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Dark Matter—Systematics of its Distribution

R. Cowsik & M. Vasanthi *Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Bombay 400005*

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Abstract. The dynamical masses of dwarf-spheroidals, spiral and elliptical galaxies, dwarf irregular binaries, groups of galaxies and clusters are shown to lie in a band about the $M \sim \rho R^3$ *line.* The value of ρ is approximately the same as that estimated for unseen matter in the solar neighbourhood. The clusters themselves lie about the $M \sim R^{-3}$ *line* derived for a self-gravitating neutrino gas; their masses are distributed around the maximum Jeans-mass, M_{Jmax}. corresponding to $m_\nu \simeq 10$ eV in an expanding universe. The present-day length scales of clusters and the dispersion in the velocities observed within them are understood in terms of a 100-fold expansion subsequent to the initial growth of the fluctuations at M_{Jmax}. These systematics on the *R*-*M* plane imply that the initial condensations in the expanding universe are on the scale of the rich clusters of galaxies, these condensations were triggered dominantly by the gravitation of the neutrinos and the constant density of all systems arises naturally due to the embedding of these systems in the large scale neutrino condensations. If the neutrino density falls off as r^{-2} beyond the cluster edge till the distributions from different clusters overlap, then the mean density of the neutrinos approximately equals the closure density of the universe.

Key words: galaxies, dark matter—galaxies, formation—neutrino, mass

1. Motivation

The study of the nature and distribution of dark matter which dominates the gravitational dynamics of large astronomical systems and the universe itself has received much attention of both theorists and observational astronomers in recent years. The primordial nucleosynthesis in the conventional big-bang models shows that this unseen matter, if baryonic, will yield too little deuterium (Peebles 1966; Yang *et al.* 1979; Olive & Turner 1981; Olive *et al.* 1981; however, see Stecker 1980; Rana 1982). Also high baryonic densities will cause distortions in the relic blackbody radiation to unacceptable levels (Silk 1967; Silk & Wilson 1981; Fall 1980) and pose insurmountable difficulties in theories of formation of galaxies. These considerations rule out the possibility that the unseen matter is in the form of black holes or condensates of cold matter like 'Jupiters' which might have arisen from first generation stars. The alternate hypotheses which have been considered are nonbaryonic relics of the big-bang such as massive neutrinos, gravitinos, photinos, monopoles, primordial monopoles, *etc.*

(Cowsik & McClelland 1972, 1973; Page's & Primack 1982; Olive & Turner 1982; Cabibbo, Farrar & Maiani 1981; Peebles 1982; Sikivie 1982).

Do the observations favour one or the other of these hypotheses? What is the typical scale of the initial condensations? Did the small objects typically of the size of globular clusters form first and later grew by hierarchical clustering (Peebles 1984) into the largest astronomical systems? Or did the large-scale condensations occur first and the density perturbations grew inside these to generate the smaller units? (See, Zeldovich 1970; Cowsik & McClelland 1973; Bond; Efstathiou & Silk 1980; Sato & Takahara 1980; Doroshkevich *et al.* 1980; Wasserman 1981.)

We have reviewed critically the relevant observations which are available and our analysis of their systematics suggests and supports the latter scenario. In our view the present day universe is made up of neutrino condensations typically of the dimensions of the clusters. Beyond the cluster edge the density falls off in such a way as to provide the closure density for the universe. The visible baryonic matter constituting less than 10 per cent of the mass is embedded in such a background and through its kinematic motions delineates the gravitational potential of the inhomogeneous neutrino background.

2. Dynamical masses of astronomical systems

The first evidence that there exists some form of 'invisible matter' dominating the dynamics of extended astronomical systems was obtained about half a century ago by Zwicky (1933) and Smith (1936) studying the Coma and Virgo clusters of galaxies. The point was that the dynamical mass of systems estimated from the dispersion in the velocities of the constituent elements:

$$M_d \simeq \langle v^2 \rangle r/G \tag{1}$$

exceeded substantially the mass estimated from visible luminosity. Since then enormous amount of careful observations has gone on not merely in the field of optical astronomy but in the radio (21 cm), infrared, UV and X-ray bands which has broadened the implications of the title 'Invisible Matter' and has shown that the dynamical effects of this unseen component is felt almost ubiquitously in all large astronomical systems. Detailed reviews on this subject are available in literature (Peebles 1979; Bahcall 1977; Faber & Gallagher 1979) and we summarise below the relevant information. Our studies of the systematics differ in an important way from earlier studies in that we consider the dependence of the dynamical mass on radius rather than that of mass to luminosity (M/L) ratio. Since the mass is dominated by the invisible component and the luminosity by the baryons, a one-to-one correspondence does not exist between them and taking ratios introduces large dispersion in their distribution; also the ratio is less amenable to direct interpretation.

2.1 *Clusters of Galaxies*

Peebles (1965) has reviewed the subtleties in the application of Equation (1) to astronomical systems in estimating their dynamical or virial masses.

Detailed study of the Coma cluster (Rood *et al.* 1972; Kent & Gunn 1982) and parameters of seven other Abell clusters are available (Dressler 1978). In several cases the authors have quoted only the M/L ratios derived from the dispersion in the velocities and effective radii derived from fits to the profiles of luminosity; since the net luminosity is independently known, the dynamical masses have been calculated in a straightforward manner. These are plotted in Fig. 1 (filled circles).

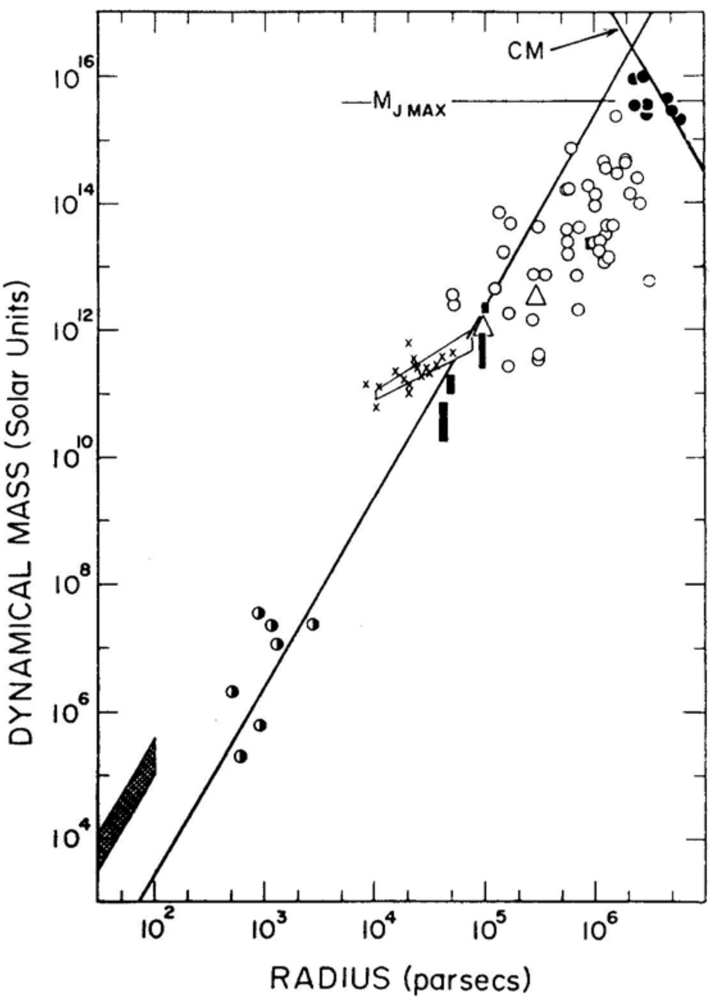

Figure 1. The dynamical masses of rich clusters of galaxies (●), groups of galaxies (○; Δ), dwarf irregular binaries (▮), spiral galaxies (×, ▱) and dwarf spheroidals (◑) are shown against the visible radius. The hatched region in the lower left hand corner is the mass of a sphere with a given radius and the density estimated from Oort-parameters in the solar neighbourhood. The dark solid line along the diagonal is the $M \sim \rho R^3$ line for $\rho \simeq 10^{-3}\ M_{\odot}/\mathrm{pc}^3 \simeq 6 \times 10^{-26}\ \mathrm{g\ cm}^{-3}$; it passes through the density estimated at the edge of our Galaxy from the rotation curve and through the M 31 group. The line marked CM is based on the dynamics of a self-gravitating neutrino cloud and is adopted from Cowsik & McClelland (1973) for $m_\nu \simeq 10\ \mathrm{eV}/c^2$. The horizontal line labelled M_{Jmax} is the maximum Jeans mass of neutrinos of mass $10\mathrm{eV}/c^2$.

2.2 *Groups of Galaxies*

A rather comprehensive and systematic study of groups of galaxies by Rood & Dickel (1978) yields data on 39 groups (Sandage & Tammann 1975; de Vaucouleurs 1975; Turner & Gott 1976) containing two to 238 members. Using Equation (1) the dynamical mass was estimated for all but two of these and the results are shown in Fig. 1 (open circles). The two which have been left out have identification numbers 40 and 91 and have 'crossing times' as long as the age of the universe. Five of these lie in the region populated by the clusters listed above but one is not shown because of overlap. We have also included data on two groups associated with M 31 and M 101 whose dynamical masses were calculated using the projected-mass method by Bahcall & Tremaine (1981).

2.3 *Binary Pairs of Dwarf Irregular Galaxies*

Lake & Schommer (1984) have studied the dynamical mass-to-light ratios of 9 binaries from the catalogues of DDO dwarfs (van den Bergh 1959; Nilson 1973) with velocities appearing in the compendium by Huchtmeier *et a!.* (1983). Out of these we select six pairs with the masses determined accurate to better than 50 per cent and with crossing times small compared to the age of the universe. The central values of the mass are uncertain typically by a factor of 2 due to projection effects and the unknown eccentricity of the orbit.

2.4 *Spiral Galaxies*

Very important insights into the structure and dynamics of the galaxies have come from systematic and meticulous observations of their rotation curves (Rubin 1979; Rubin *et al.* 1980, 1982; Rubin, Thonnard & Ford 1982; Bosma 1978). A universal feature of all rotation curves is that at large galactocentric distances they are either flat or faintly rising, and in fact there is no galaxy whose rotation curve falls. Since the rotational velocities v^2 are proportional to $GM(r)/r$, these observations imply the presence of much unseen mass up to large distances from the centre of the spirals. In Fig. 1 we have plotted a random selection of the masses of spirals measured up to the termini of the observations. For our own Galaxy we have shown how the dynamical mass increases with radius (Peebles 1979; Faber & Gallagher 1979). This progressive increase in the dynamical mass with radius is a characteristic feature of all spirals and constitutes an important clue as to the nature of the dark matter (Cowsik & Ghosh 1984a).

2.5 *Dwarf Spheroidal Galaxies*

There are seven dwarf spheroidals (DS) which have been studied recently (Aaronson 1983; Lin & Faber 1983; Faber & Lin 1983). It has been possible to measure the velocities of a few of the constituent stars in the Draco and Ursa Minor systems. These measurements as well as criteria for tidal stability have been used to estimate the dynamical masses of these objects. These are the smallest astronomical objects that bear evidence to the presence of gravitating dark matter in them. These have typical radii of ~ 1 kpc and masses of ~ 10^7 $M_\odot$. We have discussed elsewhere in some detail the luminosity profiles of these spheroidals and the nature of dark matter in these systems (Cowsik 1986; Cowsik & Ghosh 1986a). Even now, we would like to remark that the mass determination by Lin & Faber are very tentative and are subject to change. In Fig. 1 we plot the masses as published by the authors cited here. Our own mass estimates (Cowsik & Ghosh 1986b) differ by about a factor of 2 but do not generally change the basic picture presented here.)

2.6 *Oort-Bahcall Limits on Unseen Matter near the Sun*

The very early work of Kapteyn (1922) in estimating the total matter near the Sun has been put on a much firmer footing by Oort (1932, 1960), and more recently by Bahcall (1984a, b) who solves the combined Poisson-Boltzmann equation for the gravitational potential of the Galaxy modelled in terms of several isothermal 1disc components in the presence of a massive unseen halo. Fitting the distribution of F stars and K-giants Bahcall obtains definitive constraints on the matter distribution, and estimates on the total matter density and the column density near the Sun which will be called the Oort–Bahcall limit. From this we might subtract out the observed components like the stars on the main sequence, giants, white dwarfs, interstellar gas and dust, *etc*., to obtain an estimate of the mean density for the unseen matter near the Sun; this is in the range 0.09 $M_\odot$ pc^{-3} to 0.03 $M_\odot$ pc^{-3}. In Fig. 1 we plot a mass equal to (4/3) $\pi R^3 \rho_{\text{unseen}}$ (hatched region on the lower left-hand corner).

3. Discussion of the systematics

The most striking aspect of the relation between the dynamical masses of the astronomical systems and their radii shown in Fig. 1 is the strong concentration along the $M \sim R^3$ line. The solid line $M \sim 2.5 \times 10^6$ $M_\odot$ $(R/\text{kpc})^3$ is drawn to fit the dynamical mass distribution near our Galaxy best. Notice that it passes through the dwarf spheroidals, the galactic mass up to ~ 50 kpc and the M 31 group shown as Δ. This correlation, which extends from the lightest of the dwarf spheroidals (Leo II) with a mass ~ 10^5 $M_\odot$ up to the rich clusters of galaxies with masses of ~ 10^{16} $M_\odot$, indicates that the density is roughly constant implying a common dynamical basis for all these varied systems. A systematic interpretation of this correlation is presented below.

3.1 *The Rich Clusters*

These have dynamical masses in the range 10^{15}–$10^{16} M_\odot$ and have limiting radii extending from 2 to 6 Mpc. The typical radii of cores of these clusters are in the range 0.1-0.3 Mpc, *i.e*., a factor of about twenty smaller:

$$R_{\mathrm{L}} \simeq 20\, R_{\mathrm{c}}. \tag{2}$$

For understanding their properties we start with our early analysis in terms of a self-gravitating cloud of neutrinos (Cowsik & McClelland 1973). Let us define a parameter α which represents the fraction of the available phase space filled by neutrinos on the average across the whole cluster. Then following our earlier analysis we find

$$m_\nu^8 = \frac{91.3\hbar^6}{G^3 g^2 \alpha^2 R_c^3 M_c}. \tag{3}$$

One should not confuse the parameter α with the actual phase space density, but it represents its average taken up to a boundary defined by the dynamical constraints relevant to the system. The parameter g has the usual interpretation as the multiplicity of occupancy possible in each level, every flavour, particle-antiparticle state, and helicity contributing to it.

$$g = 2\, g_{\text{flavour}} \cdot g_{\text{helicity}}. \tag{4}$$

We can estimate a in the following way. As noted earlier, in our picture the neutrinos are the dominant constituents of the universe and the calculations of their Jeans-mass (see *e.g.* Bond, Efstathiou & Silk 1980) show that it goes through a maximum M_{Jmax} in the expanding universe when neutrinos become quasi-relativistic. In other words, at the time of formation of the initial condensations

$$V_\nu \text{ (at } M_{\text{Jmax}}) \sim c/2. \tag{5}$$

The redshift $(1 + z_\nu)$ at which the neutrinos attain quasi-relativistic velocity is easily estimated from the fact that their momenta also scale the same way as that of the photons in an expanding universe:

$$(1 + z_\nu) \simeq 1000\, m_\nu\, (\text{eV}/c^2). \tag{6}$$

At this epoch the energy density in the thermal radiation is still comparable to that of neutrinos

$$\frac{\rho_\gamma}{\rho_\nu} \simeq \frac{(1+z_\nu)^4 \rho_\gamma(0)}{(1+z_\nu)^3 n_\nu(0) m_\nu c^2} \simeq 2. \tag{7}$$

As the universe expands, the radiation temperature drops and the radiation which is trapped in the initial condensation diffuses out of it. Thus the net gravitational binding drops and the condensate expands. The complete dynamics of the condensate is quite complicated (Gunn & Gott 1972) and is not fully understood; we fix here the nature of the expansion empirically. Now note that the present-day velocity dispersion in the cluster is 1000 km s^{-1} so that if the expansion had gone on adiabatically by a factor $s \simeq 100$ the quasi-relativistic neutrinos would now have this V_{rms}. Now, using the subscript i to designate the initial state,

$$\alpha_i = \frac{M_{\text{Jmax}}}{m_\nu g}\left(\frac{p^3_{(\text{max})i} R^3_{ci}}{h^3}\right)^{-1} \simeq \frac{1}{2}. \tag{8}$$

The value of $\alpha_i \simeq 1/2$ follows from the very definition M_J and from the assertion that the neutrinos were generated in thermal equilibrium (Tremaine & Gunn 1979). Now since the number of neutrinos in the condensate does not change during the expansion the present day value of $\alpha = \alpha_f$ is given by

$$\frac{\alpha_f}{\alpha_i} = \frac{R^3_{ci} v^3_{(\text{max})i}}{R^3_{cf} v^3_{(\text{max})f}} \simeq \frac{R^3_{ci}}{R^3_{cf}} \cdot \frac{\{G(M_{\text{Jmax}} + M_{\text{rad}})/R_{ci}\}^{3/2}}{\{G M_{\text{Jmax}}/R_{cf}\}^{3/2}}.$$

$$\simeq \left(\frac{2R_{ci}}{R_{cf}}\right)^{3/2}. \tag{9}$$

Using $s \simeq 100$ and $M_{rad} \sim M_{Jmax}$ as estimated in Equation (7), the present value of α turns out to be

$$\alpha_f \sim 3 \times 10^{-3}\, \alpha_i \simeq 1.5 \times 10^{-3}. \tag{10}$$

We adopt this value of α in Equation (3) and substitute Equation (2) in it and get

$$M_c \simeq 1.5 \text{ x } 10^{35}\, M_{\odot}\, (R_L/\text{kpc})^{-3}\, (m_\nu(\text{eV})/c^2)^{-8}. \tag{11}$$

This relation between the mass of the cluster M_c and its limiting radius R_L is shown in Fig. 1 for $m_\nu \simeq 10$ eV $/c^2$, named CM. It would appear therefore that m_ν should be about this to fit the masses of the clusters. Even though many of the observations, and the constants that appear in the theoretical analysis, are somewhat uncertain, the eighth power on m_ν in Equations (3) and (11) does not allow much spread in the values of m_ν. Our best estimate on the allowed range of the mass of the neutrino needed to fit the cluster masses is

$$8 \text{ eV}/c^2 < m < 25 \text{ eV}/c^2$$

$$m_\nu \text{ (best fit)} \simeq 10 \text{ eV}/c^2. \tag{12}$$

Let us now return to the estimates of the maximum Jeans-mass (Bond, Efstathiou & Silk 1980; Sato & Takahara 1980; Klinkhamer & Norman 1981; Wasserman 1981) M_{Jmax} and the value of the Jeans-length L_J at that epoch

$$L_J \simeq \frac{c}{\left[12\pi G\left(\frac{\rho_\gamma(0)}{c^2}\right)(1+z)^4\right]^{1/2}} \simeq \frac{300 \text{ kpc}}{(m_\nu(\text{eV})/c^2)^2}. \tag{13}$$

If we take $m_\nu \sim 10$ eV $/c^2$, $L_J \sim 2$ kpc at the time of the initial condensation and the predicted size of the radius of the core today is $R_c \sim sL_J \sim 100$ kpc in reasonable agreement. The total mass of all the neutrinos contained in a volume of radius L_J is

$$M_{Jmax} \simeq \frac{4\pi}{3} L_J^3 m_\nu \left[n_\nu(0) + n_{\bar\nu}(0)\right](1+z_\nu)^3 \simeq \frac{1.3 \times 10^{18} M_\odot}{(m_\nu(\text{eV})/c^2)^2}. \tag{14}$$

M_{Jmax} is shown in Fig. 1, again for $m_\nu \simeq 10$ eV $/c^2$, as a horizontal line. One again sees that the observed masses of the clusters are in accord with this estimate (10^{16}–3×10^{15} $M_\odot$).

The typical distance between clusters today, expected from these considerations, is given by

$$D_\nu \sim (1+z_\nu)\, L_J \sim \frac{3 \times 10^5 \text{ kpc}}{(m_\nu(\text{eV})/c^2)}. \tag{15}$$

This gives $D \sim 30$ Mpc for $m_\nu \sim 10$ eV $/c^2$, in qualitative agreement with observed sizes of voids. Numerical simulations of the process of neutrino condensation supporting the analytic study have been performed by Bond, Szalay & White (1983) and Frenk, White & Davis (1983). It is interesting to note that the choice $m_\nu \sim 10$ eV provides the closure density for the universe. In order to obtain in this picture the requisite mean number density of neutrinos, their density should fall off roughly as r^{-2} beyond the visible edge of the clusters of galaxies, at least statistically, till the contributions from different clusters overlap at a distance of ~ 15 Mpc. This requirement of a fall in density as r^{-2} conforms exactly with that of the asymptotic behaviour of an Emden sphere.

In concluding this sub-section we have to explain why we have called the constituents of dark matter as neutrinos, when any fermion with the mass in the allowed range of

Equation (12) would do. Besides many others, we are motivated by the fact that the coupling of these particles to ordinary matter and to each other should be considerably weaker than electromagnetic, but should not be too much weaker than the Fermi-coupling, lest the decoupling from thermodynamic equilibrium be too early and their number density become too low.

3.2 *The Groups*

The fluctuations in the density of visible matter will now grow inside the neutrino condensates described in the previous section. It is to be noted that even though the scale factor of the universe changes by $(1 + z_\nu) \sim 10^4$ since the formation of the condensate, the neutrino cloud itself expands only by a factor of ~ 100 so that the growth in the fluctuations in the density of visible matter will proceed with greater relative rapidity resulting in the formation of galaxies. Thus most of the galaxies are expected to be embedded in the neutrino clouds and only very rarely will they find themselves in the voids in between the clouds. The dynamics of groups of galaxies, in so far as their mean density is small compared with that of the neutrino cloud, will clearly reveal the gravitating effects of the neutrinos. In other words, the dynamical masses calculated using Equation (1) will be $M_\mathrm{d} \sim (4\pi/3)r^3\rho_\nu + M_\mathrm{v}$ where M_v is the visible mass interior to r. It is clear from Fig. 1 that the mean density of groups ($\sim 2 \times 10^{-5}\ M_\odot\ \mathrm{pc}^{-3}$) is the same as that of the rich clusters but the dispersion in the densities is somewhat larger. This is to be expected since the radial density distribution of neutrinos in a condensate is not uniform but has a maximum at the centre, falls gently up to the core radius R_c, beyond which it steepens rapidly. Thus the mean density of the clusters is bound between the central density and that at large radius, but the sub-units reveal the effects of the density of neutrinos at the radius of their location.

Of special interest is the M 31 group (shown as A) with a dynamical mass of $\sim 10^{12}\ M_\odot$ and a radius of $\sim 10^5$ pc (Bahcall & Tremaine 1981) since this would give an estimate of the density in the neighbourhood of our Galaxy. This is about $10^{-3}\ M_\odot \mathrm{pc}^{-3} \sim 6 \times 10^{-26}\ \mathrm{g\ cm}^{-3}$**.** This value is substantiated by studying the rotation curve of our Galaxy as well as the luminosity profiles of the dwarf spheroidals as we will see in the next section. The six dwarf irregular binaries which are shown as filled oblong rectangles also yield approximately the same density.

3.3 *Spirals and Dwarf Spheroidals*

There is one essential difference between these objects and the groups we discussed in the previous section. The mass density in the central regions of these is dominated by stars rather than neutrinos. But as we move to large radii, the stellar density drops progressively and neutrinos become dominant so that mass determined only on the largest scales will bear evidence to the presence of the neutrinos. We have studied elsewhere the dynamically self-consistent distribution of the stars in these systems embedded in a large cloud of neutrinos (Cowsik & Ghosh 1986a, b). This self-consistent distribution reproduces correctly the luminosity profiles of dwarf spheroidals. Also, the flat rotation curves of the spiral galaxies arises naturally in this picture when the potentials of both the stars and neutrinos are taken into account.

4. Conclusion

From these systematics we conclude that neutrino condensates with $M \sim 10^{16}\ M_\odot$ were the first objects to be formed in the universe at a redshift of $\sim 10^4$. Subsequent to formation they expanded much slower than rest of the universe and fluctuations in the density of baryonic matter grew effectively with the formation of the galaxies. The dynamics of these galaxies and that of the stars within galaxies are sensitive to the background gas. Assuming that neutrinos have a rest mass of ~ 10 eV/c^2 the details of the dynamical motions of the galaxies and the stars can be understood quantitatively. This picture is to be contrasted with the hierarchical picture where the large clusters are built up from much smaller objects (*e.g.* Peebles 1983). An observational test of this scenario appears possible as in this picture the intensity and spatial correlations of the redshifted 21 cm line will be different from that discussed by Sunyaev & Zeldovich (1975) and by Hogan & Rees (1979). Also, further studies on the expected fluctuation spectrum of the thermal microwave background based on this picture are warranted.

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