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Vertex Operators for Axionic Wormholes.

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Abstract

The long distance effect of a small wormhole on correlation functions can be summarized as the insertion of a local 'vertex' operator. We compute, to leading order in the size of the wormhole and in Planck's constant, the vertex operator for axionic wormholes. The effects of matter fields, primarily in the form of conformally coupled scalars, are also studied. The effective bilocal action does not seem to factorize, clouding the notion of a local vertex operator.

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1. It has been recently realized that the effects of topology change in quantum gravity may be quite interesting and surprising. It is not quite clear yet what these effects are. It has been suggested, for example, that they may lead to loss of quantum coherence[1,2,3], though it is believed now that this occurs only as a sort of quantum indeterminacy of the constants of nature[4,5]. Perhaps more interesting is the proposal that the probability distribution for the constants of nature is delta-function peaked at vanishing cosmological constant[6,7,8]. It has been argued, though, that one should really interpret such results as indicating that the many universes wave function evolves typically into states of very many large and cold universes[9,10].

In trying to ascertain what the effects of topology change are, the useful concept of a vertex operator is introduced[1]. If we look at physics at distance scales much larger than the scale of the topology changing quantum fluctuations, it is rather plausible that their effects can be summarized in terms of local 'vertex' operators. It is likely that a good understanding of these vertex operators will play a role in figuring out what the ultimate effect of topology change in quantum gravity may be.

In this paper we will set up the computation of vertex operators. We will work within the context of Euclidean Quantum Gravity, and will assume the validity of a semiclassical expansion, summing over saddle points of the Euclidean action. The topology changing saddle point configurations that we will consider are the axionic wormholes, and in particular the explicit solution of Giddings and Strominger[3]. The matter sector of the theory we consider contains a complex scalar field with $U(1)$ symmetric potential and undergoes spontaneous symmetry breaking. Additional matter fields will be added, but they will play no role in stabilizing the wormhole. They merely propagate in the background of the wormhole. The different wormholes in this theory are labeled by the charge Q they carry. We will compute a vertex operator for each Q.

2. We begin by reviewing the axionic wormhole solutions. In a theory of a charged scalar $\phi = \rho/\sqrt{2} \exp i\theta$ with a symmetry under constant shifts in θ, the wormhole arises as a saddle point in the action with the constraint[11,12] that the initial and final configurations be of definite charge Q. We will present here

a quick and (not really) dirty way of reproducing the results of ref. [11]. Our method here has the added advantage of being explicitly covariant. Moreover, the result in eq. (10), which will play an important role in the computation of the vertex operators, is obtained regardless of the details of the background metric. Therefore, whatever results follow from it will be valid even for non-spherically symmetric (*i.e.*, $O(4)$ symmetric) wormholes.

Our method yields directly the euclidean (*i.e.*, imaginary time) equations of motion. It neatly resolves the paradox, explained below, that has led some to suggest that the rotation into imaginary time and the derivation of equations of motion don't commute. For this reason we first explain our method in a simpler system, a particle in two dimensions moving under the influence of a central potential. The Euclidean action is

$$I = \int d\tau \, (\frac{1}{2}\dot{r}^2 + \frac{1}{2}r^2\dot{\theta}^2 + V(r))$$

The θ equation of motion

$$\partial_\tau(r^2\dot{\theta}) = 0$$

yields

$$\dot{\theta} = \frac{C}{r^2}, \tag{1}$$

where C is a constant. Therefore, one may write

$$I_{eff} = \int d\tau(\frac{1}{2}\dot{r}^2 + V_{eff}(r))$$

where

$$V_{eff} = V(r) + \frac{C^2}{2r^2}. \tag{2}$$

This we recognize as the correct expression for the effective potential, and interpret the constant C as the conserved angular momentum. On the other hand, the equation of motion for the radial coordinate from the original action I is

$$-\ddot{r} + r\dot{\theta}^2 + V'(r) = 0 \tag{3}$$

or, using (1)

$$-\ddot{r} + \frac{C^2}{r^3} + V'(r) = 0 \tag{4}$$

This is not the equation of motion obtained from I_{eff}. While we think I_{eff} is correct, the procedure leading to eq. (4) is one generally accepted as correct. The resolution of this paradox lies in specifying what we mean by 'I_{eff} is correct'. We know from the WKB methods that if we compute a tunneling amplitude we would get a suppression factor $\exp(-I_{eff}(\bar{r}))$, with $\bar{r}$ an extremum of I_{eff} satisfying the tunneling boundary conditions. Therefore, the correct question is not what are the 'correct' equations of motion in Euclidean space, but how do we obtain the correct transition amplitudes. And of course what needs to be computed are transition amplitudes between states of definite angular momentum ℓ. We must stress that this is not an artifact introduced to save the day. In fact it has long been known that states of definite angular coordinate are ill-defined[13]. Thus one computes

$$\begin{aligned}\langle r_2\ell|r_1\ell\rangle &= \int d\theta_1 d\theta_2 \exp(i\ell(\theta_2-\theta_1))\int_{b.c.}[rdrd\theta]e^{-I(r,\theta)} \\ &= \int_{b.c.}[rdr]\int[d\theta]\exp(-I(r,\theta)+i\ell\int dt\dot{\theta}) \end{aligned} \tag{5}$$

Here 'b.c.' stands for the appropriate boundary conditions, namely, $r(\tau_i) = r_i$ and, in the first line only, $\theta(\tau_i) = \theta_i$. The integral over the angular variable is trivial, for there are no boundary conditions and the action is at most quadratic in θ. Rather than asking what are the equations of motion for θ we should ask what configuration $\bar{\theta}$ will reproduce the result of this exact integration when inserted into the argument of the exponent. The point is this. Generally, the functional integral is approximated by expanding the action about a background configuration. The obvious choice is to take that configuration to be a solution to the equations of motion. Since the action for θ is quadratic, the result will be independent of the choice of background. But what will play the role of the solution to the equations of motion will be a background such that the integral doesn't have to be performed. Thus, the background is chosen to satisfy

$$\dot{\bar{\theta}} = i\frac{\ell}{r^2} \tag{6}$$

We see now how to reconcile eqs. (2) and (4). Inserting eq. (6) into the action, *including* the surface term gives, by design, I_{eff}, and therefore is the basis for the correct semiclassical approximation. On the other hand, the r equation of motion is still given by (3), since it gets no contribution from the added 'surface' term. But now substitution of eq. (6) into (3) yields precisely the expression that is obtained from varying I_{eff}. We must stress that we are not rotating the functional integral for the angular variable into the imaginary axis.

3. The full-fledged field theoretic model is dealt with in much the same way as the simple example above. As in ref. [11] we compute the transition amplitude between geometries Σ and Σ' on which π_θ, the momentum conjugate to θ is defined. To remove the boundary conditions for the integration over the goldstone boson θ we introduce a vector π^μ satisfying

$$\begin{aligned} \pi^\mu_{;\mu} &= 0 && (7) \\ \pi^\mu n_\mu|_{\Sigma(')} &= \pi_\theta(') && (8) \end{aligned}$$

The latter of these equations might be too strong. Physically, we want to restrict only the global charge Q, so we need $\int_{\Sigma(')} d\Sigma^\mu \pi_\mu = (-)Q$. In analogy with our simple model above, we have

$$\begin{aligned} \langle \Sigma'\pi'_\theta|\Sigma\pi_\theta\rangle &= \int[d\theta_1][d\theta_2]\exp(i\int_{\Sigma'} d\Sigma\pi'_\theta\theta_2 - i\int_\Sigma d\Sigma\pi_\theta\theta_1)\int_{\text{b.c.}}[d\theta]\int[d\mu(\text{fields})]e^{-I} \\ &= \int[d\theta]\int[d\mu(\text{fields})]\exp(-I + i\int d^4x\,\sqrt{g}\,\pi^\mu\theta_{;\mu}). \qquad (9) \end{aligned}$$

Again the integration over θ can be done exactly, and the result corresponds to a 'would-be' background field $\bar\theta$ satisfying

$$\bar\theta_{;\mu} = i\frac{\pi_\mu}{\rho^2}. \qquad (10)$$

Here ρ is the modulus of the complex scalar, which we take to be a constant.

It may be proved in a variety of ways that the vertex operator for this theory is proportional to $\exp(iQ\theta(x))$. Using eq. (10) we may give two new derivations of this result. These new derivations are interesting to us because they share features with the calculations we will present later for other factors of the vertex operator. The wormholes in this theory have size $r_0 \sim \sqrt{Q/M_P\rho}$, where M_P is the Planck mass. For fixed Q, if we consider the physics on length scales $L \gg r_0$, we can imagine integrating them out. In constructing an effective theory for wavelengths longer than L we introduce the vertex operator by the requirement that it correctly reproduces expectation values when the fluctuations are restricted to not include wavelengths shorter than L. In particular, wormhole fluctuations are not included in the sum over configurations. One assumes that the expectation value in (9) factorizes, at least to leading order in r_0/L. Each factor is of the form

$$\langle \Sigma \pi_\theta | V \rangle = \int [d\theta] \int [d\mu(\text{fields})] \exp(-I + i \int d^4x \sqrt{g}\, (\pi^\mu \theta)_{;\mu}) V$$

Here the integral over the metric is thought of as dominated by fluctuations about flat space (the cosmological constant is taken to vanish, for the mean time). Again, π^μ is chosen to satisfy the boundary condition $\pi^\mu n_\mu|_\Sigma = \pi_\theta$. But now we cannot insist on $\pi^\mu_{;\mu} = 0$ everywhere in the manifold, for this is inconsistent with $\int_\Sigma d\Sigma^\mu \pi_\mu = Q \neq 0$. Instead we must have charge flowing in somewhere. Naturally we want it to flow in from the wormhole at x_0,

$$\pi^\mu_{;\mu} = Q\delta(x - x_0)/\sqrt{g} \tag{11}$$

Therefore

$$\int d^4x \sqrt{g}\, (\pi^\mu \theta)_{;\mu} = Q\theta(x_0) + \int d^4x \sqrt{g}\, \pi^\mu \theta_{;\mu}$$

and to compensate for the first term on the right hand side we must have $V \sim \exp(-iQ\theta(x_0))$.

This derivation was a bit sketchy and imprecise, but one gets away with this because the result relies ultimately on symmetry. If we want to nail down the vertex operator more precisely we will have to proceed with some more care. We will restrict our attention to the case of vanishing cosmological constant. Thus, in the semiclassical approximation we will always be able to take the wormhole to join two asymptotically flat regions. We can then ask questions about correlations for field operators inserted in these asymptotic regions. In the semiclassical approximation, if these field operators don't involve the metric, they may be computed as correlations in a fixed background metric. If the background metric is that of a wormhole, we will denote such correlations by $\langle \cdots \rangle_w$. The ellipses stand for a string of operators located at either asymptotically flat region. A prime will be used to distinguish between the two regions. For a flat background we will use $\langle \cdots \rangle_0$.

The vertex operator is obtained from the requirement that

$$\langle(\cdots)(\cdots)'\rangle_w = \langle(\cdots)\times V\rangle_0 \; \langle(\cdots)'\times V\rangle_0 \tag{12}$$

In particular, one must have

$$\langle \mathbf{1}\rangle_w = \langle \mathbf{1}V\rangle_0^2 , \tag{13}$$

where $\mathbf{1}$ is the unit operator. There is a constant term in the vertex operator, $V = c + \cdots$, and it satisfies

$$c^2 = \frac{\langle \mathbf{1}\rangle_w}{\langle \mathbf{1}\rangle_0^2}. \tag{14}$$

Comments:

1. Eq. (12) must be interpreted carefully when the wormhole background breaks symmetries. The corresponding integrals over zero modes must be extracted explicitly. The left hand side in (12) then depends explicitly on the particular configuration. Correspondingly, the vertex operator on the right hand side will generically depend on the parameters that specify such configuration. The full effect of the wormhole is then reproduced not just by introducing the vertex operator, but by summing over the corresponding parameters. A prototypical example is the translational invariance of the center of the wormhole when viewed from either end. This gives eight zero modes, and the vertex operators will depend on eight parameters x_0, x_0'. The sum over parameters corresponds to the volume integrals $\int d^4x_0 \sqrt{g} \int d^4x_0' \sqrt{g}$. In going to the integral over parameters, one must also include other appropriate factors such as $\sqrt{I_0/\hbar}$ for each zero mode, where I_0 is the value of the effective action at the saddle point. We have implicitly absorbed these into c^2.

2. Generally one is not interested in the case of vanishing cosmological constant Λ. If not vanishing, it must still be small enough for wormholes to make sense. A sensible criterion is that the de-Sitter radius $M_P/\sqrt{\Lambda}$ be much larger than the throat size r_0. The wormhole solution is much like a bridge between two half-spheres. The correlations are then computed for operators inserted on the maximal sections, three-spheres that become

our asymptotically flat spaces as $\Lambda \to 0$. In the expression (14) for the constant c^2 the denominator can therefore be replaced by $\langle 1 \rangle_{hs}^2$, where the subscript stands for 'half-sphere'. This result is reminiscent of the normalization factors usually encountered in instanton physics[15]. We find this reassuring.

3. The computation of c^2 is hard, and we do not undertake it here. It is tempting to conjecture what its phase is. First, we naïvely replace $\langle \mathbf{1} \rangle_{hs}^2$ by $\langle \mathbf{1} \rangle_s$, where the subscript now refers to the whole sphere. The phase of this object was computed by Polchinski[14], given an assumption about the contour of integration of the conformal mode. The result obtained in ref. [14] was $i^{1+(d+1)}$ in d dimensions. The first factor arises from the zero mode of the scalar laplacian, while the next $d+1$ factors correspond to similarly many conformal killing vectors of the ($O(d+1)$-symmetric) d-sphere. Using the same assumptions one may attempt to obtain the phase of the numerator. We have not done so, but it is tempting to guess that the result is $i^{1+(d)}$. The first factor is always there and comes again from the zero mode of the laplacian. The second factor is suggested by the $O(d)$ symmetry of the Giddings-Strominger wormhole. If this is correct then c^2 is $-i$ times a positive number. Perhaps this is the phase we should expect if the wormhole is the leading contribution to the decay width to states of different topology.

Of course, the constant c must appear in V multiplied by $\exp(-iQ\theta(x_0))$. We now present a second derivation of this fact. The method will be instrumental in deriving the contribution of other fields to the vertex operator. The expectation value of a string of operators $\mathcal{O}_1(x_1)\cdots\mathcal{O}_n(x_n)$ in the semiclassical approximation is given, to zeroth order in Planck's constant, by their saddle point values. Therefore we may write

$$\langle \theta(x_1)\cdots\theta(x_n)\rangle_w = \langle \mathbf{1}\rangle_w \bar{\theta}(x_1)\cdots\bar{\theta}(x_n) + \mathcal{O}(\hbar)$$

As argued above, the appropriate 'saddle point' value for the angular variable is given by $\bar{\theta}$ satisfying eq. (10). This is equal to

$$\langle \theta(x_1)\cdots\theta(x_n)V(x_0)\rangle_0 \, \langle \mathbf{1} V(x_0')\rangle_0 \tag{15}$$

From the unprimed asymptotic region point of view, the wormhole is essentially at the point x_0. Thus π^μ may be taken to satisfy eq. (11), or

$$\Box\bar{\theta}(x) = i\frac{Q}{\rho^2}\delta(x-x_0)/\sqrt{g}.$$

Up to a normalization factor of $-iQ/\rho^2$, this is the same equation satisfied by the Feynman propagator $\Delta_F(x,x_0)$ for the (properly normalized) field $\rho\theta(x)$. Therefore we have

$$\langle\theta(x_1)\cdots\theta(x_n)\rangle_w = \langle 1\rangle_w (-iQ)^n \prod_{i=1}^{n}\Delta_F(x_i,x_0). \tag{16}$$

This is reproduced by the vertex in eq. (15) if we take

$$V(x_0) = c\frac{1}{n!}(-iQ)^n\theta(x_0)^n$$

Since this is valid for arbitrary n, we may sum up the series, obtaining

$$V(x_0) = c\exp(-iQ\theta(x_0)).$$

More comments:

1. The result is independent of the details of the wormhole metric. A similar argument has been given elsewhere using the explicit form of the Giddings-Strominger background (which, of course, has the same functional dependence as a Feynman propagator). Our derivation sidesteps the need for such explicit knowledge. Moreover, the all important factor of i is often missed. It can be seen to arise, of course, only through the argument given above[1].

2. Counting $\hbar$'s is instructive. The propagator Δ_F is linear in $\hbar$, and one must therefore introduce a factor of $(1/\hbar)^n$ in eq. (16). Thus the vertex is $V = c\exp(-iQ\theta/\hbar)$. This is again expected. The integrally quantized charge q is related to the classical charge Q through $q = Q/\hbar$, which diverges as $\hbar \to 0$.

[1] Calculations of wormhole-wormhole correlations[16,17] that have been given usually don't include the factor of i. The results may be incorrect by a factor of $i^2 = -1$. We have not checked this.

4. It is now straightforward to introduce additional matter fields, ψ, not directly responsible for the stabilization of the wormhole. Since we are interested in long distance effects, we shall take such fields to be massless. This is an oversimplification, for it should suffice to consider massive fields with $m \ll r_0^{-1}$, where m is the mass of the field. But our calculations will be simpler with the choice $m = 0$. These fields have trivial saddle point configurations, so that $\langle\psi(x_1)\cdots\psi(x_n)\rangle_w$ has no contribution to zeroth order in $\hbar$. We must therefore understand the propagator of these fields in the wormhole background. To this end we specify the metric to be the conformally-flat Giddings-Strominger solution

$$ds^2 = C(x)\delta_{\mu\nu}dx^\mu dx^\nu$$

with conformal factor

$$C(x) = 1 + \frac{r_0^4}{|x - x_0|^4}$$

Here

$$r_0^4 = \frac{GQ^2}{12\pi^3\rho^2} \tag{17}$$

and G is Newton's constant. The unprimed region x is characterized by $|x - x_0| \gg r_0$ while the primed one x' is characterized by $|x' - x_0| \ll r_0$. Now the two point function $\langle\psi(x_1)\psi(x_2)\rangle_w$ is given, to good approximation, by $(\langle 1\rangle_w / \langle 1\rangle_0)\times \langle\psi(x_1)\psi(x_2)\rangle_0$, since the points x_i are both in the asymptotically flat region and satisfy $|x_i - x_0| \gg r_0$. More generally, to leading order in $r_0/|x_i - x_0|$, the correlation $\langle\psi(x_1)\cdots\psi(x_n)\rangle_w$ is properly approximated by $\langle\psi(x_1)\cdots\psi(x_n)V(x_0)\rangle_0\ \langle V(x_0')\rangle_0$, with V given as before (and only the constant term contributing). Moreover, the same is true if any number of θ fields are introduced into the expectation value.

It is more interesting to consider $\langle\psi(x)\psi(x')\rangle_w$. Now the leading contribution is at best of the order of $1/|x - x_0|^2$. Moreover, it is not obvious that such correlation factorizes according to eq. (12). To go any further we need the propagator for the field ψ in the wormhole background. This is generally difficult to compute. But since the wormhole is conformally flat, one can easily compute it for conformally coupled fields. For them[18],

$$\langle\psi(x)\psi(x')\rangle_w = \langle 1\rangle_w C^{\alpha/2}(x)C^{\alpha/2}(x')\tilde{\Delta}_F(x, x')$$

where $\tilde{\Delta}_F$ stands for the flat space Feynman propagator appropriate to the field in question and the α is the corresponding conformal weight. For example, α is -1 for scalars and $-3/2$ for spin-1/2 fermions. Take, for example, the case of scalars. The propagator is

$$\Delta_F(x,x') = -\frac{1}{4\pi^2}\frac{1}{|x-x'|^2},$$

so that, to leading order in $r_0/|x-x_0|$ and $|x'-x_0|/r_0$,

$$\langle\psi(x)\psi(x')\rangle_w \approx \langle 1\rangle_w \left(-\frac{1}{4\pi^2}\right)\frac{1}{|x-x_0|^2}\frac{|x'-x_0|^2}{r_0^2}.$$

To this order, $|x-x_0|$ is the distance from the wormhole 'throat' (defined by $|x-x_0| = r_0$) to the point x. The factor $r_0^2/|x'-x_0|$ has a similar interpretation, for

$$\int ds \approx \int_{x'}^{r_0} \sqrt{C(z)}\, dz \approx r_0^2 \int_{|x'-x_0|}^{r_0} \frac{1}{z^2}\, dz \approx \frac{r_0^2}{|x'-x_0|}$$

Therefore, the result is the product of the inverse of the square of the distance from the throat to each of the points x and x'. We can make this explicit by writing the second factor in the coordinate system

$$(y-y_0')_\mu = r_0^2\frac{(x-x_0)_\mu}{|x-x_0|^2}. \tag{18}$$

The correlation does indeed factorize. It is reproduced by

$$\langle\psi(x)V(x_0)\rangle_0\ \langle\psi(y')V(y_0')\rangle_0$$

if $V(x)$ is taken to be proportional to $\psi(x)$,

$$V(x_0)V(y_0') = c^2(-4\pi^2 r_0^2)\psi(x_0)\psi(y_0'). \tag{19}$$

It is straightforward to extend this result to the case when n fields are inserted on each side

$$\left\langle \prod_{i=1}^{n}\psi(x_i)\prod_{i=1}^{n}\psi(y_i')\right\rangle_w = \langle 1\rangle_w (-4\pi^2 r_0^2)^n n! \prod_{i=1}^{n}\frac{1}{|x_i-x_0|^2}\frac{1}{|y_i'-y_0'|^2}. \tag{20}$$

This gives

$$V(x_0)V(y_0') = c^2 \frac{1}{n!}(-4\pi^2 r_0^2)^n \psi(x_0)^n \psi(y_0')^n. \tag{21}$$

Putting it all together we obtain

$$V(x_0)V(y_0') = c^2 e^{-iQ\theta(x_0)} e^{iQ\theta(y_0')} \exp(-4\pi^2 r_0^2 \psi(x_0)\psi(y_0')). \tag{22}$$

We note that:

1. After all, the 'vertex' operator does not seem to factorize. It can be written as an infinite sum of factorized terms. For large field strengths, though, such expansion may be troublesome. We don't know how the wormhole α-parameters may be introduced in this case. This may not be as bad as it seems. After all, we must keep in mind that the amplitudes do indeed factorize.

2. We may again count $\hbar$'s. The wormhole expectation value (20) is order $\hbar^n$, while $2n$ propagators give a factor of $\hbar^{2n}$. We must therefore include a factor of $(1/\hbar)^n$ in eq. (21) or a factor of $1/\hbar$ in the argument of the exponential in the vertex operator (22).

3. The constant c^2 appears in front of every term in the bilocal vertex operator. Aside from this, the relative strengths of terms in the bilocal vertex are rather trivially computed. Fischler has suggested a rather simple interpretation for the appearance of this constant[19]. It simply corresponds to the mismatch in vacuum fluctuations for fields propagating in different backgrounds. In a first quantized formalism, a particle propagator from x_1 to x_2 is the sum over paths from one point to the other. In the background of the wormhole there are paths which go through the throat, into the second asymptotic region and back to the first one again. As $r_0 \to 0$ these paths are severed into two disconnected pieces. One goes from x_1 to x_0 and back to x_2, while the other is a closed loop in the second region. Thus, a factor of $\langle 1 \rangle_w / \langle 1 \rangle_0$ appears from the normalization of paths in the first region, while a second factor of $1/\langle 1 \rangle_0$ comes from the loops in the second region.

4. We omitted in (20) the leading terms which correspond to making contractions between fields in the asymptotic region. The lower order terms in $V(x_0)V(y_0')$ automatically reproduce such terms, to leading order in $r_0/|x_i - x_0|$. One may worry that the next to leading terms are still more important than those kept in (20), and therefore the result may be invalidated. This is not the case. Take the collection of points $\{x_i\}$ to lie on the same three-sphere $|x_i - x_0| = r \sim L \gg r_0$. Then the conformal factors $C^{-1/2}(x_i)$ are all numerically the same. The physical distance between points x_i and x_j as measured intrinsically (within the three-sphere) is then $C^{1/2}(x_i)|x_i - x_j|$. This is precisely what appears in the denominator in (20), and it is what the vertex operator in flat space reproduces.

5. If there are N species of conformally coupled massless scalars, ψ_i, our result generalizes trivially. All we need is to make the replacement $\psi(x_0)\psi(y_0') \to \sum_i \psi_i(x_0)\psi_i(y_0')$.

A similar calculation may be performed for fermions or vector bosons. If massless, these are automatically conformally coupled. The new feature that arises is that, for fixed background metric, the amplitude $\langle\psi(x)\psi(x')\rangle_w$ does not factorize. For example, in the case of fermions the two propagators from the asymptotic regions to the throat are connected through a Dirac gamma matrix. The direction of this matrix is determined by the relative angle between the points x and x'. Now, since the wormhole is $O(4)$ symmetric, *i.e.*, invariant under

$$(x - x_0)_\mu \to O_\mu^\nu (x - x_0)_\nu,$$

one must integrate over the corresponding zero modes. So even if one can write an expression for $V(x_0)V(y_0')$ analogous to (19), the result will vanish when one averages over directions (that is, when one performs the integral over $O(4)$). In the case of scalars considered above, the result was rotationally invariant. We implicitly absorbed a factor of the volume of $O(4)$ in the definition of the constant c^2. We get away with this because this volume factor is finite. Correlations of more than two fermions will involve a multilinear gamma matrix structure, and so the average over directions will not vanish. In this way the $O(4)$ symmetry ensures the Lorentz invariance of the flat space effective theory.

5. Finally we turn to the question of emission and absorbtion of gravitons by the wormhole. We address this question by computing amplitudes involving insertions of the metric in the asymptotically flat regions. Now, if we straightforwardly compute correlations involving $\prod_i g_{\mu\nu}(x_i)$ we will obtain a gauge dependent, and therefore meaningless, result. We get around this difficulty by computing correlations involving the curvature scalar $R(x)$. To leading order in Planck's constant the result is again trivially given by the background configuration,

$$\langle R(x_1)\cdots R(x_n)\rangle_w = \langle 1\rangle_w (\bar{R}(x_1)\cdots \bar{R}(x_n) + \mathcal{O}(\hbar)),$$

where, for the Giddings-Strominger wormhole,

$$\bar{R}(x) = -\frac{24r_0^4}{|x-x_0|^6(1+r_0^4/|x-x_0|^4)^3}$$

The computation of the corresponding amplitude in flat space is simplified by the observation that in the semiclassical approximation we can use the equations of motion. Therefore we write,

$$\langle R(x_1)\cdots R(x_n)V(x_0)\rangle_0 = (8\pi G\rho^2)^n \langle \theta^{,\mu}\theta_{,\mu}(x_1)\cdots \theta^{,\mu}\theta_{,\mu}(x_n)V(x_0)\rangle_0 .$$

It is easy to check, using eq. (17), that these expressions agree. In fact no computation is necessary, for we could have also used the equations of motion in the computation of the correlation in the wormhole background. Then we would be comparing correlations of θ fields only. The vertex operator was designed for these to work. It should be kept in mind that this result has been obtained to leading order in r_0/L. Beyond leading order in this parameter one expects to have to introduce into the vertex operator terms involving derivatives of the goldstone boson field $\theta_{,\mu}$ and/or the scalar curvature R.

The vertex operator we obtain seems to correctly reproduce amplitudes involving the curvature scalar $R(x)$. This is a nontrivial statement. One would expect that the vertex operator would itself have terms proportional to $R(x_0)$. In the dilute gas of wormholes approximation such term would yield a renormalization of Newton's constant. The α-parameter dependence of Newton's constant could dictate the values of low energy coupling constants[20,21]. Our computation indicates that no such (α-dependent) renormalization of Newton's constant occurs at the wormhole scale[2], to leading order in $\hbar$ and r_0/L. We should note, however, that in the semiclassical approximation it is impossible to distinguish the effects of the operators $R(x_0)$ and $8\pi G\rho^2 g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}$. They could both give cancelling contributions to the vertex operator. Moreover, there is no reason to expect that there is no such renormalization beyond leading order in $\hbar$ and r_0/L.

6. Our conclusions are as follows. The vertex operator $V(x_0)$ for axionic wormholes can be computed in the semiclassical approximation by considering correlations of fields inserted in the asymptotically flat regions of spacetime. Up to an overall constant c, the relative contributions of different operators to V can be explicitly computed. The constant c is itself computable, given a prescription for the evaluation of the path integral. It is expressed as the ratio of the partition function for the wormhole background to that of the flat background (*cf.*, eq. (14)). For a theory with an axion θ and a conformally coupled massless real scalar ψ, the vertex operator does not factorize. The effective bilocal action is given, to leading order in $\hbar$ and the wormhole size r_0, by (the integral over x_0 and y_0' of) eq. (22).

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[2]This is in agreement with the claim in ref. [17].

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