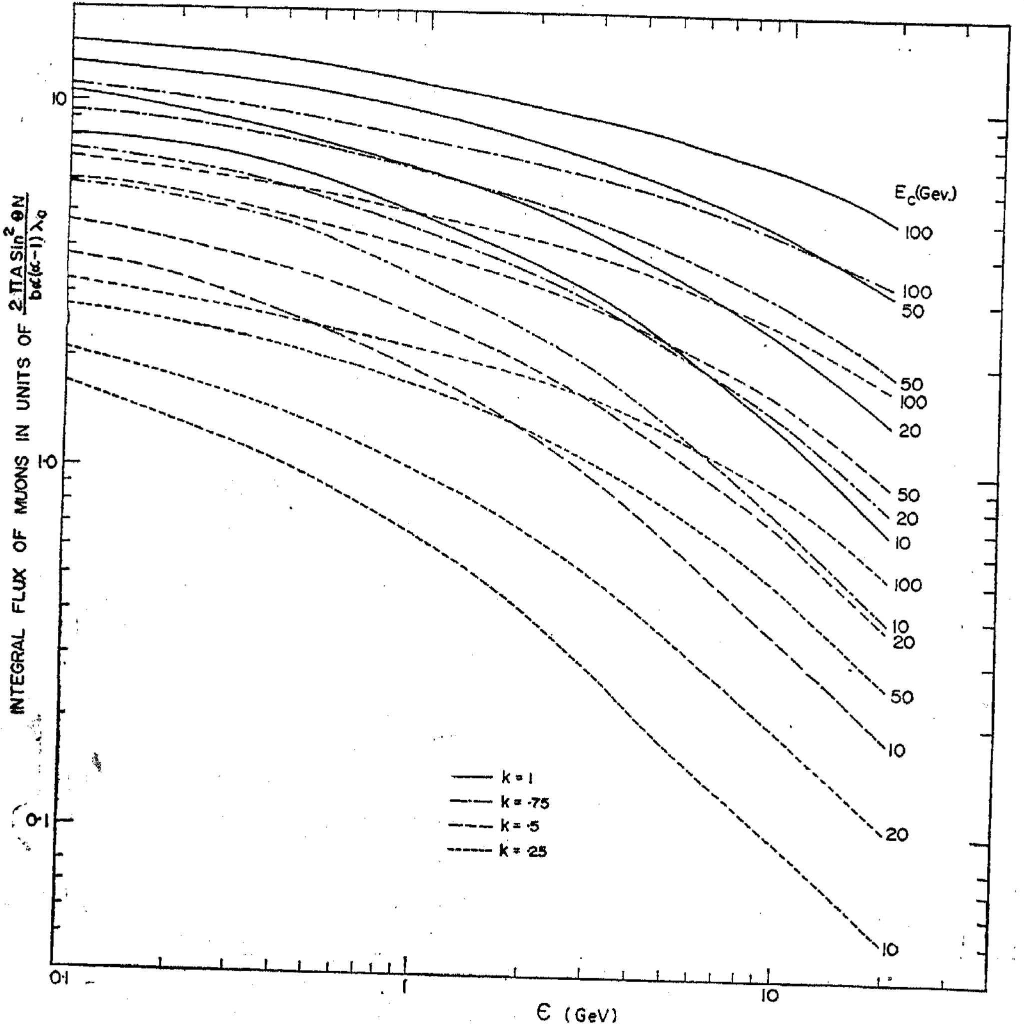

FIG. 7. Integral energy spectrum of neutrino-induced muons for linear increase of inelastic neutrino cross-section to saturation at E_c for different values of k where k is the fraction of neutrino energy transferred to the muon. The ordinate is in units of $2\pi A \sin^2\Theta\ N/b\lambda_0\alpha\,(\alpha - 1)$, (*see* text).

For $n = 1$, $E_C = 10$ GeV., $k = 1$, $\epsilon = 0 \cdot 5$ GeV. and A $\sin^2\Theta = 100\,m^2$ (vertical area), the expected counting rate due to inelastic neutrino interactions will be ~ 40/yr. The counting rate does not change very much by lowering the threshold energy of muons from 500 MeV. to 100 MeV.

B. Contribution through the Production of the Charged Intermediate Boson

If the charged intermediate boson, W, exists, then it can be produced in the earth's crust by muon neutrinos interacting coherently and incoherently with rock nuclei and also by electron neutrinos interacting with electrons in the atomic shells. All these processes would make different contributions to the muon flux deep underground; we shall consider them one by one.

Coherent Production by Muon-Neutrinos.—The cross-sections for the coherent process

$$\nu_\mu (\bar{\nu}_\mu) + {}^{63}_{29}Cu \rightarrow {}^{63}_{29}Cu + W^+ (W^-) + \mu^- (\mu^+) \qquad \text{IV (9)}$$

for M_W up to 1·8 GeV. have been calculated by Von Gehlen.[13] Using these, cross-sections for the material of the earth's crust were estimated by scaling the values down by a factor $(Z^2/A)_{rock}/(Z^2/A)_{cu}$, assuming a value of $(Z^2/A)_{rock} = 5 \cdot 3$; this procedure is justified because cross-sections become form-factor independent at high energies.[13] The cross-sections for $M_W = 2 \cdot 4$ GeV. and 5·0 Gev. were obtained by a point to point extrapolation at different energies. These extrapolated cross-sections as well as Von Gehlen's values for 1·8 GeV. mass are given in Fig. 8.

Using the flux of neutrinos $(\nu_\mu + \bar{\nu}_\mu)$ in the horizontal direction, and the cross-sections given in Fig. 8 in expressions of the type IV.2*, flux of muons deep underground arising from coherent intermediate boson production has been calculated. It is assumed that $\mu^\pm$ in reaction IV.9 carries a fraction $k = 0 \cdot 65$ of the neutrino energy; this is chosen so as to include the contribution of $W^\pm$ decay to the flux of muons.† The flux values for muons of energy greater than 500 MeV. are given in Table V, for the three values of M_W.

Incoherent Production by Muon-Neutrinos.—Again we use Von Gehlen's calculations for the process.

$$\nu_\mu (\bar{\nu}_\mu) + p \rightarrow p + W^+ (W^-) + \mu^- (\mu^+) \qquad \text{IV (10)}$$

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* At energies above a few hundred GeV. the energy loss of muons in rock increases and $\langle b \rangle$ depends on the energy of the muon; the values for the same have been taken from Ramana Murthy's Ph.D. thesis.[21] This non-linearity in range energy relation is significant especially for muons from Glashow process.

† Lee *et al.*[14] have given the energy spectrum of W produced by neutrinos, for $E >> M_W$. This gives $\langle E_W/E_\nu \rangle \approx 0 \cdot 43$ and hence $k \approx 0 \cdot 57$ for the muons. If $\Gamma(W \rightarrow \mu\nu)$ is $\frac{1}{3}$, k_{eff} becomes ~ 0·65. Contribution of muons from W decay is important for estimating the flux of muon pairs through the apparatus.

and extrapolate them to obtain cross-sectons for higher values of M_W. These are given in Fig. 8. Proceeding as before, it is found that the contribution of the incoherent process to the muon flux varies from ~50% (for $M_W = 1\cdot8$ GeV.) to ~100% (for $M_W = 5$ GeV.) of that of the coherent process.* The expected flux for different values of M_W is given in Table V.

Resonant Production in $\bar{\nu}_e$-Electron Collisions (Glashow Process). The resonant production process

$$\begin{aligned} \bar{\nu}_e + e^- \rightarrow W^- &\rightarrow \mu^- + \bar{\nu}_\mu \\ &\rightarrow e^- + \bar{\nu}_e \\ &\rightarrow \pi^- + \pi^0, \mathrm{S} \end{aligned} \qquad \mathrm{IV}\,(11)$$

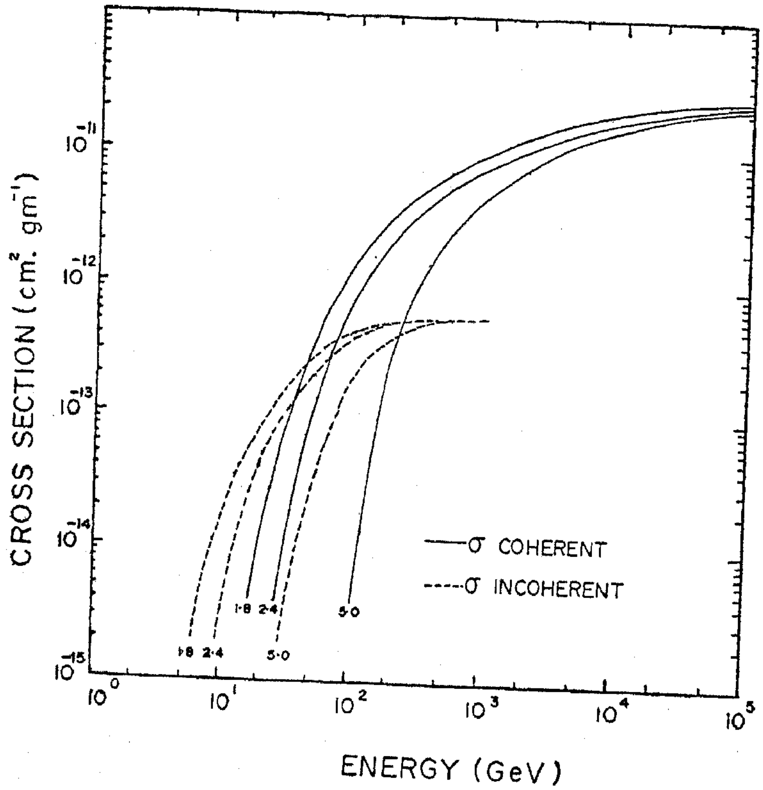

FIG. 8. Cross-sections for the coherent and incoherent production of charged intermediate-bosons for $M_W = 1\cdot8$, $2\cdot4$ and $5\cdot0$ GeV., as extrapolated from von Gehlen's[13] calculations. The values given are in units of cm.² per gm. of rock with $\langle Z^2/A\rangle = 5\cdot3$.

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* Here most of the contribution to the μ-flux comes from neutrinos of energies which may not be high enough for the asymptotic formula of Lee, *et al.*,[14] to be valid. Hence k could be slightly lower than $0\cdot65$. We have not taken into account the production of W by incoherent scattering of neutrinos on neutrons.

was first studied by Glashow.[15] Bahcall and Frautschi[16] and Bahcall[17] have stressed the importance of this process for detection of extraterrestrial neutrino sources, because the relative flux of very high energy $\bar{\nu}_e$ is expected to be much higher in neutrinos from these sources than in the cosmic ray neutrinos.

TABLE V

Counting rates for muons deep underground in the horizontal direction due to coherent and incoherent $W^{\pm}$ production

Mass of intermediate boson	Flux of muons (m.$^{-2}$ ster.$^{-1}$ year^{-1})	
	Coherent scattering	Incoherent scattering
1·8 GeV.	·27	·14
2·4 GeV.	·15	·09
5·0 GeV.	·04	·04

The intrinsic cross-section for the process IV.11 has a Breit-Wigner form with a half width directly proportional to the fourth power of the boson mass and a peak value inversely proportional to the square of the boson mass.[18] The value of the cross-section Σ_B, integrated over the resonant peak becomes

$$\Sigma_B = 2{\cdot}65 \times 10^{-28}\, M_W^2 (\text{GeV. cm.}^2/\text{electron})$$

where M_W is expressed in GeV. Muon production cross-sections will be a third of this if all the decay modes of the boson (equation IV.11) are equally probable.

The resonant neutrino energy E_R is equal to $982 \times M_W^2$ GeV. Intrinsic half width of the resonance is very small ($\sim 1{\cdot}5\ M_W^4$ MeV.); hence if the electrons were at rest, the peak value of the cross-section would be very high and the $\bar{\nu}_e$ of resonant energy would be absorbed in going through a few hundred gm./cm.^{2} of rock. However, due to the finite velocity of electrons in atomic shells the effective half width of the resonance is very large and is proportional to M_W^2. As discussed by Bahcall and Frautschi[16] for electrons of earth's crust this width is 28 M_W^2 GeV. and the peak value of the cross-section becomes 3×10^{-30} cm.^{2} per electron, corresponding to an interaction mean free path of ~ 11000 m.w.e. We would like to emphasise here the importance of using the exact electron velocity distribution in the atomic shells for calculating the attenuation of $\bar{\nu}_e$ flux at energies where reaction IV.11 is possible. For experiments operating at large depths *this attenuation is very sensitive to the electron velocity distribution.*

If we accept the mean free path of 11000 m.w.e., then in experiments operating at depths of two kilometers or more the muon flux at large zenith angles arising from resonant production of the intermediate boson would be negligible.

V. DISCUSSION

A. *Uncertainties in Neutrino Flux Estimates**

(i) An important uncertainty in the neutrino fluxes arises from lack of a precise knowledge of (K/π) ratio. However it is unlikely that this leads to more than a 15% error in the estimation of fluxes at large zenith angles. We have neglected the contribution due to pions arising from kaon decay for calculation of the neutrino flux. This contribution is estimated to be less than 5%.

(ii) Pion production spectra are fits to the observed muon spectra, which have errors $\gtrsim 20\%$ at energies above 400 GeV. Roughly speaking same uncertainties would apply to the neutrino fluxes above 100 GeV.

(iii) Below ~ 2 GeV. the neutrino flux will be affected by the geomagnetic cut-off on the primary cosmic rays. A precise evaluation of the spectrum below 2 GeV. in any direction will require a knowledge of the effective geomagnetic cut-off as also a much more detailed calculation taking into account all fluctuations in the production process. This has not been done so far. However, for a horizontally operating telescope the cut-off effects should be noticeable only below 1 GeV. if the telescope faces north-south direction and if the telescope faces east-west direction the reduction in total flux of neutrinos above 1 GeV. (traversing the telescope in both directions) would be much less than a factor of 2.

A 20% uncertainty in the neutrino fluxes is not of much consequence while calculating the expected counting rates for cosmic ray neutrino experiments, because the current estimates of the cross-section are much more uncertain and the statistical accurcy of the presently operating experiments would also be poorer.

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[*] After a reprint of this work had been circulated, Osborne *et al.*[19] published a calculation of cosmic ray neutrino fluxes where they made a comparison between their results and ours. We find that our fluxes at different energies are 10 to 20% lower than theirs. A large part of this difference arises due to slightly different pion production spectra assumed in the two calculations. Unlike them we have not fitted an exact power law to the production spectrum all through the atmosphere; this is because pion production by pions becomes operative only in the dense regions of the atmosphere and contribution of "fireball" pions is important only at low energies.[4]

B. *Possible Existence of the Intermediate Boson* (*See* the note added in proof also)

While this manuscript was in preparation, Achar *et al.*[20] reported the results of a successful four-month run of their neutrino telescope operating in Kolar Gold Mines at a depth of 7000 m.w.e. They observed four events in which muons traverse the telescope at zenith angles greater than 37° in an effective exposure of $\sim$ 2820 m.^{2} ster. days. From the zenith angle distribution of these events they conclude that possibly all four and at least three of them are due to neutrino interactions.

On the basis of the calculations presented in Section IV, we have estimated the expected number of events, arising due to different reactions, for their experimental conditions. We use the neutrino flux at 80°; this is close to the mean flux over the aperture of their telescope. The results are given in Table VI.

TABLE VI

Expected counting rates of muons due to various processes in K.G.F. telescope
$(Z^2/A)_{\text{K.G.F.}} = 6\cdot5$

Reaction	Remarks	Expected counts for 2820 m.^{2} day ster.
(1) Elastic scattering	$k = 1$	0·13
(2) Inelastic scattering	$\sigma = 3\cdot4\times10^{-39}$ E for E $<$ 100 GeV.	
	$= 3\cdot4\times10^{-37}$ for E $\geqslant$ 100 GeV.	
	$k = 0\cdot65$	0·47
	$k = 1\cdot0$	0·94
	$k = 0\cdot65$	
(3) Coherent boson production by $(\nu_\mu + \bar{\nu}_\mu)$	$M_w = 1\cdot8$ GeV.	2·09
	$M_w = 2\cdot4$ GeV.	1·08
	$M_w = 5\cdot0$ GeV.	·29
	$k = 0\cdot65$	
(4) Incoherent boson production by $(\nu_\mu + \bar{\nu}_M)$	$M_w = 1\cdot8$ GeV.	·88
	$M_w = 2\cdot4$ GeV.	·54
	$M_w = 5\cdot0$ GeV.	·22
(5) Resonant boson production by $\bar{\nu}_e$	$k = 0\cdot5$	
	$M_w = 1\cdot8$ GeV.	<0·1
	$M_w = 1\cdot8$	3·6
	$M_w = 2\cdot4$	2·2
Total	$M_w = 5\cdot0$	1·1
	$M_w = \infty$	0·60
	$k = 0\cdot5$ for inelastic scattering	

From Table VI one sees that if the inelastic scattering cross-section increases as E up to 100 GeV. and saturates thereafter (remembering that the CERN results[11] preclude a faster increase up to 10 Gev.) the expected number of counts is less than 0·9 compared to four counts obtained. Thus one is forced to conclude that the cross-section becomes larger than $0 \cdot 34 \times 10^{-38}$ E at some energy E greater than 10 GeV. Intermediate boson production can provide a channel for such an increase. Therefore it is tempting to assume that the high counting rate of Achar *et al.*,[20] is due to the existence of the postulated intermediate boson which has a mass not much greater than 1·8 GeV. The fact that in one of the four events observed by these authors two nearly parallel tracks traverse the telescope at a zenith angle of $\sim 97°$ lends support to this attractive conclusion.[20] Further results, within next few months, may be conclusive in this respect.*

Note added in proof.—When later results of the experiment of Achar *et al.* [*Proc. of the Int. Conf. on Cosmic Rays*, London (1966)] are combined with those of Reines *et al.* [*Proc. of the Int. Conf. on Cosmic Rays*, London (1966)] one obtains 12 events for a total exposure of ~ 29000 m.^{2} ster. days. We also find that the neutrino fluxes may be $\sim 40\%$ higher than those given in this paper if the 100 GeV. muon flux is taken to be that given by the depth intensity curve. If one tries to explain the observed counting rate in terms of an E dependence of the cross-section upto a certain energy E_c, then E_c may be between 100 and 1000 GeV. If this were so, no intermediate boson would be required. This shows that present statistical accuracy of experiment is insufficient to prove or disprove the existence of a boson of mass $\gtrsim 2 \cdot 5$ GeV. However a mass as low as 2 GeV. is ruled out irrespective of the decay mode of the boson.

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* At present it cannot be excluded that an extraterrestrial high energy neutrino flux larger than the cosmic ray flux is incident on earth. This could be excluded by a study of siderial time distribution for the observed events (*See* also Appendix A).

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APPENDIX A

DETECTION OF EXTRA-TERRESTRIAL NEUTRINO SOURCES

The emission of radio and optical synchrotron radiation by many stellar bodies indicates the presence of high energy electrons in their envelopes. If majority of these electrons arise in the decay of mesons produced in high energy nuclear collisions, the intensity of synchrotron radiation is related to the intensity of γ-rays and neutrinos emitted by these sources. Bahcall and Frautschi[16, 17] have used this approach to estimate the neutrino flux from some of the intense radio sources and the expected flux of muons produced by resonant scattering of $\bar{\nu}_e$ on electrons in the earth's crust. They find that for some of the sources (M 82 and 3C 48 for example) the muon flux from this process may be as high as 1–10 per meter^2 per year at a depth of 1 km.

Recently a number of experiments have put rather stringent upper limits on the flux of high energy (> 5000 GeV.) γ-rays from some of the intense radio sources in the sky; these results have been summarised by Garmire and Kraushaar.[22] Using these limits and assuming pions to be the main source for the production of γ-rays and neutrinos, it is again possible to assign upper limits to the flux of neutrinos of corresponding energy. Thus in Table VII

TABLE VII

Upper limits on γ-ray flux, $\bar{\nu}_e$ flux and the flux of muons produced by resonant scattering of neutrinos, from a few extraterrestrial sources

Source	γ-rays[22] (m.$^{-2}$ year^{-1}) (E > 5×10^3 GeV.)	$\bar{\nu}_e$ (m.$^{-2}$ year^{-1} Gev.$^{-1}$) $\times$(E/1000)$^{2\cdot67}$	Muons (m^{-2}. year^{-1}) at 7000 m.w.e.		
			M_W=1·8	2·4	5·0 GeV.
Cygnus A ..	16	·059	·042	·023	·004
Cassiopia A	16	·059	·042	·023	·004
Virgo A ..	16	·059	·042	·023	·004
3C 196 ..	16	·059	·042	·023	·004
Taurus A ..	32	·12	·084	·047	·008
3C 147 ..	32	·12	·084	·047	·008
3C 273 ..	95	·35	·25	·14	·023

we have tabulated, for various sources, the upper limit on the γ-ray flux, the corresponding limit on $\bar{\nu}_e$ flux and the limit on muon flux* at a depth of 7000 m.w.e. due to resonant production of the intermediate boson of mass 1·8, 2·4 and 5 GeV.

It is seen that this approach puts more stringent upper limits on the neutrino fluxes, than does the procedure used by Bahcall and Frautschi.[16, 17] The detection of neutrino signals from sources of the type listed in Table VII would therefore, require more ambitious experiments than those at present in operation.

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* We have used a differential spectral index of -2.67 for the γ-rays (same as for cosmic rays). However since the resonant energies are in the neighbourhood of 5000 GeV., a slight change in the value of the spectral index will not seriously affect our estimates.