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STRINGS IN A BACKGROUND: A BRS HAMILTONIAN APPROACH

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ABSTRACT

The Fradkin-Tseytlin S-matrix generating functional Σ is recast in the form of a BRS, phase-space path integral. Canonical transformations induce modulo anomalies, local space-time symmetries for Σ and corresponding Ward identities for the S-matrix.

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1. - INTRODUCTION

One of the most promising weapons for tackling dual string theory today is the first quantization of strings moving in a generic background[1)-4)]. Potentially, this approach can lead to a complete understanding of compactification, i.e., of the particular background corresponding to the ground state of the theory, of the elementary excitations above it and their scattering processes, of the coupling constant associated with string loops, etc. Inclusion of surfaces of non-trivial topology (i.e., for closed strings, of spheres with h handles) would lead to a topological expansion which is dual and unitary order by order, and which consists of one diagram at each order.

The simplicity and economy of the first quantized framework is to be contrasted with the (presently) more involved second quantized approach[5),6)], where integration over the (infinite set of) local string fields leads to the usual multiplicity of Feynman diagrams of order h: these will have to conspire in order to reproduce duality-fulfilling amplitudes. Yet the construction of a second-quantized version of string theory is deemed important in that it should reveal the geometrical meaning of the not yet fully revealed string symmetries.

In this paper we shall rather continue our efforts[7),8)] to understand these symmetries directly through the first-quantized approach. Unfortunately, technical as well as conceptual difficulties still hinder fast progress in the first-quantized line of thought.

At the technical level, one has to develop more efficient techniques for dealing with higher-order effects, both in the σ-model (α') expansion [see, e.g., Ref. 9) for a fourth-order calculation] and in the topological expansion [on which nice progress has recently been made[10)]]. Conceptually, one would like to understand better the relationship among various functionals which play a leading rôle today in the first-quantized approach, i.e.:

i) The old dual-model effective action[11)] for the massless field Γ_{DM}.

ii) The recently-discussed conformal invariance action Γ_c of Refs. 3) and 4).

iii) The Fradkin-Tseytlin (FT) generating functional[2)], Σ.

Our approach will be entirely based on the FT functional Σ which generates complete (rather than 1PI) scattering amplitudes.

The other two functionals Γ_{DM} and Γ_c, like ordinary field theory effective actions, are supposed to provide proper vertices, i.e., 1PI amplitudes with respect to the massless states (the massive ones being integrated out). Although a proof is still lacking, it seems that Γ_{DM} and Γ_c coincide up to field redefinitions, i.e., give the same physical S-matrix.

The trouble with working directly with Γ_{DM} or Γ_c is that one does not have an explicit, functional form for these actions. A more conceptual difficulty is that Γ_{DM} and Γ_c treat differently light and heavy string modes. It is well known that some of the crucial properties of dual amplitudes need the precise, coherent effects of all the intermediate states. They are likely to be spoiled by the singling out of massless exchanges implicit in Γ_{DM} and Γ_c.

It would be better, in our opinion, to base all of string theory on the study of Σ and to relate its properties eventually to those of other auxiliary objects, such as Γ_{DM} or Γ_c. It is plausible for instance[12] that the equations of motion derived from Γ_c [which correspond to the vanishing of the σ-model β-functions[3),4)]] are equivalent to

$$\left.\frac{\delta \Sigma}{\delta \Phi_i}\right|_{\Phi_i^{(0)}} = 0 \qquad (1.1)$$

a condition which puts to zero tadpoles (one-point functions), while those for a physical vertex operator[13] (anomalous dimension constraints) would correspond to finding the zero eigenvalues of the two-point function (inverse propagator)

$$K_{ij} \sim \left.\frac{\delta^2 \Sigma}{\delta \Phi_i \, \delta \Phi_j}\right|_{\Phi_i^{(0)}} \qquad (1.2)$$

The big practical advantage of working with Σ is that Σ is defined from the beginning through a path integral

$$\sum(\Phi) \equiv N^{-1}\int dX^\mu dg_{ab}\, \exp\left(-I(X^\mu, g_{ab}, \Phi)\right) \tag{1.3}$$

where N is a normalization factor (taking care of an ∞ Möbius volume as well) and $\Phi \equiv \Phi(X^\mu)$, a functional of $X^\mu(\sigma)$, represents the collection of all possible background fields parametrizing all possible two-dimensional invariant actions. For instance,

$$\Phi\left(X(\sigma)\right) = \left(\phi(x);\, G_{\mu\nu}(x),\, B_{\mu\nu}(x),\, D(x);\, F_{\mu\nu\rho\sigma}(x)\ldots\right)$$

would correspond to the action

$$I = (2\pi\alpha')^{-1}\int d^2\sigma \sqrt{-g}\,\Big(\phi(X(\sigma)) + g^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X(\sigma)) + \\ + \frac{1}{\sqrt{-g}}\epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X(\sigma)) + \frac{\alpha'}{2} R^{(2)} D(X(\sigma)) + \ldots\ldots\Big) \tag{1.4}$$

It is known[2] that expanding Σ around the usual flat vacuum with "physical" fluctuating fields

$$\Phi = \Phi^{(0)} + \tilde{\Phi} \quad ; \quad \Phi^{(0)} = (0, \eta_{\mu\nu}, 0, \text{const}, 0\ldots)$$

$$\tilde{\Phi} = \left(e^{iK_T\cdot X};\, \epsilon_{\mu\nu} e^{iKX}\ldots\ldots\right) \quad ; \quad K_T^2 = 4/\alpha',\ K^2 = 0,\ K_\mu \epsilon_{\mu\nu} = 0 \tag{1.5}$$

generates the dual S-matrix amplitudes. It is very plausible that this will also happen if the expansion is made around a not-so-trivial vacuum. It thus appears that Σ contains <u>in principle</u> much of the information we can dream of asking of a string theory.

In this paper we shall not pursue the question of the relationship between Σ and Γ_c or between first- and second-quantized approaches. Progress on these questions has been reported in Ref. 14). Our aim will rather be that of obtaining Ward identities which are obeyed by the S matrix as a consequence of some local symmetries of Σ. Our method will be based on a BRS phase space path integral reformulation of Σ. In the spirit of Refs. 7) and 8), we shall show how canonical transformations on the phase space variables (co-ordinates, ghosts and conjugate momenta) induce "gauge" invariances for Σ. We are aware of the fact that our functional manipulations are still too formal. The need to regularize the σ-model brings in anomalies, and this can lead to extra terms appearing in some of the WI's for Σ. The works of Refs. 3) and 4) are, of course, very instructive for pinning down the conformal anomaly (at least perturbatively in α'), but one needs to have under control the general case of canonical transformation before being able to uncover the nature and the meaning of the full symmetry group of Σ. Work in this direction is in progress.

The outline of the paper is as follows. In Section 2, we present the Hamiltonian, BRS quantization of closed strings in a metric and antisymmetric tensor background (and in the presence of gauge fields in the compactified case). In Section 3 we show how canonical transformations in the string phase space induce gauge invariances for the generating functional Σ; we also digress on the way to incorporate a dilaton background into our framework. In Section 4 we obtain Ward identities out of the gauge invariances proved in Section 3. Section 5 contains some conclusions and a speculation on the possible nature of the symmetry group of the string.

2. - BRS QUANTIZATION AND PHASE-SPACE PATH INTEGRAL

In this section we shall deal with the Hamiltonian, BRS quantization of the closed bosonic string in a curved background. We shall present the explicit form of the classical constraints, compute their algebra and quantize the system by a phase space path integral. We shall leave the question of conformal anomalies to to further work. We distinguish two cases:

a) Non-compact co-ordinates

Take the closed bosonic string in an arbitrary metric ($G_{\mu\nu}$) and antisymmetric tensor ($B_{\mu\nu}$) background. The action is ($2\pi\alpha' = 1$)

$$I = -\frac{1}{2}\int d^2\sigma \left\{ \sqrt{g}\, g^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu} \right\} \qquad (2.1)$$

Defining canonical momenta $P_\mu = \delta\mathcal{L}/\delta\dot{X}^\mu$, it is easy to see that, independently of any gauge choice, the following constraints hold:

$$0 = L_\pm(\sigma; G_{\mu\nu}, B_{\mu\nu}) \equiv \frac{1}{4}\left(P_\mu \pm X'^\rho G_{\mu\rho} + X'^\rho B_{\mu\rho}\right) G^{\mu\nu} \left(P_\nu \pm X'^\sigma G_{\nu\sigma} + X'^\sigma B_{\nu\sigma}\right) \tag{2.2}$$

while $H_{can} = 0$. A long, if straightforward, calculation verifies that the classical Poisson brackets of these constraints coincide with the usual ones (where $G_{\mu\nu} = \eta_{\mu\nu}$, $B_{\mu\nu} = 0$), i.e.,

$$\left\{L_\pm(\sigma), L_\pm(\sigma')\right\} = \pm\left(L_\pm(\sigma) + L_\pm(\sigma')\right) \partial_\sigma \delta(\sigma - \sigma') \tag{2.3}$$

The validity of (2.3) implies that the classical BRS charge Q has an expression in terms of constraints and ghost fields which coincides with the one in the flat background[15),16)]:

$$Q = \int \left(L_+\eta_+ + L_-\eta_- + \mathcal{P}_+\eta_+\eta_+' - \mathcal{P}_-\eta_-\eta_-'\right) d\sigma \tag{2.4}$$

The dependence on $G_{\mu\nu}$, $B_{\mu\nu}$ is therefore only implicit, through $L_\pm$.

The BRS quantization of the system can be formally defined via the usual phase space path integral[15),16)]:

$$\Sigma(G,B) = \int dX^\mu dP_\mu \, d\eta_\pm \, d\mathcal{P}_\pm \exp\left(i\int d^2\sigma\left(\dot{X}P + \mathcal{P}\dot{\eta} - H_\xi\right)\right) \tag{2.5}$$

where the Hamiltonian in the gauge ξ is defined as

$$H_\xi = \{\xi, Q\} \tag{2.6}$$

In the orthonomal gauge, $\xi = \mathcal{P}_+ + \mathcal{P}_-$, one has

$$H = H_{ON} = L_+ + L_- + 2\mathcal{P}_+\eta_+' + \mathcal{P}_+'\eta_+ - 2\mathcal{P}_-\eta_-' - \mathcal{P}_-'\eta_- \tag{2.7}$$

$G_{\mu\nu}$, its inverse $G^{\mu\nu}$ and $B_{\mu\nu}$ appear non-linearly in the Hamiltonian action as opposed to Eq. (2.1). On the other hand, the Hamiltonian action is still quadratic in the momenta P_μ. Integrating over P_μ induces the usual Lagrangian path integral (complemented with ghost terms) and adds to the naive measure a determinant factor to give:

$$d[X^\mu]\sqrt{\det G}\; d\eta\, d\mathcal{P} \tag{2.8}$$

in agreement with the prescription of Ref. 2). A nice feature of the Hamiltonian formulation is the fact that, modulo anomalies, it has explicit invariance under the BRS canonical transformation

$$\delta z = \zeta\{Q, z\}$$
$$z = (X, P, \eta, \mathcal{P}) \quad ; \quad \zeta = \text{constant Grassmann parameter.} \tag{2.9}$$

This invariance holds "off shell", i.e, without use of the equations of motion.

By the techniques of Fradkin and Vilkovisky (FV)[15], one can "prove" formally that $\Sigma_\xi = \Sigma_{\xi+\delta\xi}$, i.e., that the theory is gauge invariant. Again, the argument is too formal. Anomalies can and do creep in when one changes variables as in the FV proof. The requirement that they do not puts constraints on the number of space-time dimensions and on the allowed backgrounds. These should coincide with the constraints discussed recently by a number of authors[3),4)].

b) Compact co-ordinates

Let us consider now the case in which d out of a total D = 26 co-ordinates X^μ are compactified.

As in Refs. 17) and 8), we replace the compactified co-ordinates X^I ($I = 1,\ldots,d$) by two-dimensional Majorana spinors ψ^i ($i = 1,\ldots,2d$). Next we couple the system to background fields, including this time gauge boson fields $A^M_\mu(X)$. The way to do this is explained, for instance, in Ref. 18). For a different method based on a group manifold approach, see Ref. 19). For the sake of simplicity in the formulae, we shall only couple the right-handed spinors to gauge fields, exposing therefore only half of the actual gauge symmetry. The action becomes

$$I = -\frac{1}{2}\int d^2\sigma \Big[\sqrt{-g}\; g^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu} + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu} +$$

$$+ i e \psi_L^i e_+^a \partial_a \psi_L^i + i e \psi_R^i e_-^a \left(\partial_a \delta_{ij} + T_{ij}^M A_\mu^M \partial_a X^\mu\right)\psi_R^j \Big] \qquad (2.10)$$

Here we use the same notations as in Ref. 8). Furthermore, T^M_{ij} are the generators of the gauge group [SO(32) for d = 16, an SO(16) subgroup of E_8 for d = 8] in the fundamental representation (to which the ψ's belong). Some algebra shows that the constraints now take the form

$$0 = L_+ - L_- \equiv P_\mu X'^\mu + \Pi_R^i \partial_1 \psi_R^i + \Pi_L^i \partial_1 \psi_L^i$$

$$0 = L_+ + L_- \equiv \frac{1}{2}\tilde{P}_\mu \tilde{P}_\nu G^{\mu\nu}(X) + \frac{1}{2} X'^\mu X'^\nu G_{\mu\nu}(X)$$

$$-\Pi_L^i \partial_1 \psi_L^i + \Pi_R^i \partial_1 \psi_R^i - \Pi_R^i T_{ij}^M A_\mu^M \psi_R^j X'^\mu$$

where, as in Ref. 8), $\Pi^i_{L,R} = i/2\ \psi^i_{L,R}$ and

$$\tilde{P}_\mu = P_\mu + X'^\lambda B_{\mu\lambda} + \Pi_R^i T_{ij}^M A_\mu^M \psi_R^j \qquad (2.11)$$

A lengthy calculation confirms once more that the Dirac brackets of L_+, L_- give the algebra (2.3). Thus the BRS charge and the Hamiltonian again take the forms (2.4) and (2.7) respectively, with the background appearing only implicitly through the constraints.

We remark in passing that it should be possible to add further non-trivial backgrounds to the system, since we know[8)] that the spectrum also contains a massless spin-zero multiplet $\phi^{M,N}$ belonging to the adjoint representation of the left × right gauge group. On the basis of some recent work[20)], we believe that around specific values for these backgrounds, new gauge symmetries of the type first proposed by Narain[21)] might emerge.

3. - CANONICAL TRANSFORMATIONS AND GAUGE INVARIANCE OF Σ

We shall now argue that, modulo anomalies which are still to be analyzed, canonical transformations on the phase-space path integral (2.5) determine the local symmetries of Σ under transformations of the background field $G_{\mu\nu}$, $B_{\mu\nu}$ and A^M_μ. We shall again distinguish the cases of non-compact and compact co-ordinates and conclude with a digression on the way the dilaton field can be introduced into our framework.

a) Non-compact case

Consider two kinds of infinitesimal canonical transformations ($\delta z = \{z,\Phi\}$, Φ = generator) induced by

$$\text{i)} \quad \Phi_G \equiv \int d\sigma \; P_\mu \, \xi^\mu(X) \tag{3.1a}$$

$$\text{ii)} \quad \Phi_B \equiv \int d\sigma \; X'^\mu \Lambda_\mu(X) \tag{3.1b}$$

The transformation corresponding to (3.1a) is

$$\delta_G X^\mu = \xi^\mu(X) \quad , \quad \delta_G P_\mu = - P_\nu \, \xi^\nu{}_{,\mu}(X)$$
$$\delta_G(\eta, \mathcal{P}) = 0 \tag{3.2}$$

where, as usual, the comma means ordinary differentiation. The backgrounds $G_{\mu\nu}(X)$ and $B_{\mu\nu}(X)$ are given functions and thus transform as a consequence of the change in X:

$$\delta_G G_{\mu\nu} = G_{\mu\nu,\lambda} \, \xi^\lambda \quad ; \quad \delta_G B_{\mu\nu} = B_{\mu\nu,\lambda} \, \xi^\lambda \tag{3.3}$$

One can verify that the Hamiltonian action appearing in (2.5) and (2.7)

$$S_H = \int d^2\sigma \left(\dot{X} P + \mathcal{P} \dot{\eta} - H_{ON} \right) \tag{3.4}$$

satisfies

$$\delta_G S_H = - \delta^{GCT} S_H \tag{3.5}$$

where now $\delta^{GCT} S_H$ is the variation of S_H under the standard general co-ordinate transformation (GCT) of $G_{\mu\nu}$, $B_{\mu\nu}$:

$$\delta^{GCT} G_{\mu\nu} = - G_{\mu\lambda} \xi^{\lambda}{}_{,\nu} - G_{\nu\lambda} \xi^{\lambda}{}_{,\mu} - G_{\mu\nu,\lambda} \xi^{\lambda}$$

$$\delta^{GCT} B_{\mu\nu} = - B_{\mu\lambda} \xi^{\lambda}{}_{,\nu} - B_{\lambda\nu} \xi^{\lambda}{}_{,\mu} - B_{\mu\nu,\lambda} \xi^{\lambda} \tag{3.6}$$

Similarly, the transformation corresponding to (3.1b) gives

$$\delta^{B\text{-gauge}} S_H = - \delta_B S_H \tag{3.7}$$

where the B-gauge transformation is defined as:

$$\delta^{B\text{-gauge}} B_{\mu\nu} = \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \; ; \; \delta^{B\text{-gauge}} G_{\mu\nu} = 0 \tag{3.8}$$

If we assume the phase-space (Liouville) measure to be invariant under the canonical transformations induced by (3.1a) and (3.1b), we immediately arrive at the desired result:

$$\Sigma(G, B) = \Sigma\left(G + \delta^{GCT} G,\ B + \delta^{GCT} B + \delta^{B.g.} B\right) \tag{3.9}$$

b) Compact case

The generator is now taken to be

$$\Phi_\psi \equiv \int d\sigma\, \psi_R^i \psi_R^j\, T_{ij}^M\, \xi^M(X) \tag{3.10}$$

which generates the transformation discussed in Ref. 8). It is again simple to check that

$$\delta_\psi S_H = -i/2\, \delta^{Gauge} S_H \tag{3.11}$$

where δ^{gauge} is the gauge transformation acting just on $A_\mu^M(X)$:

$$\delta^{Gauge} A_\mu^M = \xi^M{}_{,\mu}(X) + f^{MNP} A_\mu^N(X)\, \xi^P(X) \tag{3.12}$$

with f^{MNP} the group's structure constants:

$$[T^M, T^N] = i f^{MNP}\, T^P \tag{3.13}$$

Arguing as in (a), we arrive at the conclusion that, modulo anomalies, Σ is gauge invariant:

$$\Sigma(G,B,A) = \Sigma(G,B,A+\delta^{Gauge} A) \tag{3.14}$$

c) The dilaton in the Hamiltonian formulation

At this point we would like to make a digression. It is clear that, because of its invariances, $\Sigma(G,B)$ can only describe a "spin-two" and one "pseudoscalar" but not a "scalar" dilaton. It is well known, however, that such a dilaton is present in the physical massless spectrum; it will appear as soon as poles in graviton amplitudes are analyzed.

A similar problem has arisen in the covariant, second-quantized approach[5)]:

it was resolved[5),6)] by the introductions of further fields, associated with a functional $\Phi(X,\eta)$ of both the string and the ghost co-ordinates.

In the FT approach[2)], the extra fields are instead associated with new terms appearing in the first-quantized Lagrangian action I which contain the two-dimensional curvature $R^{(2)}$, i.e., for the dilaton:

$$I_D = \frac{1}{4\pi}\int \sqrt{-g}\, R^{(2)} D(X)\, d^2\sigma \tag{3.15}$$

This term is not conformal invariant by itself at the classical level. Nevertheless, the total action leads to a conformal invariant theory at the quantum level if G, B and D satisfy suitable constraints [following from the action Γ_c of Refs. 3) and 4)].

Clearly, from the point of view of the BRS, Hamiltonian approach, the term (3.15) is to be converted into a term involving ghost fields. Such a term has been constructed within the Lagrangian BRS approach[22)]. Inspired by Ref. 22), we were able to construct an additional contribution to the gauge fixed Hamiltonian involving ghosts and the dilaton background:

$$\begin{aligned} H_D = {} & 8/3\, \mathcal{P}_+ \eta_+ \left(P_\mu + X'^\rho B_{\mu\rho} - X'^\rho G_{\mu\rho}\right) G^{\mu\nu} \partial_\nu D(X) \\ & + 8/3\, \mathcal{P}_- \eta_- \left(P_\mu + X'^\rho B_{\mu\rho} + X'^\rho G_{\mu\rho}\right) G^{\mu\nu} \partial_\nu D(X) \\ & + 64/9\, \mathcal{P}_+ \eta_+ \mathcal{P}_- \eta_-\, \partial_\mu D\, G^{\mu\nu} \partial_\nu D \end{aligned} \tag{3.16}$$

The total Hamiltonian, $H_T = H_{ON} + H_D$, satisfies the following two properties:

i) The canonical transformation generated by $\Phi_{ghost} \equiv \int (\mathcal{P}_+\eta_+ + \mathcal{P}_-\eta_-)\, \epsilon(X)$, i.e.,

$$\delta \eta_\pm = \epsilon(x)\, \eta_\pm \quad ; \quad \delta \mathcal{P}_\pm = -\epsilon(x)\, \mathcal{P}_\pm$$

$$\delta X^\mu = 0 \quad ; \quad \delta P_\mu = -(\partial_\mu \epsilon)\cdot(\mathcal{P}_+ \eta_+ + \mathcal{P}_- \eta_-) \tag{3.17}$$

induces the same change in the action as the shift

$$\delta_{\text{ghost}} D(X) = -3/8\, \epsilon(X) \tag{3.18}$$

Hence, classically, D can be rotated away by a field redefinition. We know, however, that this is not true at the quantum level[22)].

ii) Integrating out P_μ (which is still appearing quadratically) reproduces exactly the ansatz of Ref. 22) in the Lagrangian formulation.

We thus feel that the addition of H_D should be the proper way to include a background dilaton field in the phase-space approach, at least up to total derivatives (surface terms). The fact that extra surface terms may be necessary can be argued from the fact that, for D = constant, the FT term (3.15) gives D times the Euler number $\chi = 2 - 2h$ of the surface [and thus $\exp(-2\langle D\rangle)$ is the string-loop-counting parameter], while our H_D, or the term considered in Ref. 22), vanishes in this case.

4. - WARD IDENTITIES

Given the invariance properties of Σ, it becomes a relatively trivial matter to derive Ward Identities (WI's) for the S-matrix elements which it generates. We can thus make contact with the results of Refs. 7) and 8) and extend them. Consider as a first example the case of GCT. As an immediate consequence of (3.3), one has

$$0 = \overset{GCT}{\delta} \Sigma = \left\langle \int d^D x \left[\frac{\delta S_H}{\delta G_{\mu\nu}(X)} \overset{GCT}{\delta} G_{\mu\nu}(X) + \frac{\delta S_H}{\delta B_{\mu\nu}(X)} \overset{GCT}{\delta} B_{\mu\nu}(X) \right] \right\rangle_{G,B} \tag{4.1}$$

where the symbol $\langle \ \ \rangle_{G,B}$ means functional integration with the measure $\exp(iS_H(G,B))$. Recalling that S_H has the generic form

$$S_H = \int d^2\sigma \, \mathcal{L}_H \left(X', P, \eta', \mathcal{P}; G(X(\sigma)), B(X(\sigma)) \right) \tag{4.2}$$

one finds

$$\frac{\delta S_H}{\delta G_{\mu\nu}(X)} = \int d^2\sigma \, \delta(x - X(\sigma)) \frac{\delta \mathcal{L}_H}{\delta G_{\mu\nu}(X)} \equiv \int d^2\sigma \, \delta(x - X(\sigma)) V_G(X) \tag{4.3}$$

and similarly for $B_{\mu\nu}$. $V_G^{\mu\nu}$ and $V_B^{\mu\nu}$ can be called the graviton and antisymmetric tensor vertices in a background. One finds easily:

$$V_G^{\mu\nu} = \frac{1}{2} X'^{\mu} X'^{\nu} - \frac{1}{2} G^{\mu\rho} G^{\nu\sigma} P_\rho P_\sigma - P_\rho G^{\mu\rho} G^{\nu\sigma} B_{\sigma\tau} X'^{\tau}$$

$$- \frac{1}{2} X'^{\rho} B_{\alpha\rho} B_{\beta\sigma} X'^{\sigma} G^{\mu\alpha} G^{\nu\beta}$$

$$2V_B^{\mu\nu} = P_\rho G^{\rho\mu} X'^{\nu} + X'^{\nu} B_{\rho\sigma} X'^{\sigma} G^{\mu\rho} - (\mu \leftrightarrow \nu) \tag{4.4}$$

Using (3.6) and (4.3), Eq. (4.1) becomes

$$\Big\langle \int d^2\sigma \Big[V_G^{\mu\nu}(X) \left\{ G_{\mu\lambda}(X) \xi^\lambda{}_{,\nu}(X) + G_{\nu\lambda} \xi^\lambda{}_{,\mu}(X) + G_{\mu\nu,\lambda} \xi^\lambda(X) \right\} +$$

$$+ V_B^{\mu\nu}(X) \left\{ B_{\mu\lambda}(X) \xi^\lambda{}_{,\nu} - B_{\nu\lambda}(X) \xi^\lambda{}_{,\mu}(X) + B_{\mu\nu,\lambda}(X) \xi^\lambda(X) \right\} \Big] \Big\rangle_{G,B} = 0 \tag{4.5}$$

WI's for multigraviton and antisymmetric field processes follow by taking a functional derivative with respect to $\xi^\lambda(X)$ at $\xi^\lambda = 0$ and an arbitrary number of derivatives with respect to $G_{\mu\nu}(y)$ and $B_{\mu\nu}(Z)$ at the ground state values of $G_{\mu\nu}$ and $B_{\mu\nu}$.

In order to make contact with Ref. 7) [or with Ref. 23), where a light-cone approach is followed], let us take the case in which the vacuum background is taken to be $D = 26$ flat Minkowski space and we consider an n-graviton amplitude. We thus write

$$\frac{\delta^n}{\delta G_{\mu_1\nu_1}(y_1)\cdots\delta G_{\mu_n\nu_n}(y_n)} \left\langle \int d^2\sigma\, V_G^{\mu\nu}\left\{G_{\mu\lambda}\partial_\nu\delta(x-X) + (\mu\leftrightarrow\nu) + G_{\mu\nu,\lambda}\delta(x-X)\right\}\right\rangle_G\Bigg|_{G=\eta} = 0 \tag{4.6}$$

This is analogous to standard forms of gravitational WI's[24]. The derivatives in (4.6) either act on $V_G(X)$ or on $G_{\mu\lambda}$, $G_{\nu\lambda}$, $G_{\mu\nu,\lambda}$ or, finally, on the action $S_H(G)$ implicit in the definition of $\langle\ \rangle_G$. In the first two instances, one gets contact terms ($Y_i = X$), while in the last case one gets extra vertices. In conclusion, the WI, after a Fourier transform, takes the form:

$$\left\langle \prod_{i=1}^{n}\int d^2\sigma_i\, V_G^{\mu_i\nu_i}(X(\sigma_i))\,e^{ik_iX(\sigma_i)} \cdot \int d^2\sigma\, 2k_\nu\, V_G^{\lambda\nu}(X(\sigma))\,e^{ikX(\sigma)}\right\rangle_{G=\eta} =$$

$$= \sum_{i=1}^{n} k_i^\lambda \left\langle \int d^2\sigma_i\, V_G^{\mu_i\nu_i}(X(\sigma_i))\,e^{i(k+k_i)X(\sigma_i)} \prod_{j\neq i}\int d^2\sigma_j\, V_G^{\mu_j\nu_j}(X(\sigma_j))\,e^{ik_jX(\sigma_j)}\right\rangle_{G=\eta} + \text{(other contact terms)} \tag{4.7}$$

which relates the divergence of an (n+1)-graviton amplitude to lower point functions. It has precisely the form obtained in Ref. 7) [and also in Refs. 23) and 24)], with the necessary modifications due to the spin of the external gravitons. Since all our manipulations have been formal and neglected anomalies, the WI is presumably only valid if the background preserves BRS (conformal) invariance and if the fluctuations considered correspond to on-shell external particles. For off-shell Green's functions, other gauge-dependent terms are expected to occur.

In a very similar way, one can consider the case of compact co-ordinates and gauge WI's. One thus recovers the results of Ref. 8), extends them to the case of a curved background and obtains generalized vertex operators for the latter case. Here we just quote the result for the case of an extended gauge and graviton background:

$$\left\langle \int d^2\sigma \, V^{\mu}_{A^P}(X(\sigma)) \left\{ \partial_\mu \delta(X-x) \delta_{MP} + f^{NMP} A^N_\mu(X(\sigma)) \delta(X-x) \right\} \right\rangle_{G,A} = 0 \tag{4.8}$$

where

$$V^{\mu}_{A^M} \equiv \frac{\delta S_H}{\delta A^M_\mu} = \psi^i_R T_{ij}{}^M \psi^j_R \left(G^{\mu\nu} \tilde{P}_\nu - X'^\mu \right) \tag{4.9}$$

and $\tilde{P}_\nu$ is defined in (2.11).

5. - CONCLUSIONS AND OUTLOOK

In this paper we have tried to convey our belief that the BRS, Hamiltonian formulation of strings propagating in non-trivial backgrounds can be a powerful tool for addressing a variety of fundamental questions about strings.

The distinctive advantages of a phase-space approach to strings may originate from their fundamental symmetry (duality itself!) under the exchange of the two world-sheet co-ordinates σ and τ. This leads to a symmetry under the exchange of P and X', a sort of Born's reciprocity principle[25)] obeyed by strings.

In this connection it has recently been proposed[26)] that string theories naturally lead to a system of units in which co-ordinates and momenta are measured in the same units, and $\hbar$ is replaced by a fundamental length λ ($\hbar \to \lambda^2$). In these units, the symmetry between co-ordinates and momenta becomes even more transparent; keeping that symmetry manifest calls for a Hamiltonian, phase-space framework. We could in fact ask ourselves if it is possible to construct a string theory directly in phase space. This would amount to taking an algebra of constraints [e.g., Eqs. (2.3) for the bosonic string], as the <u>definition</u> of a particular string theory and to looking for a representation of that algebra in terms of functions $L_{\pm}(X',P,X)$ of the phase-space variables. Alternatively, one could look for representations of the BRS operator itself.

In these optics, bosonic string theories in flat or curved backgrounds can be seen as special representations of the algebra of constraints. We conjecture, for instance, that the representation (2.2) is the most general one, which is bilinear in X'^{μ} and P_{μ}.

The next step would be quantization by a (properly-defined) phase-space path integral. This leads to $\Sigma(\Phi)$, the partition function seen as a functional of all the fields that define a representation of the algebra of constraints. Such a functional is just the Hamiltonian version of the FT functional[2)]. It should provide all the information about a specific string theory following the general framework developed by Faddeev and co-workers[27)].

Here the question of anomalies arises. At the quantum level, Σ is two-dimensional gauge(BRS)-invariant only if the background fields Φ are conveniently chosen [so as to make the β-functions of Ref. 3) and 4) vanish]. Such a condition defines a manifold $\mathcal{M}_c$ of acceptable backgrounds (c for conformal). Analogously, given a background Φ_c in $\mathcal{M}_c$, the on-shell S-matrix elements correspond to derivatives of Σ at $\Phi = \Phi_c$ along directions which preserve two-dimensional gauge invariance [these correspond to the physical, BRS-invariant vertices[13)]]. The meaning of $\Sigma(\Phi)$ outside $\mathcal{M}_c$ is unclear, since there Σ is a two-dimensional gauge-dependent quantity[28)]. What is really important is to find the local invariances of $\Sigma(\Phi)$ within the manifold $\mathcal{M}_c$ of conformal invariant backgrounds. As already suggested in Ref. 7), these invariances should be looked for within the huge group G_{can} of all canonical transformations. There will be a subgroup $G^q_{can} \subset G_{can}$ defining those canonical transformations which are not affected by anomalies. As we have shown in Sections 2 - 4, for particular cases, non-anomalous canonical transformations imply the invariance of Σ under local changes of the backgrounds. In the absence of anomalies, these transformations map conformal backgrounds Φ_c into other conformal backgrounds Φ'_c ($\{Q(\Phi),H(\Phi)\} = 0$ implies classically $\{Q(\Phi'),H(\Phi')\} = 0$), i.e., they do not take Φ out of $\mathcal{M}_c$.

It is obviously the present lack of understanding of anomalies in canonical transformations that is impeding progress in at least two important issues:

i) D = 10 supersymmetry for the superstring

It is straightforward to set up a Hamiltonian, BRS treatment of the superstring in the old Neveu-Schwarz-Ramond formulation. Following the results of Ref. 29), it is also possible to guess[30)] the canonical transformation that will induce D = 10 supergravity WI's, at least in a flat, D = 10 Minkowski background. The problem is to understand directly in the BRS language why that particular canonical transformation stands out. This is probably related to the requirement that it be free of anomalies and that (consequently?) the gravitino vertex associated with it be BRS invariant.

ii) Higher string symmetries

It is very likely that other local symmetries of Σ exist under transformations mixing together backgrounds of different mass-levels. Their understanding might lead to an explanation of the huge degeneracy of the dual-model spectra which has long been a great mystery.

Generalizations of the canonical transformations generated by (3.1) could be those induced by

$$\tilde{\Phi} = \int d\sigma \, (P+X')_\mu (P+X')_\nu (P-X')_\rho \, \xi^{\mu\nu\rho}(X)$$

$$\tilde{\tilde{\Phi}} = \int d\sigma \, (P+X')'_\mu (P-X')_\rho \, \xi^{\mu\rho}(X)$$

and these would correspond geometrically to transformations in which strings are mapped into strings in a way which is local in string (phase) space, rather than in ordinary point space. Thus, strings that touch before the transformation would fail to do so afterwards. Are these General String Transformations (GST), as opposed to general co-ordinate transformations, the geometrical symmetries underlying string theories? The answer to this question seems to depend once more on the understanding of anomalies in general canonical transformations.

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