

# Ward Identity for Membranes

A. Ghosh\*

*CERN, Theory Division, CH-1211 Geneva 23, Switzerland*

J. Maharana†

*Max-Planck Institute for Gravitation, Albert-Einstein Institute, Potsdam, Germany*

*and Institute of Physics, Bhubaneswar 751005, India‡*

(February 1, 2008)

## Abstract

Ward identities in the case of scattering of antisymmetric three form RR gauge fields off a D2-brane target has been studied in type-IIA theory.

---

\*e-mail: Amit.Ghosh@cern.ch

†e-mail: maharana@nxth04.cern.ch

‡permanent address

Recent progress in understanding the nonperturbative aspects of string theory [1,2] has provided a unified description of the dynamics of the five (perturbatively) consistent superstring theories. The p-branes and D-branes, which appear naturally as solutions of the string effective action, have a crucial role to play in these developments [3]. It is believed that there is an underlying fundamental theory and the five string theories are various phases of this theory [4,5] and the low energy effective action of the fundamental theory, named M-theory, can be identified with that of  $D = 11$  supergravity. There is mounting evidence that M-theory encompasses and unifies string theories and governs string dynamics in diverse dimensions. The bosonic sector of the low energy effective action of this theory contains the graviton and the antisymmetric 3-rank tensor and consequently, the membrane that couples to the three index antisymmetric tensor field is expected to be the natural extended fundamental object [6,7] in eleven dimensions. It is natural to explore whether the fundamental (super)membrane can provide the degrees of freedoms of M-theory. It is not evident that there exists a consistent quantum mechanical theory of the (super)membrane [8]. This issue is very closely related with the large  $N$  behaviour of the  $U(N)$  matrix model [9]. The proposal of the M(atrrix) model [10] has led to very interesting developments [11]. The M(atrrix) theory reveals various salient features of the M-theory. In this approach, the dynamics of the eleven dimensional M-theory is described by the many body quantum mechanics of  $N$   $D0$ -branes of the type IIA theory in the limit  $N \rightarrow \infty$ . The compactified M(atrrix) theory gets related to super Yang-Mills theories through dualities. Furthermore, it provides a theoretical basis to the understanding of the microscopic dynamics of M-theory and holds the promise of exploring various aspects of string theories nonperturbatively [12].

Since the membrane theory provides an intimate connection with M-theory, there have been attempts to study the supermembrane action in curved space with antisymmetric tensor field background [13] in spite of the above mentioned shortcomings. It is interesting to investigate how much the world volume theories know about the spacetime [14]. One of us [15] has investigated how the bosonic membrane theory is encoded with the target space local symmetries such as general coordinate transformation and gauge transformation; the former being associated with the graviton and the latter with antisymmetric tensor field. The strategy followed to expose these symmetries was based on the one proposed by Veneziano [16], in the context of string theory, to derive gravitational Ward identities, and by Veneziano and one of the present author [17,18] to derive Ward identities for various massless excitations of strings: both compactified and noncompactified [19]. Let us recall the technique adopted in ref.[16-18]. A Hamiltonian phase space framework is adopted to obtain a Hamiltonian form of the action. Next, a set of generating functionals associated with the local (target space) symmetries of the theory are introduced. It was explicitly demonstrated that the variation of the action, under these canonical transformations, can be compensated by appropriate (gauge) transformation of the massless backgrounds. The Ward identities are derived with the argument that the Hamiltonian path integral measure remains invariant under canonical transformations, at the classical level at least.

A similar approach was adopted in the case of the membrane and it was shown that it is possible to introduce canonical transformations associated with general coordinate transformation and gauge transformation in the target space of the M-theory. Of course, the results for the membrane are to be understood as classical one in view of the preceding remarks regarding the quantum theory of membranes. It is worthwhile to mention that the

the Ward identities derived for membrane [15] are not easy to verify explicitly, unlike the Ward identities in string theory where one can obtain explicit expressions for the vertex operators for some simple backgrounds and the conformal invariance imposes constraints on the form of various vertex operators. In the covariant formulation, the vertex operators are required to be BRST invariant.

The purpose of this note is to derive Ward identities for scattering of the antisymmetric tensor gauge field from a membrane, specifically D2-brane, that appear in the type IIA string theory. In fact, the type IIA theory is of special significance since the strong coupling limit of this theory is intimately related to the  $D = 11$  supergravity [20]. Thus the membrane appearing in type IIA theory is very closely connected with the membrane of the eleven dimensional theory. We shall see that our test of Ward identity will be at the level of scattering amplitudes similar to the works in [16-18], although the techniques will be slightly different. Let us recall how the Ward identities are derived in the string theory [16-18]:

$$Z[J] = \int [dX dP][d\mathcal{G}] \exp(-iS_H[X, P, \mathcal{G}, J]) \quad (1)$$

here,  $Z[J]$  is the generating functional,  $X$  represents the string coordinate,  $P$  represents the conjugate momentum,  $\mathcal{G}$  are the ghosts and  $S_H$  is the Hamiltonian action which is a functional of coordinates, momenta, ghosts and the massless background field  $J$ , which corresponds to graviton, antisymmetric tensor or gauge potential according to the case at hand. As mentioned earlier, one introduces a suitable generator of the canonical transformation,  $\Phi_J[X, P]$ . Then it was shown explicitly that for the above backgrounds, the following relation holds:

$$\delta_{\Phi_J} S_H = \delta_{Gauge}^J S_H \quad (2)$$

In other words, first one computes the variations of  $X$  and  $P$  under  $\Phi_J$  to obtain the variation of  $S_H$ . Then, one checks from the l.h.s of the above equation that it is the same as implementing general coordinate transformation or Abelian gauge transformation associated with graviton or antisymmetric tensor field. In the case of compactified string the nonabelian gauge fields are also permissible backgrounds and in that case the corresponding nonabelian gauge transformation is to be considered in the r.h.s of the equation. Now the argument is that the Hamiltonian phase space measure remains invariant under  $\Phi_J$  and the variation of  $S_H$  under  $\Phi_J$  can be compensated by appropriate gauge transformation of background  $J$  leading to the relation

$$0 = \delta_{Gauge}^J Z[J] = \left\langle \int d^D x \left[ -i \frac{\delta S_H}{\delta J(x)} \delta_{Gauge}^J J(x) \right] \right\rangle_{J_{bg}} \quad (3)$$

Here  $\langle \dots \rangle$  means that the expression is averaged with the path integral factor  $\int [dXdPd\mathcal{G}] \exp(-iS_H)$ . The interpretation of the above equation is as follows: from the preceding arguments  $Z[J] = Z[J + \delta_{Gauge}^J J]$  leading to the equation (3). Here  $J_{bg}$  means that the massless fields  $G_{\mu\nu}, B_{\mu\nu}$  or  $A_\mu$  takes their background values after the functional derivative of the Hamiltonian action is taken. Notice that

$$\frac{\delta S_H}{\delta J(x)} = \int d^2 \sigma \delta(x - X(\sigma)) \frac{\delta \mathcal{L}_H}{\delta J(X)} = \int d^2 \sigma \delta(x - X(\sigma)) V_J(X, P) \quad (4)$$

where,  $V_J(X, P)$  is the corresponding vertex operator for  $G_{\mu\nu}$ ,  $B_{\mu\nu}$  or  $A_\mu$  depending on what type of WI one is interested in. As an example, let us consider a quick derivation of the gravitational WI, note that

$$\delta^{GCT} G_{\mu\nu} = -G_{\mu\lambda} \xi^\lambda{}_{,\nu} - G_{\nu\lambda} \xi^\lambda{}_{,\mu} - G_{\mu\nu,\lambda} \xi^\lambda \quad (5)$$

Using (3) and (4) we arrive at

$$\langle \int d^2\sigma \left[ V_G^{\mu\nu}(X, P) \{ G_{\mu\lambda}(X) \xi^\lambda(X)_{,\nu} + G_{\nu\lambda}(X) \xi^\lambda(X)_{,\mu} + G_{\mu\nu}(X)_{,\lambda} \xi^\lambda(X) \} \right] \rangle_{G_{bg}} = 0 \quad (6)$$

here  $\xi^\lambda(X)$  is the local parameter associated with infinitesimal general coordinate transformation. Since it is an arbitrary parameter, if we differentiate the above equation with respect to  $\xi^\lambda$  and set  $\xi = 0$  the r.h.s will be still zero. Furthermore, we can take functional derivatives of the whole expression with respect to a string of G's and set eventually the metric to be flat space metric (for simplicity) to arrive at

$$\frac{\delta^n}{\delta G_{\mu_1\nu_1}(y_1) \dots \delta G_{\mu_n\nu_n}(y_n)} \langle \int d^2\sigma V_G^{\mu\nu} \{ G_{\mu\lambda} \partial_\nu \delta(x - X) + G_{\nu\lambda} \partial_\mu \delta(x - X) + G_{\mu\nu,\lambda} \delta(x - X) \} \rangle_{G=\eta} = 0 \quad (7)$$

Note that the functional derivatives of metric act in three ways: first when it acts on the path integral factor buried in  $\langle \dots \rangle$  it brings down the vertex operator  $V_G^{\mu\nu}(X)$ , in this case; second, if there is any G dependence in the vertex operator in the above expression, it removes that G and introduces a factor of  $\delta(y_i - X)$  and thirdly it kills the factor of G which exists inside the curly bracket. The WI is rather transparent if one takes the Fourier transform. The derivatives of delta function will give factors of momenta. Thus  $(n+1)$ -graviton amplitude gets related to lower point amplitudes. We also know that BRST invariance will impose constraints on the vertex operators.

We shall adopt following prescription to derive WI for the scattering of three index antisymmetric tensor field of type IIA theory from the D2-brane. Notice that the three-index potential (whose field strength is four index antisymmetric tensor) comes from the RR sector. First, we obtain the vertex operator for the three index antisymmetric tensor field in the type IIA theory. The amplitude for scattering of the gauge boson from the D2-brane is obtained using the techniques of conformal field theory. As is well known the vertex operators must be BRST invariant and the prescription of deriving them in the covariant formulation of superstring was given by Friedan, Martinec and Shenker [21]. Finally, one can explicitly check that the WI are satisfied when the scattering amplitude, after separating out the 'kinematical' factors, is contracted with the momenta of the incoming or outgoing gauge bosons.

Let us recall that the massless excitations of the type II theory arise from the product of the left and right moving sectors involving NS-NS and RR oscillators. We can represent this as

$$|\mu\alpha\rangle_R \times |\nu\beta\rangle_L \quad (8)$$

here  $\mu, \nu$  refer to the NS-NS sector and  $\alpha, \beta$  correspond to the RR sector; the former being spacetime indices take values 0,1...9 and the latter are spinor indices. The familiar bosonic

fields are graviton, dilaton and the antisymmetric tensor coming from the NS-NS sector. Furthermore, the bosonic fields originating from RR sector appear as bispinors in the vertex operator

$$V_{RR} = F_{\alpha\beta} U^{\alpha\beta} \quad (9)$$

the bispinor  $F_{\alpha\beta}$  can be expanded in terms of a complete basis of the ten dimensional gamma matrices (antisymmetric products) as follows

$$F_{\alpha\beta} = \sum_{k=0}^{10} \frac{i^k}{k!} F_{\mu_1\mu_2\dots\mu_k} (\gamma^{\mu_1\dots\mu_k})_{\alpha\beta} \quad (10)$$

where the  $k$ -dimensional tensor  $\gamma^{\mu_1\dots\mu_k}$  is constructed from the ten dimensional gamma matrices and it is antisymmetric with respect to all its indices. Therefore, the tensors  $F_{\mu_1\dots\mu_k}$  appearing the above equation are antisymmetric in their indices and one concludes that the massless RR fields are antisymmetric Lorentz tensors. As the bispinors have definite chirality projections, thus all the components of the  $F$ 's are not independent. As a consequence, type IIA theory has only field strengths corresponding to even integers of  $k$  and type IIB contains field strengths with odd integers of  $k$ . The former admits only even branes and the latter only odd ones, as is well known.

In the covariant formulation of superstring [21] the vertex operators involving RR fields contain the spin field  $S^\alpha$ , the bosonized ghost  $\phi$  and of course the 'plane wave' piece  $e^{ik \cdot X}$ . Their combinations have to be such that the vertex operator commutes with the BRST charge. Generically, we can write

$$U_\alpha^\beta = V_{(-\frac{1}{2})\alpha}(z) \bar{V}_{(-\frac{1}{2})}^\beta(\bar{z}) \quad (11)$$

with

$$V_{(-\frac{1}{2})\alpha}(z) = e^{-\frac{1}{2}\phi(z)} S_\alpha(z) e^{ikX(z)} \quad (12)$$

Note that the bar on the argument of the vertex operator refers to complex conjugation here and everywhere. Moreover, the  $\gamma$  matrices are  $32 \times 32$  dimensional and have the representation

$$\gamma^\mu = \begin{pmatrix} 0 & \gamma^{\mu\alpha\beta} \\ \gamma^\mu_{\alpha\beta} & 0 \end{pmatrix} \quad (13)$$

satisfying the anticommutation relation  $\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu}$  with  $\eta^{\mu\nu} = \text{diag}(-1, 1\dots 1)$ ; furthermore,  $\gamma^{11} = \gamma^0 \dots \gamma^9 = \text{diag}(1, -1)$ . The standard method for the construction of  $S^\alpha$  is [21] to bosonize the worldsheet fermions, and introduce a cocycle operator. In the case of D-brane some of the coordinates (and therefore also worldsheet fermions) satisfy Dirichlet or Neumann boundary condition. Thus  $S(z)$  and  $\bar{S}(\bar{z})$  get related depending on the Dp-brane one is considering. In our case, IIA,

$$\bar{S}^\alpha(\bar{z}) = M^{\alpha\beta} S_\beta(z) \quad (14)$$

with  $M = \gamma^0 \dots \gamma^p$  for Dp-brane which are even.

Let us consider scattering of massless R-R 3-form states in Type-IIA off a D2-brane target. In components form the 4-form field-strength is given by

$$F_{\mu\nu\rho\lambda} = \partial_\mu C_{\nu\rho\lambda} - \partial_\nu C_{\rho\lambda\mu} + \partial_\rho C_{\lambda\mu\nu} - \partial_\lambda C_{\mu\nu\rho}$$

where  $C_{\mu\rho\lambda}$  is the 3-form potential. Now, in the absence of the antisymmetric tensor field  $B_{\mu\nu}$ , the Chern-Simons like term ( $F \wedge F \wedge B$ ), where  $B$  is the NS-NS 2-form potential, is not present in the low energy effective action; therefore, the equation of motion for the RR field strength is given by

$$\partial^\mu F_{\mu\nu\rho\lambda} = 0 \tag{15}$$

Now we choose a plane wave ansatz for the gauge potential

$$\begin{aligned} C_{\mu\nu\lambda} &= \epsilon_{abc} \epsilon_\mu^a \epsilon_\nu^b \epsilon_\lambda^c e^{ik \cdot X}, \quad a, b, c = 1, 2, 3; \quad \epsilon_{123} = +1 \\ F_{\mu\nu\rho\lambda} &= i\epsilon_{abc} [k_\mu \epsilon_\nu^a \epsilon_\rho^b \epsilon_\lambda^c - k_\nu \epsilon_\rho^a \epsilon_\lambda^b \epsilon_\mu^c + k_\rho \epsilon_\lambda^a \epsilon_\mu^b \epsilon_\nu^c - k_\lambda \epsilon_\mu^a \epsilon_\nu^b \epsilon_\rho^c] e^{ik \cdot X} \end{aligned} \tag{16}$$

where,  $k$  is the momentum, satisfying the on-shell condition ( $k^2 = 0$ ); and  $\epsilon_\mu^a$  are polarization vectors subject to constraints  $\epsilon^a \cdot k = 0$  as a consequence of eq.(15). The amplitude for scattering of the three index gauge field off the D2-brane involves computation of the correlation functions involving two of the  $V_{RR}$  operators:

$$A = \int \frac{dz_1 d\bar{z}_1 dz_2 d\bar{z}_2}{(Vol)_{conformal}} \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle, \tag{17}$$

where  $(Vol)_{conformal}$  represents the conformal group volume factor that has to be factored out and  $V_1, V_2$  correspond to the vertex operators of asymptotically incoming and outgoing massless R-R states. The precise form of these vertex operators for this case are given below

$$V = \frac{1}{(4!)^2} F_{\mu_1 \mu_2 \mu_3 \mu_4} U^{\alpha\beta} (\gamma^{[\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4]})_{\alpha\beta}. \tag{18}$$

The computation of the correlation function in (17) involve the correlators of four spin-fields  $V_{-\frac{1}{2}\alpha}(z)$ ,  $V_{-\frac{1}{2}\beta}(\bar{z})$ ,  $V_{-\frac{1}{2}\gamma}(w)$  and  $V_{-\frac{1}{2}\delta}(\bar{w})$ , defined through (12), with the indices  $\alpha, \beta, \gamma$  and  $\delta$  are appropriately contracted with the products of  $F_{\mu\nu\rho\lambda}$  and the gamma matrices (see eq.10). This is known and has been calculated in [22]. The amplitude can be computed using the techniques of [21] as was considered by [23]. When the gauge potential is taken to be of plane wave form, then the amplitude is given by the following expression

$$A = \frac{\Gamma(s)\Gamma(t)}{\Gamma(s+t+1)} [(s+t)P_1 + sP_2] \tag{19}$$

where,  $s = 2k_{1\parallel}^2 = 2k_{2\parallel}^2$  and  $t = k_1 \cdot k_2$ ,  $k_1$  and  $k_2$  are incoming and outgoing momenta respectively of the massless plane wave states and  $k_{i\parallel}$  are the components of  $k_i$  parallel to D2-brane. Furthermore,  $P_1$  and  $P_2$  appearing in (17) are given by the traces of gamma matrices.

$$P_1 = \frac{1}{4} \frac{1}{4!} \frac{1}{4!} \left[ \epsilon_{abc} [k_{1\mu} \epsilon_\nu^a \epsilon_\rho^b \epsilon_\lambda^c - k_{1\nu} \epsilon_\rho^a \epsilon_\lambda^b \epsilon_\mu^c + k_{1\rho} \epsilon_\lambda^a \epsilon_\mu^b \epsilon_\nu^c - k_{1\lambda} \epsilon_\mu^a \epsilon_\nu^b \epsilon_\rho^c] A^{\mu\nu\rho\lambda\sigma} A^{\mu'\nu'\rho'\lambda'\sigma'} \eta_{\sigma\sigma'} \right. \\ \left. \epsilon_{a'b'c'} [k_{2\mu'} \epsilon_{\nu'}^{a'} \epsilon_{\rho'}^{b'} \epsilon_{\lambda'}^{c'} - k_{2\nu'} \epsilon_{\rho'}^{a'} \epsilon_{\lambda'}^{b'} \epsilon_{\mu'}^{c'} + k_{2\rho'} \epsilon_{\lambda'}^{a'} \epsilon_{\mu'}^{b'} \epsilon_{\nu'}^{c'} - k_{2\lambda'} \epsilon_{\mu'}^{a'} \epsilon_{\nu'}^{b'} \epsilon_{\rho'}^{c'}] e^{i(k_1+k_2)\cdot X} \right] \quad (20)$$

where,  $A^{\mu\nu\rho\lambda\sigma} = \text{Tr} [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\lambda \gamma^0 \gamma^1 \gamma^2 \gamma^\sigma]$  and

$$P_2 = \frac{1}{2} \frac{1}{4!} \frac{1}{4!} \left[ \epsilon_{abc} [k_{1\mu} \epsilon_\nu^a \epsilon_\rho^b \epsilon_\lambda^c - k_{1\nu} \epsilon_\rho^a \epsilon_\lambda^b \epsilon_\mu^c + k_{1\rho} \epsilon_\lambda^a \epsilon_\mu^b \epsilon_\nu^c - k_{1\lambda} \epsilon_\mu^a \epsilon_\nu^b \epsilon_\rho^c] C^{\mu\nu\rho\lambda\sigma\mu'\nu'\rho'\lambda'\sigma'} \eta_{\sigma\sigma'} \right. \\ \left. \epsilon_{a'b'c'} [k_{2\mu'} \epsilon_{\nu'}^{a'} \epsilon_{\rho'}^{b'} \epsilon_{\lambda'}^{c'} - k_{2\nu'} \epsilon_{\rho'}^{a'} \epsilon_{\lambda'}^{b'} \epsilon_{\mu'}^{c'} + k_{2\rho'} \epsilon_{\lambda'}^{a'} \epsilon_{\mu'}^{b'} \epsilon_{\nu'}^{c'} - k_{2\lambda'} \epsilon_{\mu'}^{a'} \epsilon_{\nu'}^{b'} \epsilon_{\rho'}^{c'}] e^{i(k_1+k_2)\cdot X} \right] \quad (21)$$

where,  $C^{\mu\nu\rho\lambda\sigma\mu'\nu'\rho'\lambda'\sigma'} = \text{Tr} [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\lambda \gamma^0 \gamma^1 \gamma^2 \gamma^\sigma \gamma^{\mu'} \gamma^{\nu'} \gamma^{\rho'} \gamma^{\lambda'} \gamma^0 \gamma^1 \gamma^2 \gamma^{\sigma'} (1 + \gamma^{11})]$ . Now let us look at the expressions for  $P_1$  and  $P_2$ . Each one can be written as a product of a tensor involving only trace of the gamma matrices times another piece which contains polarization tensors and momenta. Let us denote them as  $T_{\mu\nu\dots\mu'\nu'}^{(1)}$  and  $T_{\mu\nu\dots\mu'\nu'}^{(2)}$  for  $P_1$  and  $P_2$  respectively. It is easy to check that when the tensors  $T^{(1)}$  and  $T^{(2)}$  are contracted with  $k_{1\mu}$  or  $k_{2\mu'}$ , the corresponding momenta of incoming and outgoing gauge bosons, then both  $P_1$  and  $P_2$  vanish separately and therefore the scattering amplitude,  $A$ , given by eq.(17), vanishes. This is the gauge invariance of the scattering amplitude reflected through the Ward identity.

We recall that when the BRST invariance condition is imposed in the first quantized version of string theory on its massless backgrounds one obtains the equations of motion for those backgrounds. For example, this requirement, in the NS-NS massless sector of the closed string, imposes constraints on the polarization tensors of graviton and antisymmetric tensor field in addition to the mass-shell condition. Similarly, if we consider the scattering of massless RR fields the BRST invariance restricts the form of the vertex operators as has been investigated by Polyakov [24]. For Abelian  $U(1)$  gauge field he derived the Maxwell equation by imposing BRST invariance on the corresponding RR vertex operator for vector bosons in type II theory. For the case at hand, the BRST invariance gives rise to constraints on polarization tensor and mass-shell condition; in other words  $k^\mu F_{\mu\nu\rho\lambda} = 0$  is a sufficient condition for the BRST-invariance of the vertex operator (9). Furthermore, it has been shown [24] that one can calculate three point function, using the conformal field theory techniques, involving the dilaton and RR gauge fields. This interaction does not show up in the string effective action expressed in the string frame metric. However, if one first goes over to the Einstein frame by a conformal transformation (involving the dilaton) and the redefines the RR gauge fields, the interaction terms involving the dilaton and RR gauge fields can be exhibited and the correspondence with the three point function mentioned above can be established. In the light of our investigation we can conclude that the derivation of the Ward identities for the scattering amplitude of D2-brane and 3-form potential is a consistency check of the current conservation as is also reflected in the Ward identities associated with conventional gauge theories.

We note that in the conformal field theoretic calculation of the scattering process involving D2-brane and the three-index antisymmetric gauge fields, we do not see the effects of the  $(F \wedge F \wedge B)$ -like term at this order. The presence of this CS-like term, at the tree level

calculations, can be seen if one looks at the  $(B - F - F)$ -vertex<sup>1</sup>.

We mention in passing that one might envisage our results from the perspectives of M theory. It is well known that if we start with the M-theory membrane and compactify one of the transverse directions on a circle we obtain the type-IIA D2-brane with equal tension. Furthermore, the antisymmetric 3-form in d=11 supergravity gives rise to the RR 3-form under Kaluza-Klein reduction. Therefore, the symmetries uncovered by the Ward identities in the scattering of RR 3-form from D2-brane in IIA theory should also be obeyed in the d=11 theory when one considers 3-form membrane scattering amplitude. We interpret it, at the present level of our understanding, that this is an indirect evidence for the abelian gauge invariance in quantized M-theory. Another check would be to calculate directly the scattering amplitude of the 3-form field in eleven dimensional supergravity limit off the M-theory membrane as the target. Similar calculations have been done in ten dimensional supergravity using extreme black p-branes as target and the amplitude has been compared with the string calculation when the probe energy is small or the impact parameter is large compared to the string scale. They seem to agree perfectly in this limit [23]. At this stage it is not possible to compute the scattering of three form gauge field from the membrane in the 11-dimensional theory in a reliable manner, as compared to the scattering of 3-form gauge field from D2-brane in the case of type IIA theory using the conformal field theory techniques. It will be interesting to see whether such perfect matchings come out of a M(atrrix) theory computation. We may mention that the type-IIA D2-brane being a dynamical object asks for a consistent effective quantum description of the M-theory membrane by which one can hope to provide direct checks of these symmetries in d=11, most possibly using similar techniques discussed at the beginning of this paper and in [15].

In view of these results, it will be possible to derive Ward identities for the entire massless supergravity multiplets of the ten dimensional type II theories and supergravity and super Yang-Mills theories obtained from other string theories, although a partial results were derived by Veneziano and JM a few years ago [25,26]. We hope to report our results in future.

**Acknowledgements:** We would like to thank Gabriele Veneziano for valuable suggestions during the course of this work. One of us (JM) would like to acknowledge useful discussions with H. Nicolai and would like to thank Max-Planck Institute for Gravitation for warm hospitality.

---

<sup>1</sup>We thank to S. Mukhi for discussions related to this issue



## REFERENCES

- [1] A. Sen, *Int. J. Mod. Phys. A*9 (1994) 3707, hep-th/020002; A. Sen, An Introduction to Non-perturbative String Theory hep-th/9802051; A. Giveon, M. Porrati and E. Rabinovici, *Phys. Rep. C*244 (1994) 77; M. J. Duff, R. R. Khuri and J. X. Lu, *Phys. Rep. C*259 (1995) 213, hep-th/9412184; E. Alvarez, L. Alvarez-Gaumé and Y. Lozano, An Introduction to T-duality in String Theory, hep-th/9410237. These are some of the review articles.
- [2] E. Witten, ‘Some comments on String Theory Dynamics’, *Proc. String ’95, USC, March 1995*, hep-th/9507121; *Nucl. Phys. B*443 (1995) 85, hep-th/9503124; P. K. Townsend, *Phys. Lett. B*350 (1995) 184; C. M. Hull and P. K. Townsend, *Nucl. Phys. B*438 (1995) 109, hep-th/9410167.
- [3] J. Polchinski, Lectures on D-branes, in *TASI 1996*, hep-th/9611050; C. Bachas, Lectures on D-branes, hep-th/9806199.
- [4] J. H. Schwarz, Lectures on Superstring and M-theory Dualities, *ICTP and TASI Lectures*, hep-th/9607201.
- [5] P. K. Townsend, Four Lectures on M-theory, Trieste Summer School, 1996, hep-th/9612121; M-Theory from its Superalgebra, 1997 Cargèse Lectures, hep-th/9712004; H. Nicolai, On M-Theory, hep-th/9801090.
- [6] E. Bergshoeff, E. Sezgin and P. K. Townsend, *Phys. Lett. B*189 (1987) 75, *Ann. Phys.* 185 (1988) 330.
- [7] M. J. Duff, Supermembranes, hep-th/9611203.
- [8] B. de Wit, J. Hoppe and H. Nicolai, *Nucl. Phys. B*305 (1988) 545.
- [9] J. Goldstone (unpublished); J. Hoppe, in *Proc. Int. Workshop on Constraint’s Theory and Relativistic Dynamics*, Eds. G. Longhi and L. Lusanna, World Scientific (1987).
- [10] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, *Phys. Rev. D*55 (1997) 5112, hep-th/9610043.
- [11] T. Banks, Matrix Theory, hep-th/9710231; D. Bigatti and L. Susskind, Review of Matrix Theory, hep-th/9712072.
- [12] R. Dijkgraaf, E. Verlinde and H. Verlinde, Notes on Matrix and Microstrings, hep-th/9709107.
- [13] B. de Wit, K. Peters and J. C. Plefka, Supermembranes and Supermatrix Models, Talk Presented at the Valencia Workshop Beyond the Standard Model; from Theory to Experiment, October 1997, hep-th/9712082.
- [14] P. K. Townsend, talk at the Workshop on Nonperturbative Aspects of Strings, Branes and Field Theory, CERN, December 1997.
- [15] J. Maharana, Hidden Symmetries of M theory, hep-th/9801181, *Phys. Lett. B* (in press).
- [16] G. Veneziano, *Phys. Lett. B*167 (1986) 338.
- [17] J. Maharana and G. Veneziano, *Phys. Lett. B*169 (1986) 177.
- [18] J. Maharana and G. Veneziano, *Nucl. Phys. B*283 (1987) 126.
- [19] J. Maharana, *Phys. Lett. B*211 (1988) 431.
- [20] E. Witten, *Nucl. Phys. B*443 (1995), hep-th/9503124.
- [21] D. Friedan, E. Martinec and S. Shenker, *Nucl. Phys. B*271 (1986) 93
- [22] V.A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, *Nucl. Phys. B*288 (1987) 173.

- [23] S. S. Gubser, A. Hashimoto, I. R. Klebanov and J. M. Maldacena, Nucl. Phys. B472 (1996) 231;  
A. Hashimoto and I. R. Klebanov, Scattering of Strings from D-branes, hep-th/9611214.
- [24] D. Polyakov, RR - Dilaton Interaction in a Type IIB Superstring, hep-th/9512028.
- [25] J. Maharana and G. Veneziano, Unpublished results on Ward identities for Supergravity particles from string theory, January, 1986.
- [26] Some of the related works are: M. Evans and B. Ovrut, Phys. Rev. D39 (1989) 3016, B. Ovrut and S. Kalyana Rama, Phys. Rev. D45 (1992) 550; H. Gasparakis, hep-th/9308156.