CERN-TH.4344/85

GAUGE WARD IDENTITIES OF THE COMPACTIFIED BOSONIC STRING

J. Maharana*) and G. Veneziano
CERN - Geneva

ABSTRACT

The BRS quantization of the closed bosonic string in d compact and (D-d) non-compact dimensions is shown to require D = 26 and to restrict the form of allowed compactifications. Suitable canonical transformations in the phase space path integral generate the Ward identities of E_8 (d = 8) and of $E_8 \times E_8$ or Spin(32)/Z_2 (d = 16).

---

*) Permanent address: Institute of Physics, 751005 Bhubaneswar, India.

CERN-TH.4344/85
December 1985

1. One of the most striking results obtained during the dormant decade ('74-'84) of dual-string theory[1] has been the discovery [2] of huge internal symmetry groups resulting from compactification of a number d (out of the total D) of string coordinates on particular tori.

This development is a crucial ingredient of new anomaly free superstrings [3], which offer better phenomenological prospects[4] than the one originally proposed by Green and Schwarz [5].

In the case of closed strings the groups thus generated correspond to unbroken local symmetries of the theory and manifest themselves, in particular, through the presence of massless vector-bosons in the adjoint representation of a gauge group G.

While the emergence of a $U(1)^d$ gauge group is expected on the ground of standard Kaluza-Klein-type considerations ,if the uncompactified theory is invariant under general coordinate transformations (GCT), the completion of $U(1)^d$ to a much larger group of rank d(or 2d) is still highly mysterious.

Recently, one of us [6] has proposed a simple method for deriving the Ward identities (WI) of GCT (and possibly more) from suitable canonical transformations in a (first quantized) phase-space path integral formulation of the theory.

In this note we shall extend that approach to the above mentioned gauge symmetries showing that, under particular conditions, E_8, $Spin(32)/Z_2$ and $E_8 \times E_8$ WI's are indeed generated and give the vertex operators of the corresponding massless gauge bosons.

We shall only deal here with purely bosonic closed strings in D=26. A crucial ingredient of the construction is that left and right moving compact coordinates $X_L^I(\tau-\sigma)$ and $X_R^J(\tau+\sigma)(I,J=1...d)$ are to be treated as completely independent. We thus follow the framework of the heterotic string [3] except for the fact that we do not replace right movers by fermionic-string coordinates (generalization to the heterotic case looks however straightforward).

The allowed eigenvalues of $p_L(p_R)$ in $X_L(X_R)$ will play both the role of ordinary momenta and that of winding numbers. This, plus self-consistent interactions, forces [3] the allowed momenta to lie in an even, self dual Euclidean lattice, which exists only if d=8n . Surprisingly, if we don't fix d a priori and proceed to a BRS quantization [7,8], we find a similar constraint on d ,if we require integer Regge intercepts (massless states).

The path integral over X involves arbitrary functions of σ,τ. It is only the classical solutions which obey the second-order wave equation $\ddot{X}=X''$. Similarly, in the case of independent left and right moving compact coordinates, we should have the constraints $\dot{X}_{R,L}=\pm X'_{R,L}$ only as classical equations. This suggests the usefulness of a formulation [3] involving fermionic coordinates which obey the two-dimensional Dirac-equation. Fortunately,this formulation is also the one best suited for generalizing the approach of ref [6].

2. BRS formulation with fermionic compactified coordinates

The fermionic coordinates needed in order to describe the compact bosonic coordinates X^I, X^J (I,J=1...d) are a set of two dimensional Majorana spinors $\psi^i(\sigma,\tau)$ (i=1..2d, two-dimensional Dirac index understood). An invariant action can be written easily [9,3] as ($\alpha'=1/2$):

$$S=-\frac{1}{2}\int d^2\sigma\, e\left[\partial_\alpha X^\mu \partial_\beta X^\nu g^{\alpha\beta}\eta_{\mu\nu}+i\,\bar{\psi}^i\gamma^a e_a^\alpha\partial_\alpha\psi^i\right] \tag{1}$$

where μ, ν = 0,1...25-d; i=1...2d; α,β = 0,1 and a,b,μ,ν indices are raised and lowered by the flat Minkowski metric with $\eta_{00}=-1$. Furthermore

$\sigma_\alpha=(\sigma,\tau)$, $e=\det e^a_\alpha=\sqrt{-g}$, $g^{\alpha\beta}=e^\alpha_a e^{a\beta}$ and γ^a are two dimensional γ-matrices with $(\gamma^1)^2=-(\gamma^0)^2=\gamma_5^2\equiv(\gamma^1\gamma^0)^2=1$ and $\bar\psi=\psi^T\gamma^0$. We have also rescaled X, ψ to get rid of a trivial factor $1/\pi$ in S. Note that the two components $2\psi_L{}^i{}_R=(1\pm\gamma_5)\,\psi^i$ decouple from each other.

S is invariant under σ_α reparametrizations since the additional spin connection term needed to transform ∂_α into D_α vanishes [9] between Majorana fermions. It is also invariant [9] under conformal (Weyl) transformations $e^\alpha_a\to Ce^\alpha_a$ with $X\to X$, $\psi\to\sqrt{C}\psi$.

In order to perform correctly a canonical quantization we need to find the first-class constraints of the system. Defining as usual canonical momenta $p_i\equiv\frac{\delta\mathcal{L}}{\delta\dot q_i}$, we find the algebraic constraints

(2) $(P_e)^a_\alpha=0$; $2P_{\psi^i}\equiv 2\pi^i=-ie\,\bar\psi^i\gamma^a e^0_a$

and also, upon use of $\delta S/\delta e^\alpha_a\equiv T_\alpha{}^a=0$,

(3)
$$L_+\equiv\frac{1}{4}(P_\mu+X'_\mu)^2+\pi_R{}^i\,\partial_1\psi_R{}^i=0$$
$$L_-\equiv\frac{1}{4}(P_\mu-X'_\mu)^2-\pi_L{}^i\,\partial_1\psi_L{}^i=0$$

L_+, L_- obey the usual Poisson bracket algebra

(4) $\{L_\pm(\sigma),L_\pm(\sigma')\}=\pm\left(L_\pm(\sigma)+L_\pm(\sigma')\right)\partial_\sigma\,\delta(\sigma-\sigma')$

hence constitute first-class constraints. Finally, the canonical Hamiltonian H_0 vanishes.

Since local symmetries of the theory are also imposed at the quantum level (see below) we can choose the orthonormal gauge $e^\alpha{}_a=\delta^\alpha{}_a$ in which the action becomes

(5) $$S=\frac{1}{2}\int d^2\sigma\left[\dot X^2-X'^2+i\left(\psi_L{}^i\,\partial_+\psi_L{}^i+\psi_R{}^i\,\partial_-\psi_R{}^i\right)\right]$$

showing that left (right) handed fermions describe left (right) movers ($\partial_\pm = \partial_0 \pm \partial_1$).

Using the general procedure of Fradkin and Vilkovisky [10] as in ref [8], quantization requires anti-commuting ghost coordinates $\eta^\pm$ with conjugate momenta $\mathcal{P}_\pm$ out of which the BRS charge can be written as

$$Q = \int d\sigma \left(L_+ \eta^+ + L_- \eta^- + \mathcal{P}_+ \eta^+ \partial_1 \eta^+ - \mathcal{P}_- \eta^- \partial_1 \eta^- \right) \tag{6}$$

and the total Hamiltonian density is given by

$$H_\phi = \{ \phi, Q \} \tag{7}$$

where ϕ is a gauge fixing function.

In the orthonormal gauge the non ghost part of the action should reduce to (5) which is what happens if we take $\phi = (\mathcal{P}_+ + \mathcal{P}_-)$. We thus obtain:

$$\begin{aligned} H = \frac{1}{2}\left(P^2 + X'^2\right) + \Pi_R^i \partial_1 \psi_R^i - \Pi_L^i \partial_1 \psi_L^i + \\ + \mathcal{P}_+ \partial_1 \eta^+ - \mathcal{P}_- \partial_1 \eta^- \end{aligned} \tag{8}$$

Furthermore we have to impose the constraints (2) giving

$$\Pi_{L,R}^i = \frac{i}{2} \psi_{L,R}^{T,i} \tag{9}$$

In conclusion, the phase-space functional integral in the orthonormal gauge takes the form:

$$Z \exp(iW) = \int dX^\mu dP_\mu d\psi_{L,R}^i d\Pi_{L,R}^i d\eta^\pm d\mathcal{P}_\pm \times \tag{10}$$
$$\delta\left(\Pi_{L,R}^i - \frac{i}{2}\psi_{L,R}^i\right) \exp\left(i \int d^2\sigma \left(P \cdot \dot{X} + \Pi_L \dot{\psi}_L + \Pi_R \dot{\psi}_R + \mathcal{P}_+ \dot{\eta}^+ + \mathcal{P}_- \dot{\eta}^- - H + \text{sources} \right)\right)$$

whose non ghost part reduces to eq.(5) after integration over P, Π (in the absence of sources).

Notice that we are in phase-space for the non-compact coordinates but, after integrating out $\Pi_{L,R}$, we appear to be in coordinate space for the compact coordinates. This is however misleading: we should think of the ψ^i ($i=1..2d$) as replacing the phase-space original coordinates X^I, P^I ($I=1..d$) (both in the left and in the right moving sectors). This is quite obvious if one looks at the details of the fermionization procedure (see also eq.(23) below). Eq. (10) is the starting point of our derivation of the WI's.

Before turning to that, let us discuss an amusing aspect of the BRS quantization of the system. We use canonical (anti)commutators for $(\eta, \mathcal{P})$ X, P while for ψ we use (in order to take (9) into account) the modified anticommutator following from the Dirac brackets:

(11)
$$\left\{ \psi_L^i(\sigma) , \psi_L^j(\sigma') \right\}_{DB} = i\,\delta^{ij}\delta(\sigma-\sigma') \; ; \; \text{same for } L \to R$$

We also recall [2,11] that the ψ-fields can be either of the Neveu-Schwarz (antiperiodic) or of the Ramond (periodic) type according to the lattice of momenta they operate on. Defining the quantum BRS charge as[8]

(12)
$$\hat{Q} = \,:Q:\, - \beta_+ \eta_0^+ - \beta_- \eta_0^- \; ; \; \eta_0^\pm \equiv \frac{1}{2\pi}\int_{-\pi}^{+\pi} d\sigma\, \eta^\pm$$

a straightforward calculation similar to that of ref. (8) yields:

(13)
$$24\pi \hat{Q}^2 = \left\{ \sum_{n=1}^{\infty} (D-26)n^3 + (2-D+24\beta_+ + 3d_+^P)n \right\} \cdot \int \eta_+^2 d\sigma + \left(\beta_+, \eta_+, d_+^P \to \beta_-, \eta_-, d_-^P\right)$$

where $2d^P_\pm(2d^A_\pm)$ is the number of periodic (antiperiodic) $\psi^i_{L,R}$ fields ($d^P_\pm + d^A_\pm = d$). Since Q^2 must be 0 for conformal invariance [7,8] we get

(14)
$$D = 26 \; ; \; \beta_\pm = 1 - \frac{1}{8} d_\pm^P$$

$\beta_\pm$ are related to the spectrum by the mass shell conditions:

(15) $$\left(L^{(0)}_{\pm} - \beta_{\pm}\right) \equiv -\frac{1}{8} M^2 + N_B^{\pm} + N_F^{\pm} - \beta_{\pm} = 0$$

where $N_{B,F}$ are (integer) occupation numbers for the bosonic (noncompact) and fermionic (compact) oscillators. Integer intercepts (massless states) are only obtained for $d^P_{\pm} = 8n$,a condition naturally satisfied if d=8n, which is just the constraint that follows from the need of an even self-dual lattice!

3. Ward Identities for internal Gauge symmetries

Following ref [6] it is obvious how to obtain the WI's corresponding to GCT of the 26-8n non compactified coordinates. It is enough to start from the canonical change of variables induced by the generating function:

(16) $$\Phi_{GCT} = P_\mu \, \xi^\mu(X) \quad ; \mu = 0, 1, \ldots (25-8n)$$

The gauge WI's we are interested in follow from a similar canonical transformation generated this time by

(17) $$\Phi^L_G = \psi^i_L \psi^j_L \, \xi^{ij}(X) \; ; \; \xi^{ij} = -\xi^{ji} \; ; \; i,j = 1,2 \ldots 16n$$

and similarly for the right movers. The associated change of variables is

(18) $$\delta_G X^\mu = \delta_G \psi^i_R = 0$$

$$\delta_G P_\mu = -\psi^i_L \psi^j_L \, \xi^{ij}{}_{,\mu}(x) \; ; \; \delta_G \psi^j_L = 2i \, \psi^i_L \, \xi^{ij}(X)$$

A straightforward calculation then gives

(19)

$$\delta_G \int (P\dot{X} + \Pi \dot{\psi} - H) d^2\sigma = \int d^2\sigma \, (P^\mu + X'^\mu) \psi^i_L \psi^j_L \, \xi^{ij}{}_{,\mu}(X)$$

Taking as in ref [6] $$\xi^{ij}(X) = \epsilon^{ij} \exp(ik \cdot X) \; ; \; k^2 = 0$$

one gets the WI in the form (here Ω is any operator)

(20) $$\epsilon^{ij} k^\mu \left\langle \int d^2\sigma \, (P^\mu + X'^\mu) \psi^i_L \psi^j_L(\sigma) e^{ikX} \cdot \Omega \right\rangle = \langle \delta_G \Omega \rangle$$

This is the desired WI where the vertex for the gauge bosons G^{ij}

(21) $$\int d^2\sigma \, (P_\mu + X'_\mu)\, \psi_L^i \psi_L^j \exp(ik\cdot X(\sigma)) \quad ; \; k^2 = 0$$

has appeared on the left-hand side. The right-hand side of (20) vanishes if we take S-matrix elements [6].

Let us now discuss in some detail the structure of the gauge group. Apparently we have just generated the WI's of a SO(16n) group (times an analogous one from right movers). The fermionic operators ψ_L^i transform as the fundamental (vector) respresentation of such group :they are known to be related to the original fields X_L^I, P_L^I by the "fermionization" [2]

(22) $$\psi_L^i = \left(B_L^{e_I} + B_L^{-e_I} \; ; \; -i\left(B_L^{e_I} - B_L^{-e_I}\right)\right)$$

$$B_L^{\pm e_I} = z^{1/2} : \exp(\pm i\, e_I X_L) : \quad ; \; z \equiv \exp(i(\tau - \sigma))$$

(similarly for ψ_R^i)

where e_I=(0,0..,1,...,0) are an orthonormal Cartesian basis in d=8n dimensions. The $B_L^{e_I}$ satisfy [2,11]:

(23) $$: B_L^{e_I} B_L^{-e_I} : \; = P_L^I = i z \frac{d}{dz} X_L^I$$

Comparing eqs(16), (17) and (23) we see the perfect analogy between the $U(1)^{8n}$ (Cartan) subgroup of SO(16n) and that of GCT. This is just the usual Kaluza-Klein mechanism expected upon compactification. We see however that a larger symmetry taking advantage of the full dimensionality (16n) of phase space has appeared.

In fact this is not yet the end of the story. Let us consider separately the two cases n=1 and n=2.

n=1. Here d=8 and the gauge group obtained so far is SO(16). Its roots are $\pm e_I \pm e_J$ (I,J=1...8). The lattice generated by these roots is even but not self-dual. It becomes self-dual if we add to it the points [2,3]

(24) $$\frac{1}{2}\sum_{I=1}^{8} \pm e_I \qquad (\text{even \# of } -\text{'s})$$

which, together with the previous ones, represent the root lattice of E_8.

Can we get the extra WI's of E_8? At first sight this looks hard and indeed, as the authors of ref. [11] have shown, there is no algebraic construction of the extra operators in terms of the ψ-fields. On the other hand, if we just want to derive the extra WI's, we can just observe that the E_8 root lattice is left invariant by a finite group of discrete SO(4)xSO(4) rotations in root space (not to be confused with SO(16) rotations!), which transform the vector weights = $\pm e_I$ into the (first or second kind) spinor weights [11]

(25) $$f_K = \frac{1}{2}\left(\sum_1^4 \pm e_I\right)_{\substack{\text{odd \#}\\ \text{of } -\text{'s}}} \;;\; g_K = \frac{1}{2}\left(\sum_1^4 \pm e_I\right)_{\substack{\text{even \#}\\ \text{of } -\text{'s}}} \;;\; (\text{same for } I = 5,\cdots 8)$$

This is nothing but the well known triality of SO(8) [12] . The extra missing WI's are obtained precisely as before but fermionizing the theory after having performed such discrete rotations in the compactified space. This gives two more SO(16) subgroups of E_8 sharing with each other and with the original SO(16) a common SO(8)xSO(8) for a total of 3x120-2x(28+28)=248 generators which exhaust E_8.

Another way of arriving at the same conclusion is by computing the lowest states of the theory keeping an even number of periodic or antiperiodic ψ_L and ψ_R oscillators.

Antiperiodic ψ_L ,ψ_R give through eqs. (14,15) $\beta_\pm$ =1, hence a singlet tachyon (M^2=-8). At M=0 one finds an SO(17,1) graviton, a $[(120,1)+(1,120)]_{\text{spin}\,1}$ and a $(120,120)_{\text{spin}\,0}$ of $SO(16)_L \times SO(16)_R$. Adding periodic ψ_L ,ψ_R and mixed periodicity states, and using again eqs (14,15) one gets the extra massless states

$[(128,1)+(1,128)]_{spin 1} + [(120,128)+(128,120)+(128,128)]_{spin\ 0}$

Altogether these states combine to give:

$(1,1)_{\mu\nu} + [(248,1)+(1,248)]_{\mu} + (248,248)$ of $E_8 \times E_8$

where the μ,ν indices refer to SO(17,1) Lorentz transformations.

n=2 -. In this case we have two distinct possibilities. In the first ($E_8 \times E_8$ gauge group) one applies the above procedure twice and independently to two eight- dimensional spaces.

There is however a second even self-dual lattice in d=16, obtained by combining the roots of SO(32)

(26) $\pm e_I \pm e_J$

with the points of length 4

(27) $\frac{1}{2} \sum_{1}^{16} \pm e_I$ (even # of $-$'s)

The latter are the weights of a spinor representation of SO(32) which together with those in (26) lead to the weight lattice of Spin(32)/Z_2.

Using antiperiodic ψ_L ,ψ_R we get again a singlet tachyon, a massless graviton and gauge bosons in the adjoint representation of $SO(32)_L \times SO(32)_R$. For periodic ψ_L ,using (14) ,one gets $\beta_{\pm}=-1$. Consequently we do not produce further massless states but only a massive multiplet ($M^2=8$) belonging to a spinor representation of $SO(32)_L$ (and to nontrivial representations of $SO(32)_R$).

In conclusion, we have shown that the canonical Hamiltonian formulation of the compactified bosonic string, together with the approach of ref [6], leads to a simple derivation of the gauge Ward identities of these theories. This could be important in order to understand the possible connection between bosonic, super or heterotic strings recently discussed by many authors [13]. As already emphasized in [6], the question still remains of possible new terms (anomalies) affecting the naive canonical change of variables at the quantum level. Such terms were found to occur [14] in general beyond one loop in simpler theories . A similar analysis in string theories may turn out to be very rewarding.

We are grateful to S. Fubini for illuminating discussions on SO(8) triality and octonions and for bringing Ref.[14] to our attention. We also thank A. Jevicki for a useful conversation and A. Schwimmer for discussions and useful correspondence on the fermionic construction of E_8.

REFERENCES

[1] Dual Theory, Collection of Phys. Reports, Vol.1 ,M. Jacob ed. (North Holland, Amsterdam, 1974); J.Scherk, Rev. Mod. Phys.47(1975); J.H.Schwarz, Phys. Reports 89(1982) 223; M. B. Green, Surveys in High Energy Phys.3(1983)127

[2] I.B.Frenkel and V.G.Kac, Inven. Math. 62(1980)23; G. Segal Comm Mat Phys. 80(1982)301; P. Goddard and D.Olive, Vertex Operators in Mathematics and Physics (J. Lepowsky et al.eds.) Springer Verlag (1984) 419; P. Goddard, Lectures given at the summer workshop on high energy Physics and Cosmology, Trieste(1985).

[3] D.J.Gross, J.A.Harvey, E.Martinec and R.Rohm, Nucl. Phys. B256(1985)253

[4] P.Candelas, G.T.Horowitz, A.Strominger and E.Witten, Nucl. Phys. B258 (1985)46

[5] M.B.Green and J.H.Schwarz, Phys. Lett.149B(1984)117

[6] G.Veneziano, CERN preprint TH.4318(1985)

[7] M.Kato and K.Ogawa,Nucl. Phys.B212(1983)443

[8] S.Hwang,Phys. Rev.,D28(1983)2614

[9] S.Deser and B.Zumino,Phys. Lett.65B(1976)369.

[10] E.S.Fradkin and G.A.Vilkovisky,Phys.Lett.55B(1975) 224.

[11] P.Goddard, D.Olive and A.Schwimmer, Imperial College Preprint TP84-85/23(1985).

[12] See e.g. E.Cartan, Leçons sur la Théorie des Spineurs, Vol. 2 (Hermann & Cie. Éditeurs ,Paris ,1938) page 53.

[13] P.G.O.Freund, Phys.Lett.151B(1985)387; A.Casher, F.Englert, H.Nicolai and A.Taormina, CERN Preprint TH.4220 (1985).

[14] J.L. Gervais and A. Jevicki, Nucl. Phys. B110(1976) 93.