

# From Big Crunch to Big Bang: A Quantum String Cosmology Perspective

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The scenario that the Universe contracts towards a big crunch and then undergoes a transition to expanding Universe in envisaged in the quantum string cosmology approach. The Wheeler-De Witt equation is solved exactly for an exponential dilaton potential. S-duality invariant cosmological effective action, for type IIB theory, is considered to derive classical solutions and solve WDW equations.

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It is natural to expect that cosmology must be ultimately founded on quantum gravity. Since string theory/M-theory synthesise quantum mechanics and general theory of relativity; therefore, the evolution of the Universe in early epochs and fundamental issues concerned with initial singularity may be resolved in the frame work of string theory. In recent years, there has been considerable amount of interest in cosmology from the string theory point of view [1,2]. The pre-big bang (PBB) scenario [3], which has drawn a lot of attention, proposes an alternative mechanism for inflation when contrasted with the original paradigm of inflation [4,5] and promises a possible formulation of nonsingular cosmology. One of the postulates of PBB cosmology is that the Universe, in the remote past,  $t \rightarrow -\infty$ , is described by weak coupling, low curvature and cold state and it undergoes an accelerated expansion by the kinetic energy term of the dilaton while proceeding towards the singularity lying in its future. Subsequently, there is a transition from the accelerating to the FRW like branch in the  $t > 0$  region. However, one encounters no-go theorems for the branch change while dealing with the tree level effective action [6,7]. There are several other attempts to understand diverse aspects of cosmology in the frame work of string theory [8–12].

Recently, Khoury et al. [13] have put forward a proposal where the Universe contracts towards a big crunch and then makes a transition to an expanding big bang Universe. The scenario envisaged in [13] holds the promise to explore new class of cosmological models. Furthermore, it has been pointed out that this idea leads to interesting connection with recently proposed ekpyrotic model [14] which has generated considerable activities [15]. The essential ingredients in ref. [13] is to consider an effective action with graviton and a massless scalar field, dilaton, describing the evolution of the Universe. This model incorporates some of the ideas of PBB proposal in that the evolution of the Universe began in the far past. However; it also differs from the scenario of the PBB that the Universe followed the accelerated expanding branch for  $t < 0$

and then it exits to the expanding, decelerating branch for which the singularity lies in its past.

The purpose of this note is to present an investigation of the mechanism for the transition from big crunch to big bang from the quantum mechanical perspective. We derive the Wheeler-De Witt equation for the case at hand and impose appropriate boundary conditions which describes the emergence of the FRW type Universe starting from initial state which corresponds to big crunch classically. To be specific, we adopt an exponential potential and choose a suitable metric to facilitate solution for the case under considerations. We note that *raison de etre* for the exponential potentials have been argued by Moore, Peradze and Salina [16] from M-theoretic analysis. Furthermore, exponential potentials arising from M-theory cosmology might explain quintessence as expounded in [17].

The  $D = 4$  tree level string effective action is

$$S = \int d^4x \sqrt{-g} \left( \mathcal{R}_g - \frac{1}{2}(\partial\tilde{\phi})^2 - V(\tilde{\phi}) \right) \quad (1)$$

Here  $g_{\mu\nu}$  is the Einstein frame metric,  $g$  is its determinant and  $\mathcal{R}_g$  is the Ricci scalar derived from  $g_{\mu\nu}$ . Note that  $\tilde{\phi}$  is the dilaton and  $V(\tilde{\phi})$  is the dilaton potential term. The metric and dilaton are taken to be time dependent in the cosmological scenario and we shall consider isotropic, homogeneous and spatially flat FRW metric. In ref [13], the following form of metric was adopted:  $ds^2 = a(\tau)^2(-N(\tau)^2 d\tau^2 + \sum_{i=1}^3 dx_i^2)$ ,  $\tau$  is conformal time. We shall choose a different form of metric [18]

$$ds^2 = -\frac{N(t)^2}{a(t)^2} + a(t)^2 \delta_{ij} dx^i dx^j \quad (2)$$

Here  $N(t)$  is the lapse function. It is easy to show that  $\sqrt{-g}\mathcal{R}_g = -6N(t)^{-1}a(t)^2\dot{a}(t)^2 + \text{total derivative term}$ . It is very useful for our purpose [18] to rescale  $a(t)$  and  $\tilde{\phi}$  to  $\phi$  to bring the action to more convenient form

$$L = N(t)^{-1} \left( -\frac{1}{2}a^2\dot{a}^2 + \frac{1}{2}a^4\dot{\phi}^2 \right) - Na^2V(\phi) \quad (3)$$

where the dot denotes time derivative. The equation of motion for  $N$  leads to the Hamiltonian constraint and the equation of motion for  $a$  and  $\phi$  can be derived easily. We choose an exponential potential i.e.  $V(\phi) = V_0 e^{\alpha\phi}$ ,  $V_0$  being a constant. The motivation for choosing the exponential dilaton potential has been alluded to earlier. We look for solutions where scale factor,  $a$ , has a power law growth in time:  $a(t) = |t|^p$ ,  $p > 0$ . The Hubble parameter,  $H$ , its time derivative, the dilaton,  $\phi$  and its time derivative have the following form as can be inferred from the equations of motion.

$$H = \frac{\dot{a}}{a} = \frac{p}{t}, \quad \dot{H} = -\frac{p}{t^2} \quad (4)$$

$$\phi = \phi_0 + \sqrt{\frac{p(1-p)}{3}} \ln|t|, \quad \dot{\phi} = \sqrt{\frac{p(1-p)}{3}} \frac{1}{t} \quad (5)$$

Here  $a_0$  and  $\phi_0$  are arbitrary constants. The parameter  $\alpha$  appearing in the potential gets fixed in terms of exponent  $p$  and so is the ratio of  $V_0$  and  $a_0^2$ ,

$$\alpha = -2\sqrt{\frac{3(1-p)}{p}}, \quad \text{and} \quad \frac{V_0}{a_0^2} = \frac{4p^2 - p}{6} \quad (6)$$

Cosmological solutions with exponential potentials, for different form of metric, has been considered in the past [19] and they have been topics of discussion more recently in the context of ekpyrotic model [20]. Note that,  $G_N$  or the Planck mass,  $m_P$  does not appear in our action and therefore, these constants are also absent in our solutions of equations of motion. From now on we choose  $\alpha = -2$  so that

$$V = V_0 e^{-2\phi}, \quad \text{correspondingly} \quad p = \frac{3}{4} \quad (7)$$

Note that we have also absorbed a factor of  $e^{-2\phi_0}$  in the definition of  $V_0$ . We have mentioned earlier the justification for adopting an exponential form of potential; however, it is possible that this form of potential might not be adequate all the way to very small values of  $t$ . Let us define, new set of variables:  $u = \frac{a^2}{2} \cosh 2\phi$  and  $v = \frac{a^2}{2} \sinh 2\phi$ ; then the Lagrangian is

$$L = \frac{1}{2N} (\dot{v}^2 - \dot{u}^2) - \frac{N}{2} V_0 (u - v) \quad (8)$$

for our choice of specific exponential potential. The corresponding Hamiltonian is

$$\mathcal{H} = N \left( \frac{1}{2} p_v^2 - \frac{1}{2} p_u^2 + \frac{V_0}{2} [u - v] \right) \quad (9)$$

varying  $N$  yields the constraint  $H = 0$  in  $N = 1$  gauge. The momenta  $p_v$  and  $p_u$  are conjugate to  $v$  and  $u$  respectively. Furthermore,  $[p_u + p_v, \mathcal{H}]_{PB} = 0$ , implying existence of a conserved charge,

$$Q = \frac{e^{-2\phi}}{a(t)^2} [-H + \dot{\phi}^2] \quad (10)$$

$H$  is the Hubble parameter. Note that  $\phi = \phi_0 + \frac{1}{4} \ln|t|$ , where  $\phi_0$  is a constant which is value that dilaton assumes at some constant,  $t = t_0$ . Thus  $Q$  contains a factor  $e^{-2\phi_0}$  in its definition, revealing the coupling constant dependence.

Now we obtain the Wheeler-De Witt equation and impose appropriate boundary conditions on the wave function. It has been advocated that quantum string cosmology might be useful to address the issue of graceful exit [21–23] and to study evolution of the early Universe [24].

The Wheeler-De Witt equation takes the following form for the Hamiltonian (set  $N = 1$ ) (9),

$$\left[ -\frac{\partial^2}{\partial v^2} + \frac{\partial^2}{\partial u^2} + \frac{V_0}{2} (u - v) \right] \psi(v, u) = 0 \quad (11)$$

We may solve (11) by separation of  $v$  and  $u$  and  $\psi(u, v)$  is product of two Airy functions. However, it is useful to reexpress the WDW equation in terms of another set of variables and the wave function thus derived has a more direct interpretation in terms of a scenario that the Universe evolved towards (classical path of) big crunch in negative  $t$  region and then undergoes a transition to the expanding phase for  $t > 0$ . Define,

$$\xi = \frac{1}{6} \ln 4XY, \quad \zeta = \frac{1}{6} \ln \frac{X}{Y} \quad (12)$$

with  $X = \frac{1}{4}(u + v)$  and  $Y = \frac{1}{8}(u - v)^2$ . The Wheeler-De Witt equation is expressed as (in the gauge  $N^{-1} = \frac{1}{2}[u - v]$ )

$$\mathcal{H}\psi(\xi, \zeta) = \left( \frac{\partial^2}{\partial \xi^2} - \frac{\partial^2}{\partial \zeta^2} + 9V_0 e^{6\xi} \right) \psi(\xi, \zeta) = 0 \quad (13)$$

The wave function is a product of a plane wave in  $\zeta$  variable of the form  $e^{\pm ik\zeta}$ , where  $k$  is the separation constant. Note that  $k$  is also identified with eigenvalue of the momentum operator  $i\frac{\partial}{\partial \zeta}$  acting on the plane wave solution (see the arguments in [21] for the choice of the sign in defining operator  $p_\zeta$ ) and this is a constant of motion since  $[p_\zeta, H] = 0$ . Indeed, the conserved momentum is related to the charge  $Q$ , defined in (10). The solution to the equation of  $\xi$  variable is Bessel function; therefore,  $\psi$  is given by

$$\psi_k(\xi, \zeta) = e^{\pm ik\zeta} \mathcal{F}_{\pm \frac{ik}{3}}(z), \quad z = \sqrt{V_0} e^{3\xi} \quad (14)$$

where  $\mathcal{F}_\nu(z)$  is one of the Bessel functions [25],  $J_\nu(z)$ ,  $Y_\nu(z)$  or Hankel functions  $H_\nu^{(1,2)}(z)$ . The relevant Bessel function is chosen in order to fulfill desired boundary conditions. We demand that there is only right moving wave in the negative  $t$  region, with positive eigenvalue of  $p_\xi$ , and are led to choose  $J_{-\frac{ik}{3}}(z)$ . In this domain, for small geometries, as  $a \rightarrow 0$ ,  $z \rightarrow 0$  and

$$\lim_{z \rightarrow 0} J_{-i\frac{k}{3}}(z) \sim \frac{1}{2}(z)^{-i\frac{k}{3}} \sim e^{-ik\xi} \quad (15)$$

Thus, in this limit, the wave function behaves as

$$\psi_k(\xi, \zeta) \sim e^{-ik(\xi+\zeta)} \quad (16)$$

The Universe expands in the  $t > 0$  region. The scale factor grows with time and we are led to consider behavior of  $\psi$  when  $\xi \rightarrow \infty$ . The asymptotic form of  $\psi$  is expressed as sum of two components:

$$\psi_k(\xi, \zeta) = e^{-ik\zeta} J_\nu(z) =_{\xi \rightarrow \infty} \psi_{k(+)} + \psi_{k(-)} \quad (17)$$

where  $\nu = -i\frac{k}{3}$  and the two components are given by

$$\psi_{k(\pm)} = \sqrt{\frac{2}{\pi z}} e^{-ik\zeta} e^{\mp i(z - \frac{\pi}{2} - \frac{1}{2}\nu\pi - \frac{\pi}{4})} \quad (18)$$

Moreover,  $\psi_{k(\pm)}$  satisfy following relations when operated up on by the momentum operator,  $p_\xi = -i\frac{\partial}{\partial \xi}$

$$p_\xi \psi_{k(\pm)} = \mp z \psi_{k(\pm)} \quad (19)$$

Thus, the choice of the wave function describes the evolution of the Universe as follows: the right moving wave propagates from the -ve  $t$  region. The wave function in the positive time domain is superposition of the left and right moving components. The boundary conditions adopted to choose the wave function is in the spirit of Vilenkin's proposal [26]. The probability for the transition from the branch with wave function of the form  $e^{-ik\zeta - ikz}$  to the branch which has wave function  $e^{-ik\zeta + ikz}$ , in the asymptotic limit, is given by

$$P_k = \frac{|\psi_{k(-)}|^2}{|\psi_{k(+)}|^2} = e^{-\frac{2}{3}\pi k} \quad (20)$$

Recall that  $k$  is the conserved momentum. Note that the the conserved charge (10) arises due to conservation of momentum. Therefore,  $k = \text{const } e^{-2\phi_0}$  and we are not in a position to determine the exact value of the multiplicative constant; although our arguments show how a factor of  $e^{-2\phi_0}$  appears in definition of  $k$ . Thus the probability gets suppressed for in the weak coupling phase.

Next, we proceed to discuss cosmological solutions, in the present context, for graviton-dilaton-axion system which appear in the type IIB effective action. The dilaton,  $\phi$  and RR scalar,  $\chi$ , called axion, belong to  $SL(2, R)$  S-duality group and they parametrize the coset  $\frac{SL(2, R)}{SO(2)}$ .

The relevance of this problem in the present context can be perceived as follows. The presence of axion introduces the kinetic energy term  $a^4 e^{2\phi} \dot{\chi}^2$ , in the Einstein frame, in the action besides the curvature scalar and kinetic energy term of the dilaton. When, one considers the  $SL(2, R)$  invariant for of the type IIB action, it is not possible to introduce a nontrivial  $SL(2, R)$  invariant potential (depending on dilaton and axion) as has been argued in [28]. Thus, the equation of motion for the axion is a conservation law i.e. there is a conserved charge  $Q_a = a^4 e^{2\phi} \dot{\chi}$ .

Therefore, while solving coupled set of equations of motion, we may eliminate the kinetic energy term of axion ( or  $\dot{\chi}$  wherever it appears) and end up with a term which looks like an exponential dilaton potential. Furthermore, while solving the Wheeler-de Witt equation, we solve the wave equation with scale factor, dilaton and a potential depending on scale factor and exponential of dilaton. However, it is much more convenient to express the type IIB action in terms of  $SL(2, R)$  matrices so that the action is manifestly S-duality invariant. Therefore, I shall adopt the form of the action which could be obtained through toroidal compactification of ten dimensional type IIB action to four dimensions as was shown by me a few years ago [28].

The 10-dimensional effective action can expressed in manifestly S-duality invariant form [27] in the Einstein frame. The toroidal compactification of that action to lower dimensions, preserving S-duality invariance, was presented by the author and Roy [28,29] which has been useful to obtain classical solutions of IIB theory. We consider a simple version of the 4-dimensional type IIB effective action: the compactification radii of the tori are taken to be constant (set to one), only graviton, dilaton and axion are retained in the reduced action and furthermore, rest of the scalar, vector and tensor fields are set to zero. We refer the interested reader to [28] where the full reduced action is derived. Our starting point is

$$S_4 = \int d^4x \sqrt{-g} \left( \mathcal{R}_g + \frac{1}{4} \text{Tr}[\partial_\mu \mathcal{M} \Sigma \partial^\mu \mathcal{M} \Sigma] \right) \quad (21)$$

This is the action in the Einstein frame with,

$$\mathcal{M} = \begin{pmatrix} \chi^2 e^\phi + e^{-\phi} & \chi e^\phi \\ \chi e^\phi & e^\phi \end{pmatrix}, \quad \text{and} \quad \Sigma = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \quad (22)$$

Here  $\Sigma$  is the  $SL(2, R)$  metric in our conventions [28]. The above action is manifestly invariant under S-duality transformations

$$\mathcal{M} \rightarrow \Lambda \mathcal{M} \Lambda^T, \quad g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad \Lambda \Sigma \Lambda^T = \Sigma \quad (23)$$

where  $\Lambda \in SL(2, R)$  with unit determinant. Some of the useful relations these matrices satisfy are

$$\Sigma \Lambda \Sigma = \Lambda^{-1}, \quad \mathcal{M} \Sigma \mathcal{M} = \Sigma, \quad \text{and} \quad \Sigma \mathcal{M} \Sigma = \mathcal{M}^{-1} \quad (24)$$

Note that  $\mathcal{M} \in SL(2, R)$  and is symmetric. For the cosmological case, with our form of FRW metric (after some rescaling) the Lagrangian is

$$L = -\frac{1}{2N} a^2 \dot{a}^2 - \frac{a^4}{4N} \text{Tr}[\dot{\mathcal{M}} \Sigma \dot{\mathcal{M}} \Sigma] \quad (25)$$

$\mathcal{M}$ -equation of motion deserves some care, since  $\mathcal{M} \in SL(2, R)$  and is a conservation law as expected [28,30].

$$\partial_0(\sqrt{-g} g^{00} \mathcal{M} \Sigma \partial_0 \mathcal{M} \Sigma) = 0 \quad (26)$$

and thus we conclude  $\sqrt{-g} g^{00} \mathcal{M} \Sigma \partial_0 \mathcal{M} \Sigma = A$ , where  $A$  is a constant  $2 \times 2$  matrix; is symmetric and satisfies

$A\Sigma\mathcal{M} = -\mathcal{M}\Sigma A$  which follow by using the relations between  $\mathcal{M}$  and  $\Sigma$  mentioned above. Another important relation is (using  $g^{00}\sqrt{-g} = a^4$ )

$$\text{Tr}(\dot{\mathcal{M}}\Sigma\dot{\mathcal{M}}\Sigma) = -\frac{1}{a^8}\text{Tr}(A\Sigma A\Sigma) \quad (27)$$

The time dependence in the *r.h.s.* is buried in the presence of  $a$ . The Hamiltonian constraint relates the Hubble parameter and scale factor appearing through relation (27)

$$H^2 = \frac{1}{2a^8}\text{Tr}(A\Sigma A\Sigma) \quad (28)$$

resulting in the time dependence:  $a = a_0 t^{\frac{1}{4}}$ ,  $a_0$  being a constant including the factor coming from  $\text{Tr}A\Sigma A\Sigma$ . Notice that the Einstein-Friedman equation derived from (25) is also satisfied when (28) solved. We can solve (26) once  $a(t)$  is determined

$$\mathcal{M}(t) = e^{A\Sigma \ln(t-t_0)} \mathcal{M}(t_0) \quad (29)$$

where  $t_0$  is an arbitrary constant and  $\mathcal{M}(t_0)$  is value of the matrix at  $t_0$ .

Let us derive the WDW equation for the  $SL(2, R)$  invariant system. First of all define  $a(t) = e^{\alpha(t)}$  and then a new time variable through the relation  $d\tau = e^{-4\alpha} dt$  and the derivatives with respect to  $\tau$  are denoted by prime.

$$L = \int d\tau \left[ -\frac{1}{2}\alpha'^2 - \frac{1}{4}\text{Tr}(\mathcal{M}'\Sigma\mathcal{M}'\Sigma) \right] \quad (30)$$

$\alpha'$  appearing here not to be confused with inverse string tension. The canonical momenta are

$$p_\alpha = -\alpha', \quad \Pi_{\mathcal{M}} = -\frac{1}{2}\Sigma\mathcal{M}'\Sigma \quad (31)$$

The canonical Hamiltonian is

$$\mathcal{H} = -\frac{1}{2}p_\alpha^2 - \text{Tr}(\Sigma\Pi_{\mathcal{M}}\Sigma\Pi_{\mathcal{M}}) \quad (32)$$

The WDW equation assumes the following form

$$\left( \frac{\delta^2}{\delta\alpha^2} + \text{Tr}\left(\Sigma\frac{\delta}{\delta\mathcal{M}}\Sigma\frac{\delta}{\delta\mathcal{M}}\right) \right) \Psi(\alpha, \mathcal{M}) = 0 \quad (33)$$

We may factorize  $\Psi(\alpha, \mathcal{M}) = \mathcal{F}(\alpha)\mathcal{G}(\mathcal{M})$ . From the conservation law of the  $\mathcal{M}$ -matrix evolution equation, the quantum mechanical relation

$$\mathcal{M}\Pi_{\mathcal{M}}\mathcal{G} = i\mathcal{M}\frac{\delta}{\delta\mathcal{M}}\mathcal{G} = (2A\Sigma)\mathcal{G} \quad (34)$$

follows immediately. The wave equation satisfied by  $\mathcal{F}$  is

$$\left( \frac{\delta^2}{\delta\alpha^2} + \frac{1}{4}\text{Tr}(A\Sigma)^2 \right) \mathcal{F}(\alpha) = 0 \quad (35)$$

which leads to a 'plane wave' solution

$$\mathcal{F}(\alpha) = e^{\pm i\frac{\alpha}{2}[\text{Tr}(A\Sigma)^2]^{\frac{1}{2}}} \quad (36)$$

To solve for  $\mathcal{G}$  with constraint (35), one needs to specify the matrix  $A$ . For example, when  $\chi = 0$ ,  $\mathcal{M}$  is diagonal and  $\mathcal{G}$  is pure dilatonic plane wave. More general form of  $A$  will lead to interesting class of solutions respecting S-duality. An important point is that an arbitrary potential  $V(\phi)$  added to Lagrangian (25) breaks S-duality symmetry; moreover, the choice of S-duality invariant potentials are severely restricted [28]. Therefore, such symmetry considerations might play important roles in the study of cosmological solutions [31].

It is worth while to recall the scenario proposed in [13] where the Universe evolves from large negative  $t$  towards the big crunch. Subsequently it undergoes a transition to a FRW-like branch in the  $t \geq 0$  domain. When we contrast this proposal with the PBB [3] picture, the Universe proceeds towards the singularity as it accelerates in the  $t < 0$  domain. Then, it is expected to go through a transition to expanding, decelerating phase for positive  $t$ . These two branches are related to each other through scale factor duality symmetry. In the proposal, where the Universe is proceeding towards big crunch, it also approaches weak coupling regime and therefore, the perturbation theory could be trusted in that neighborhood. Of course, there might be additional terms which could give additional contributions near the singularity. In this investigation of quantum string cosmology, we do not take into account those effects.

In summary, we have presented a quantum string cosmological investigation of the scenario that the initial state of the Universe is the one which evolves towards big crunch in negative regime and subsequently, the wave function gets reflected. The probability is exponentially suppressed in the weak coupling regime. At this stage, this suppression is shown to be true for the typical potential we have chosen here; however, this result might be valid under general conditions although no such proof exists at the moment. Furthermore, we considered an S-duality invariant action and solved the classical equations in a general setting and discussed the structures of the wave functions of the corresponding WDW equations.

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- [1] For recent reviews see J. Lidsey, D. Wands and E. Copeland, Phys. Rep. **C337**(2000) 343, hep-th/9909061.
  - [2] G. Veneziano, String Cosmology: The Pre-Big Bang Scenario, hep-th/0002094. Also see the homepage
  - [3] G. Veneziano, Phys. Lett. **B265**, 287(1991); M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317(1993)

- [4] A. H. Guth, Phys. Rev. **D23**(1981) 347.
- [5] A. D. Linde, Phys. Lett. **108**(1982) 389; A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**(1982) 1220.
- [6] R. Brustein and G. Veneziano, Phys. Lett. **B329**, 429(1994), hep-th/9403060.
- [7] N. Kaloper, R. Madden and K. Olive, Nucl. Phys. **B452**(1995) 677(1995), hep-th/9505027; R. Easther, K. Maeda and D. Wands, Phys. Rev. **D53**(1996) 4247; hep-th/9509074.
- [8] T. Damour and A. M. Polyakov, Nucl. Phys. **B423**(1994) 532, hep-th/9401069.
- [9] J. Horne and G. Moore, Nucl. Phys. **432**(1994) 532, hep-th/9403058.
- [10] T. Banks, M. Berkooz, G. Moore, S. Shenker and P. J. Steinhardt, Phys. Rev. **D52**(1995) 3548, hep-th/9503114.
- [11] T. Damour, M. Henneaux, B. Julia and H. Nicolai, Phys. Lett. **B509**(2001) 323, hep-th/0103094.
- [12] T. Banks, W. Fischler and L. Motl, JHEP **9901**(1999) 091, hep-th/9811194.
- [13] J. Khoury, B. Ovrut, N. Seiberg, P. J. Steinhardt and N. Turok, From Big Crunch to Big Bang, hep-th/0108187.
- [14] J. Khoury, B. Ovrut, P. J. Steinhardt and N. Turok, hep-th/0103239.
- [15] R. Kallosh, L. Kofman and A. D. Linde, hep-th/0104073; R. Kallosh, L. Kofman, A. D. Linde and A. Tseytlin, hep-th/0106241; K. Enqvist, E. Keski-Vakkuri and S. Räsänen, hep-th/0106282, and references therein.
- [16] G. Moore, G. Perdze and N. Saulina, Nucl. Phys. **B607**(2001) 117, hep-th/0012104.
- [17] P. K. Townsend, hep-th/0110072.
- [18] L. J. Garay, J. J. Halliwell and G. A. Mena Marugan, Phys. Rev. **D43**(1991) 2572.
- [19] J. J. Halliwell, Phys. Lett. **B185**(1987) 341; E. D. Stewart and D. H. Lyth, Phys. Lett. Phys. Lett. **B302**(1993) 171.
- [20] D. H. Lyth, hep-ph/0106153; R. Brandenberger and F. Finelli, hep-th/0109004; J. Hwang, astro-ph/0109045,
- [21] M. Gasperini, J. Maharana and G. Veneziano, Nucl. Phys. **B472**(1996) 394, hep-th/9602087; M. Gasperini and G. Veneziano, Gen. Rel. Grav. **28**(1996) 1301.
- [22] J. Maharana, S. Mukherji and S. Panda, Mod. Phys. Lett. **A12**(1997) 447, hep-th/9701115;
- [23] M. Cavaglia and C. Ungarelli, Class. Quant. Grav. **16**(1999) 1401, gr-qc/9902004; O. Bertolami and R. Schiappa, Class. Quant. Grav. **16**(1999) 2545.
- [24] J. Lidsey, Phys. Rev. **D55**(1997) 3305, gr-qc/9605017;
- [25] M. Abramowicz and I. A. Stegun, Handbook of Mathematical Functions (Dover, New York, 1972).
- [26] A. Vilenkin, Phys. Rev. **D37**(1988) 888, Phys. Rev. **D50**(1994) 2581.
- [27] C. M. Hull, Phys. Lett. **B357**(1995) 545; J. H. Schwarz, Phys. Lett. **B360**(1995) 13.
- [28] J. Maharana, Phys. Lett. **B402**(1997) 64.
- [29] S. Roy, Int. J. Mod. Phys. **A13**(1998) 4445.
- [30] K. A. Meissner and G. Veneziano, Mod. Phys. Lett. **A6**(1991) 3397.
- [31] E. J. Copeland, J. Lidsey and D. Wands, Phys. Rev. **D57**(1998) 625, hep-th/9708154.