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M THEORY AND P-BRANES

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Abstract

Ten dimensional type IIA and IIB theories with p-branes are compactified to 8-dimensions. It is shown that resulting branes can be classified according to the representations of $\mathbf{SL(3,Z)} \times \mathbf{SL(2,Z)}$. These p-branes can also be obtained by compactification of M theory on three torus and various wrappings of membrane and five brane of the eleven dimensional theory. It is argued that there is evidence for bound states of the branes in eight dimensions as is the case in the interpretation of $\mathbf{SL(2,Z)}$ family of string solutions obtained by Schwarz.

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Recently, a considerable progress has been made in our understanding of non-perturbative phenomena in string theory [1-4]. The extended objects, the 'p-branes', have played a key role in these developments. They appear as non-perturbative solutions of the low energy string effective field theory [5,6] and they have been instrumental in providing an understanding of duality symmetry conjectures in string theory. Furthermore, p-branes carrying Ramond-Ramond charges have important implications for investigations of dualities and string dynamics in diverse dimensions [4,7]. It is now accepted that all the five string theories are intimately related and there is only one underlying theory and different theories are manifestations of various phases of a unique theory.

The conjectured $\mathbf{SL(2,Z)}$ symmetry [8,9] of ten dimensional type IIB superstring theory has interesting consequences. The strong and weak coupling regimes are related by the duality similar to $N = 4, D = 4$ heterotic string. We recall that in type IIB theory strings carrying two gauge field charges are related (this interchange is analogous to T-duality) in contrast to the four dimensional case where electric and magnetic charge carrying particle states are connected. When we consider theory toroidally compactified to less than ten spacetime dimensions, the $\mathbf{SL(2,Z)}$ together with T-duality group results in U-duality group [8].

The existence of a family of string solutions of type IIB theory in 10-dimensions has been demonstrated by Schwarz [10] recently. Of special significance are the BPS saturated solutions which form an $\mathbf{SL(2,Z)}$ multiplet, and these are labelled by a pair of integers (m,n), where m and n are relatively prime. When the type IIB theory is compactified on a circle and the spectrum of the $D = 9$ theory is compared with eleven dimensional supergravity compactified on a torus, several interesting results follow. The $\mathbf{SL(2,Z)}$ duality of type IIB theory, in $D = 10$, corresponds to the modular group of the torus; moreover, one can interpret type II theories as wrapped supermembranes of $D = 11$ supergravity. In sequel [11], nine dimensional type IIB theory on $\mathbf{R^9} \times \mathbf{S^1}$ with p-branes was considered along with M theory on $\mathbf{R^9} \times \mathbf{T^2}$. The eleven dimensional M theory admits only a membrane and a five membrane [12]. It was found that the p-brane tensions of type IIB theory could be related to the tensions of M theory using simple heuristic arguments. Similar relations were also derived for the p-branes appearing in type IIA theory. In view of these developments, it is of interest to study p-branes in 10-dimensions, their compactifications to lower dimensions and various duality symmetries. We shall show that the study of p-branes in eight dimensions results in revealing many interesting features which can be understood from the perspectives of type II theories as well as that of M theory.

In an interesting paper, Polchinski [13] has shown that p-branes carrying RR charges can be described by an exact conformal field theory. Moreover, IIA theory admits even p-branes and IIB theory couples to odd p-branes. Witten [14], in a beautiful paper, has shown that the $\mathbf{SL(2,Z)}$ family of string solutions obtained by Schwarz [10] can be interpreted as the bound states of strings and D-strings and the existence such of BPS saturated (m,n) states is equivalent to existence of vacua of $1+1$ dimensional supersymmetric Yang-Mills theories with a mass gap. Subsequently, several authors have have addressed the problem of bound states for D-branes [15,16].

In ten spacetime dimensions, we recall that for p-branes carrying RR charges, type IIA theory admits even branes whereas type IIB couples to odd branes. When we turn to NS-NS sector, both theories admit a string [17] and a five brane [18]. We mention in passing that more detailed discussions of p-branes can be found in references [5,6,18]. If we turn our attention to the p-branes in 8-dimensions [19], we have not only to take into account the compactification from $D = 10$ to $D = 8$, but also consider how the branes wrap around various geometries. The M theory provides another perspective of the p-branes in eight dimensions. Note that the eleven dimensional theory admits only membrane and five brane; therefore the p-branes in 8-dimensions will arise from dimensional reductions and various kinds of wrappings as we go from $D = 11$ to $D = 8$.

To illustrate the point, let us consider membrane and 4-brane which can arise in ten dimensional IIA theory. When we go to 8-dimensions the dimensional reduction [20] of membrane will take a membrane to a membrane whereas the double dimensional reduction [21,6] of the 4-brane will also result in a membrane. Furthermore, the interpretation of these two membranes in $D = 11$ M theory is different: one membrane arises from dimensional reduction of membrane of this theory and another comes from wrapping of the five brane around three torus. We shall see that when we consider the p-branes in eight dimensions several interesting results follow.

As a definite case, let us consider the ten dimensional theory with 4-brane. The relevant the action in ten dimensions that admits 4-brane solution is given by

$$\tilde{I}_{10}(5) = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-\tilde{g}}\Big\{\tilde{R} - \frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{2}\frac{1}{6!}e^{-\frac{1}{2}\tilde{\phi}}\tilde{F}_6^2\Big\} \tag{1}$$

The 4-brane couples to the worldvolume as

$$\begin{aligned}\tilde{S}_5 = \tilde{T}_5\int d^5\xi\Big\{&\sqrt{-\tilde{\gamma}}\tilde{\gamma}^{ij}\partial_i X^M\partial_j X^N\tilde{g}_{MN}e^{\frac{1}{10}\tilde{\phi}} + \frac{3}{2}\sqrt{-\tilde{\gamma}}\\ &-\frac{1}{5!}\epsilon^{i_1i_2i_3i_4i_5}\partial_{i_1}X^M\partial_{i_2}X^N\partial_{i_3}X^P\partial_{i_4}X^Q\partial_{i_5}X^R\tilde{A}^{MNPQR}\Big\}\end{aligned} \tag{2}$$

The actions $\tilde{I}_{10}(5)$ and $\tilde{S}_5$ are defined with canonical metrics; here the fields with tilde refer to objects in ten spacetime dimensions. Note that $\tilde{F}_6$ refers to the 6-form field strength and the corresponding 5-form gauge potential is denoted by $\tilde{A}$. In what follows, we recapitulate the essential steps to obtain brane solution and its 'dual' solitonic solution, in the context of 4-brane and we refer to the review article of Duff et. al [6] for details. While looking for the 4-brane solution, we split the coordinates as $x^M = (x^\mu, y^m)$, where $\mu = 0,1,2,3,4$ and $m = 5,6....9$ and the metric ansatz is

$$ds^2 = e^{2\tilde{A}}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2\tilde{B}}\delta_{mn}dy^m dy^n \tag{3}$$

and the ansatz for the five form gauge potential is $\tilde{A}_{\mu_1..\mu_5} = \epsilon_{\mu_1..\mu_5}e^{\tilde{C}}$. We demand invariance under Poincare transformations in directions 0,1,2,3 and 4 and $SO(5)$ rotational invariance in y coordinates. Then, $\tilde{\phi}$,$\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ are functions of $y = \sqrt{\delta_{mn}y^m y^n}$.

The solution we look for is the 'electric' 4-brane since we solve the field equation for combined action $\tilde{I}_{10}(5)+\tilde{S}_5$ and the solution is interpreted as an elementary brane with $e^{-2\tilde{\phi}}=(1+\frac{k_5}{y^3})^{1/2}$ and $k_5=2\kappa_{10}^2\frac{\tilde{T}_5}{3\Omega_4}$, Ω_4 being the volume of four sphere. The 'electric' charge is given by

$$g_5^{(e)}=\frac{1}{\sqrt{2}\kappa_{10}}\int_{S^4}e^{-\frac{1}{2}\tilde{\phi}}\;{}^*\tilde{F}_6 \tag{4}$$

where S^4 is the four sphere surrounding the 4-brane. Similarly, the 'magnetic' charge is

$$g_3^{(m)}=\frac{1}{\sqrt{2}\kappa_{10}}\int_{S^6}\tilde{F}_6 \tag{5}$$

$g_3^{(m)}$ is nonzero when $\tilde{I}_{10}(5)$ has a solitonic membrane solution. The solitonic membrane is obtained by solving the equations of motion in the absence of source and adopting an ansatz of combined $P_3\times SO(7)$ invariance just as 4-brane had $P_5\times SO(5)$ invariance, P refers to Poincare transformation. The mass per unit volume of the 4-brane is

$$M_5=\frac{1}{\sqrt{2}}|g_5^{(e)}|e^{\frac{1}{4}\tilde{\phi}_0}=\sqrt{2}\kappa_{10}\tilde{T}_5e^{\frac{1}{4}\tilde{\phi}_0} \tag{6}$$

$\tilde{\phi}_0$ being the asymptotic constant value of dilaton. The corresponding mass density of the membrane is

$$\tilde{M}_3=\frac{1}{\sqrt{2}}|g_3^{(m)}|e^{-\frac{1}{4}\tilde{\phi}_0} \tag{7}$$

where $g_3^{(m)}=2\pi n(\sqrt{2}\kappa_{10}\tilde{T}_5)^{-1}$ by the Dirac quantisation rule. We mention in passing that the masses and charges obey the same equality as the supersymmetric case [6] when one chooses the ratio of the coefficients of the kinetic energy term and the WZW terms appearing in eq.(1) and (2) as adopted here. The solitonic states are also BPS mass saturated states. We also note that if λ_5 is the coupling constant (now we are in σ-model metrics) associated with 4-brane and λ_3 is the one for membrane then, one can check that the relation $(\lambda_5)^5=(\lambda_3)^{-3}$ holds.

Let us proceed to envisage the scenario in 8-dimensions when we adopt double dimensional reduction to obtain a membrane from the 4-brane; however, the membrane, when it is dimensionally reduced will be a membrane in eight dimensions. It is convenient to use a prescription where the determinant of the five-world-volume metric is equal to the determinant of the three-world-volume metric which we get after the reduction. The 8-dimensional action takes the following form

$$I_8(3) = \frac{1}{2\kappa_8^2}\int d^8x\sqrt{-g}\Big\{R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}\frac{1}{4!}e^{-\phi}F_4^2\Big\} \tag{8}$$

The membrane source term becomes

$$S_3 = T_3\int d^3\xi\Big\{\sqrt{-\gamma}\gamma^{ij}\partial_i X^M\partial_j X^N g_{MN}e^{\frac{1}{3}\phi} + \frac{1}{2}\sqrt{-\gamma} - \frac{1}{3!}\epsilon^{ijk}\partial_i X^M\partial_j X^N\partial_k X^P A_{MNP}\Big\} \tag{9}$$

Now the spacetime indices take values $M, N = 0, 1, 2, 5, 6, 7, 8, 9$ (we compactified x^3 and x^4) and the world volume indices run over 0,1 and 2. In eq.(8), F_4 refers to the 4-form field strength associated with the 3-form potential appearing in (9) which arises as dimensional reduction of 5-form potential, $\tilde{A}$, in ten dimension. The membrane tension T_3 appearing in the above equation is proportional to $\tilde{T}_5$ with a two-volume factor as we come down from five dimensional world volume to three dimensional one. The constant $\kappa_8^2 = (2\pi R)^2\kappa_{10}^2$ where R is the radius of the circles along directions x^3 and x^4. The 'electric' charge (we still have the source term) is given by $g_3^{(e)} = (\sqrt{2}\kappa_8)^{-1}\int_{S^4} e^{-\phi *}\ F_4$ and the magnetic charge is $g_3^{(m)} = (\sqrt{2}\kappa_8)^{-1}\int_{S^4} F_4$ and they satisfy the quantization condition $g_3^{(e)}g_3^{(m)} = 2\pi n$, where n is an integer. The solution to the field equation can obtained in a straight forward manner by demanding invariance under $P_3 \times SO(5)$ transformations and choosing appropriate ansatz for the break up of the metric (now x^μ take three values and y^m go over five values) and taking the gauge potential to be 1 to three index ϵ tensor times a function of y. It is evident that we shall have electrically and magnetically charged membranes.

Now let us turn our attention to the study of p-branes in 8-dimensions and look for their origin in type IIB theory in ten dimension, a path taken by Schwarz [10] while considering strings in IIB theory in ten and nine dimensions. The type IIB theory, in RR sector has 3-form and 1 5-form field strengths thus admitting a string and a 3-brane; in addition a five brane (we shall refrain from considering higher p-branes, $p > 7$, here). The NS-NS sector has a string and a 5-brane in D=10. We note that with each p-brane we can associate a $p+2$ form field strength. Again, to be specific, let us look at the 4-branes in $D = 8$. When we consider RR sector, the 7-form field strength $\tilde{H}^{(7)}$ will give rise to two 6-form field strengths $H^{(6)}_{\alpha}$, $\alpha = 3, 4$ being compact directions. From the NS-NS sector, we have four 6-form field strengths: two coming from the 7-form in NS-NS sector and the other two come from the dual of two of the 2-form field strengths (which come from dimensional reduction of 3-form field strength in ten dimensions). Thus there are altogether six 6-form field strengths and we conclude that *in eight spacetime dimensions there are six 4-branes* . There is another way to cross check our accountings. We know, when the type IIB theory is toroidally compactified to 8-dimensions, there are six gauge bosons: two pairs come from the dimensional reduction of the ten dimensional metric and antisymmetric tensor fields (these are from NS-NS sector) and two more from the antisymmetric tensor of the RR sector. Notice that the existence of the six gauge bosons implies that there are six 0-branes. Since, in $D = 8$, the dual of a 0-brane is a four brane, we should have six 4-branes. Since IIB theory has two five branes (each from NS-NS and RR sector), in ten dimensions, the four branes will arise when the five branes wrap around the two torus to give rise to the 4-branes we have been discussing. How do we understand these 4-branes from eleven dimensional M theory? There is one five brane in D=11 and we come down to a D=8 compactifying on three torus. The 4 will give three of the 4-branes and the rest will come from the KK modes. Now to complete the discussion of the 4-branes, let us 2 their origin from type IIA view point since IIA and IIB theories are equivalent in 8-dimensions. In ten dimensions, the graviton and antisymmetric tensor field originate from NS-NS sector and the lone gauge field and 3-form potential arise from the RR sector. When we count the number of 6-form field strengths in $D = 8$, they add up to six. We can follow same line of arguments for counting of other branes in 8-dimensions.

There are interesting consequences of these results. It is well known that when IIB theory is compactified to 8-dimensions, the resulting fields can be grouped into $\mathbf{SL(3,)} \times \mathbf{SL(2,)}$ representations. The gauge six gauge fields, alluded to above, belong to $(\mathbf{3}, \mathbf{2})$ representations and consequently, the 0-branes and their duals the 4-branes also belong to this representations. We can adopt and generalize the arguments of Schwarz [10], and propose that these 4-branes/0-branes will carry $\mathbf{SL(3,Z)} \times \mathbf{SL(2,Z)}$ charges, and those objects carrying charges with relatively prime integer will be stable. Such 4-branes/0-branes can be interpreted as bound states of other branes. We know that as we come to $D = 8$, the group is product of $\mathbf{SL(2,Z)}$, which was in ten dimensions and $O(2,2;\mathbf{Z})$ which arises as a result of dimensional reduction. The $O(2,2;\mathbf{Z})$ has $\mathbf{SL(2,Z)} \times \mathbf{SL(2,Z)}$ as its subgroup. One of these $\mathbf{SL(2,Z)}$, the one which parametrizes $B_{34} + i\sqrt{detG_{\alpha\beta}}, \alpha, \beta = 3,4$ combines with the $\mathbf{SL(2,Z)}$ coming from $D = 10$ and the $\mathbf{SL(3,Z)}$ is a subgroup of the product of these two $\mathbf{SL(2,Z)}$ groups. Here B_{34} and $G_{\alpha\beta}$ refer to the internal components of antisymmetric tensor and the metric as we come from ten to eight dimensions.

When we turn out attentions the membranes in 8-dimensions, we find that there is only a pair of them. This can be seen by counting 4-form fields, after dimensional reduction, either in type IIA, or in IIB or in M theories. It is easy to see that each of the theory contains only two such field strengths. Therefore, we conclude that the 4-form field strength F_4 and its dual ${}^*F_4$ belong to $(1,2)$ representation of $\mathbf{SL(3,Z)} \times \mathbf{SL(2,Z)}$. Indeed, the membranes will be characterized by a pair of integers (m,n) and the dyonic solutions of Izquierdo et. al [18] now finds a natural interpretation in this perspective. Now we can invoke the arguments of Schwarz, [10] for $\mathbf{SL(2,Z)}$ family of strings, and claim that if a membrane carries charges (m,n), m and n relatively prime, they will be stable. Therefore, bound states of membrane should exist as stable membranes.

It follows from results of ref.11 that the p-brane tensions are related to the membrane and fivebrane tensions of M theory. It is well known that the IIA theory has a simple interpretation as the M theory on $R^{10} \times S^1$. If $g^{(M)}$ is the eleven dimensional metric, and $L = 2\pi R$ is the circumference in that metric, then the string metric, $g^{(A)}$, of type IIA theory is $g^{(A)} = e^{2\phi_A/3} g^{(M)}$ and the dilaton of the IIA theory is identified as ϕ_A and the coupling constant is the vacuum expectation value of e^{ϕ_A}. The set of relations derived by Schwarz [11] are ($T_1^{(A)}$, $T_2^{(A)}$ and $T_4^{(A)}$ denote IIA theory tensions for string, membrane and 4-brane in what follows and similar parameters with superscript M refer to the M-theory counter parts):

$$T_2^{(A)} = g_A^{-1} T_2^{(M)}, \qquad T_4^{(A)} = g_A^{-\frac{5}{3}} L T_5^{(M)} \tag{10}$$

for the even p-branes coming from the RR sector and

$$T_1^{(A)} = g_A^{-\frac{2}{3}} L T_2^{(M)}, \qquad T_5^{(A)} = g_A^{-2} T_5^{(M)} \tag{11}$$

for string and 5-branes in the NS-NS sector. Therefore, when we come down to $D = 8$, the tensions of four branes and two branes can be expressed in terms of M theory tensions and the volume factors. It is quite interesting to see all the intimate connections not only between IIA and IIB theories, but also with M theory in the 8-dimensional world.

It is evident that we can consider theories compactified to lower dimensions and systematically study p-branes in those theories starting from type II theories or M theory. Then, dualities and classification of 0-branes, strings and membranes can be studied by adopting this procedure. Recently, membrane solutions have been obtained for type IIA and heterotic strings in 6-dimensions. It is natural to expect that there will be many more solutions than the types of solutions obtained by Johnson et. al [22].

To summarize, we have studied p-branes in 8-dimensional theory from compactification of ten dimensional type II theories. They can be viewed from the M theory perspective, where the eleven dimensional theory is compactified to 8-dimensions. The appearance of branes, classified according to $\mathbf{SL(3,Z)} \times \mathbf{SL(2,Z)}$, tells us that we expect to have stable 0-branes as well as 4-branes when their charge assignments satisfy suitable constraints and from these considerations, we can conjecture that there is evidence for stable bound states of such branes. Our results can also be viewed as a way to verify the predictions of U-duality [8]. Furthermore, we have shown intimate connections between the type II theories and the M theory in these studies of p-branes and we demonstrated simple and elegant descriptions of the eight dimensional p-branes from the view point of M theory.

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