A Bound on Violations of Lorentz Invariance

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Recently Coleman and Glashow [1] have developed a model which allows the introduction of a small violation of Lorentz invariance. Observational signatures arise because this interaction also violates flavor conservation and allows the radiative decay of the muon, $\mu \rightarrow e + \gamma$, whose branching ratio increases as $b\ \gamma^4$ where γ is the Lorentz factor of the muon with respect to the reference frame in which the dipole anisotropy of the universal microwave radiation vanishes. In this paper we place a bound of $b < 10^{-25}$ based on observations of horizontal air showers with $n_e \geq 5 \times 10^6$. With such small values of b the proposed radiative decay of the muon will not affect the functioning of the muon collider. (THIS IS A PRELIMINARY VERSION)

To test by experiments the limits of validity of Lorentz invariance or indeed any of the fundamental principles of physics we need a theoretical model which assumes a specific form for the violation and makes predictions of physical phenomena which can be searched for by the experiments [2-5]. The recent model of Coleman and Glashow incorporates tiny departures from Lorentz invariance which do not also respect flavor conservation [1]. One of the signatures of such a flavor non conservation is the transition $\mu \rightarrow e + \gamma$ whose rate increases rapidly with the energy of the muon as measured in a preferred frame such as the one in which the 2.7^oK universal microwave background does not have any dipole anisotropy. Following their suggestion we calculate the possible contributions of such a process to the flux of "horizontal air showers" and μ-less showers which provide useful estimates for the possible strength of such an interaction and also provide a good bound on such violations.

The idea on which the bound on flavor violating interactions is derived becomes clear by noting that the primary cosmic rays consist mainly of nuclei which interact strongly when they are incident on the top of the earth's atmosphere. The amount of shielding provided by the atmosphere in the vertical direction above the earth is about 1000 g cm^{-2} and increases as the secant of the zenith angle θ upto $\sim 80^o$. The total grammage in the horizontal direction is about 36500 g cm^{-2}. The primary cosmic rays interact in the atmosphere and create a 'nuclear active' cascade. Since the atmosphere is tenuous with a scale height h $\approx 7 \times 10^5$ cm pions and kaons in the cascade decay producing the cosmic-ray muonic component. Nuclear interactions of pions and kaons with the atmosphere compete with their decay and become increasingly dominant as the particle energy increases, so that the spectrum of the muonic component at high energies is steeper than that of the nuclear active component by a factor E^{-1}. Also the muon component at high energies increases as $\sim$ sec θ, as the scale height of the atmosphere also has this dependence. Since the interaction mean free path of the hadronic components is ~ 70 g cm^{-2}, after reaching their maximum development, they are absorbed with an absorption mean free path of ~ 100 g cm^{-2}. In contrast the muons suffer only electromagnetic interactions and propagate with hardly any reduction in flux. Now note that as we move away from the vertical towards the horizontal direction, with increasing sec θ the nuclear active components get severely absorbed but the high energy muonic component increases as $\sim$ sec θ! Thus at large angles we have a nearly pure beam of high energy muons, traversing distances of the order of few times the scale height $h_\theta \sim hsec\theta$. Now should the muon decay radiatively the decay products e and γ will induce an electromagnetic cascade which can easily be observed signalling the violation of flavor conservation, as described in the model of Glashow and Coleman. Indeed as the energy of the muon increases the observability of the eγ-cascade increases as it penetrates deeper, spreads wider and produces more observable electrons and photons. The electromagnetic cascade has a very broad peak at about 500 g cm $^{-2}$ from the point of initiation for an electron or γ of energy E $\sim 10^4$ GeV and the depth of maximum increases logarithmically with energy. The total number of electrons at the peak of an electromagnetic cascade is approximately equal to the energy of the initiating electron or gamma ray in GeV units. Thus any array of particle detectors deployed to detect extensive air showers will be able to detect such showers generated by the radiative decay of the muon. There will be negligible amount of nuclear active particles and muons in these showers. The background due to showers induced by the primary cosmic ray nuclei become negligible as we go to large zenith angles. Thus 'μ-less' showers appearing in near horizontal directions constitute a signal of the new process described by Coleman and Glashow.

To quantify these ideas we note that the spectrum of muons at high energies near the earth may be parametrized as

$$\mu(E) = \frac{\kappa_i sec\theta}{E_i^{\beta+1}} cm^{-2} s^{-1} sr^{-1} GeV^{-1} \tag{1}$$

with

$$\kappa_1 = 10, \beta_1 = 2.7 \;\; for \;\; 1000 GeV < E < 10^5 \;\; GeV \tag{2}$$

and

$$\kappa_2 = 10^4, \beta_2 = 3.3 \quad for \quad 10^5 GeV < E < 3 \times 10^7 GeV \tag{3}$$

Here β_1 and β_2 are the power law exponents of the primary cosmic ray spectrum at energies of 10 to 30 times the energy of the muon.

According to Coleman and Glashow[1] the total decay probability per unit time, Γ, of a muon of Lorentz factor γ is given by:

$$\Gamma = \Gamma_w + \Gamma_r = \frac{1 + b\gamma^4}{\gamma\tau_o} = \frac{1}{\gamma\tau_o} + \frac{b\gamma^3}{\tau_o} \tag{4}$$

Here $\tau_o \approx 2.2 \times 10^{-6}s$ is the life-time of the muon and b is a very small parameter describing the violation of Lorentz invariance and flavor conservation. For a muon decay close to the earth say within a distance d of about 5 km ($\sim 700gcm^{-2}$ from the air shower array), it has to survive decay during its flight though the atmosphere beyond this i.e. a distance of few times h_θ, the scale height in that direction. Thus the number of muons decaying in the 5 km stretch is given by

$$s(E) \approx \kappa sec\theta E^{-\beta-1} exp\left\{-jh_\theta \Gamma/c\right\} \Gamma d/c \tag{5}$$

where j is a number of the order of 2 to 3. Noting that Γ is a small number and that at high energies $\Gamma \sim \Gamma_r$, the exponential in eq. 5 may be set to unity and eq. 5 is rewritten as

$$s(E) \sim \kappa sec\theta E^{-\beta-1}.\Gamma_r d/c \approx \frac{\kappa sec\theta b d m_\mu^{-3}}{c\tau_o} E^{2-\beta} \tag{6}$$

$$\equiv \kappa b\eta \ E^{2-\beta}$$

where $\eta = dm_\mu^{-3} sec\theta/c\tau_o \ GeV^{-3} \approx 5 \times 10^4 \ GeV^{-3}$. The products of the radiative decay of the muon generate an extensive air shower which contains a large number of electrons near the maximum, n, related to the muon energy through the simple relation

$$n_e \approx E/\epsilon \tag{7}$$

where $\epsilon \approx 1$ GeV for an electromagnetic shower of primary energy in the range 10^4 GeV - 10^6 GeV. The number spectrum of particles that will be seen by an air shower array is given by

$$f(n) \approx \epsilon^{3-\beta}.\kappa b\eta\, n^{2-\beta}$$

Or the number of showers F, of size larger than n is given by

$$F(n) = \int_n^\infty f(n')dn' \tag{9}$$

$$F_2(n) = \frac{\epsilon^{3-\beta}\kappa_2 b\eta}{\beta_2 - 3} n^{3-\beta} \quad for\, n \geq 10^5 \tag{10}$$

$$F_1(n) = \frac{\epsilon^{3-\beta}\kappa_1 b\eta}{3-\beta_1}\left[10^{5(3-\beta)} - n^{3-\beta}\right] + F_2(10^5) \quad for\ n < 10^5 \tag{11}$$

We compare the integral number spectrum of horizontal air showers obtained by Nagano et al [6] with the Akeno array in Fig. 1 for $b = 10^{-23}$ (curve a) and $b = 10^{-25}$ (curve b). Note that $b \sim 10^{-23}$ excluded even by the lower energy data at $n_e \sim 10^5$ and the bound

$$b < 10^{-25} \tag{12}$$

obtains when we consider the fluxes of horizontal air showers quoted by Nagano et al for $n_e \sim 5 \times 10^6$. Clearly these bounds are considerably more stringent than those derived by looking at the depth intensity curves for muons and as such small values of branching ratio for radiative decay will not have any detrimental effects on the functioning of muon colliders (Coleman and Glashow 1998). It is interesting to note that in the Coleman Glashow model this limit translates to

$$\mid 1 - c \mid \leq 6 \times 10^{-21} \tag{13}$$

This limit is of course several others weaker than those reviewed in their paper.

References

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Figure Caption

Fig. 1 The integral flux of horizontal air showers given by Nagano et al. is compared with the expectation from the Coleman-Glashow process for the two values of b, 10^{-23} and 10^{-25} respectively.