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On the Compactification of type IIA String Theory

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ABSTRACT

The ten dimensional type IIA string effective action with cosmological constant term is dimensionally reduced on a d-dimensional torus to derive lower dimensional effective action. The symmetries of the reduced effective action are examined. It is shown that the resulting six dimensional theory does not remain invariant under $SO(4,4)$ symmetry whereas the reduced action, in the absence of the cosmological constant respects the symmetry as was shown by Sen and Vafa. New class of black hole solutions are obtained in five and four dimensions in the presence of cosmological constant. For the six dimensional theory, a four brane solution is presented.

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Recently, considerable progress has been made in our understanding of the nonperturbative features of superstring theories [1-3]. It is now realised that the five consistent superstring theories might be envisioned as various phases of a single unique theory [4]. Dualities play a cardinal role in revealing the intimate connections between different string theories in diverse spacetime dimensions and provide deeper insight into string theory dynamics. We recall that the predictions of T-duality are subject to tests in the perturbative frame work ; whereas, the predictions and tests of S-duality are beyond the realms of perturbation theory [5]. The p-branes, which appear as classical solutions of the string effective action, have been instrumental in our understanding of various duality conjectures in string theory [6]. The RR p-branes are interpreted as D-p-branes of type II theories [7]. The type IIA string admits even D-branes, $p = 0, 2, 4, 6$ and type IIB theory, on the other hand, has the odd ones, i.e. $p = 1, 3, 5$ with the identification that -1-brane is the instanton of the theory. Furthermore, for 10-dimensional spacetime, dual of a p-brane is the $(6 - p)$ brane and consequently, those p-branes with $p \leq 6$, have duals with $p \geq 0$. Thus, for $D = 10$, the 8-brane and 7-brane appearing in type IIA and type IIB string theories respectively have special roles different from the other branes alluded to above.

A p-brane couples to $(p + 1)$-form potential; therefore, the 8-brane will couple to the potential A_9 whose corresponding field strength is the ten form F_{10}. In standard type IIA supergravity, the presence of the potential A_9 is rather obscure. From the perspective of type IIA string theory, we know that the theory admits 8-D-brane [7,8]. Notice that the equations of motion arising from the kinetic energy term F_{10}^2 only give rise to a conservation law and the presence of this term does not introduce any new dynamical degree of freedom. However, the effect of this additional term amounts to introduction of cosmological constant, when we introduce the Poicare dual of ten form field strength instead. In this context it is worthwhile to mention that it had been realised several years ago that the introduction of a four-form field strength in four spacetime dimensions amounts to having a cosmological constant term in that supergravity theory [9]. Romans [10], subsequently, constructed the massive ten dimensional type IIA supergravity theory and a complete construction was given in ref.11.

The study of type IIA superstring effective action in the presence of F_{10}, or alternatively the theory with cosmological constant has drawn attention of several authors [12-15] in the recent past and it has been argued that the cosmological constant takes only quantized values. We mention in passing, another interesting feature of the presence of cosmological constant in the four dimensional heterotic string effective action. It was shown that the equations of motion are not invariant under S-duality transformations in the presence of cosmological constant [16], whereas the equations of motion do respect the symmetry when the constant is set to zero. Then, a weaker form of the naturalness criterion, due to 't Hooft [17], was invoked to argue that the cosmological constant should remain small since when it required to vanish there is enhancement of symmetry at the level of equations of motion, derived from string effective action. We recall that the usual Einstein-Hilbert action does not have any enhanced symmetry in the absence of the cosmological constant as was recognised by 't Hooft, when he introduced the idea of naturalness [17]. Since we expect string theory to provide answers to deep questions in quantum gravity, it is hoped that the cosmological constant problem will be solved by string theory. Recently, Witten has proposed a resolution of the cosmological constant problem [18]. The starting point is to envisage three dimensional theory with a string vacuum, with unbroken supersymmetry and dilaton whose exponential is related to the string coupling constant, g_s. In the weak coupling regime, the string perturbation theory is valid and cosmological constant vanishes due to unbroken supersymmetry. When one passes to the strong coupling limit, $g_s \to \infty$, the resulting theory is a Poincare invariant theory in $3+1$ dimensions. The cosmological constant remains zero in this four dimensional theory since it continues to take vanishing value for all g_s. We speculate that the stringy symmetries might provide a clue for the resolution of the cosmological constant problem (see discussions below).

It is well known, for the massless theory, that type IIA compactified on S^1 with radius R is T-dual to type IIB compactified on another circle with reciprocal radius [19]. Thus the issue of compactification of massive type IIA theory to $D = 9$ has been addressed in the context of its T-duality to type IIB theory. It has been argued that the ten dimensional type IIB theory, when compactified according to the generalised Scherk-Schwarz [20] prescription, yield a massive theory in 9 dimensions and then one can explore the T-duality. Moreover, there have been attempts to obtain various brane solutions in type IIA, IIB and M-theory [21-24] and relate these solutions in lower dimensions by adopting sequential steps in dimensional reductions.

The purpose of this article is to present dimensionally reduced string effective action for massive type IIA superstring action when we compactify it on a d-dimensional torus. We investigate the symmetry properties of the reduced effective action. In particular, we show that the six dimensional effective action for the case of the massive theory does not respect the $SO(4,4)$ symmetry of the corresponding six dimensional massless theory. Furthermore, we find new black hole solutions in five and four space time dimensions from the reduced effective action in the presence of cosmological constant term, and we also present four-brane solutions in six dimensions.

The bosonic part of massive type IIA supergravity action, in ten dimensions, is of interest to us. The action was introduced by Romans [10] and we write an action in the string frame metric

$$S_m = \int d^{10}x\sqrt{-g}\ \Big[e^{-2\Phi}\Big(R_g + 4\ \partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2\cdot 3!}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\Big) - \frac{1}{2\cdot 2!}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\cdot 4!}G_{\mu\nu\lambda\rho}G^{\mu\nu\lambda\rho} - \frac{1}{2}m^2\Big], \tag{1}$$

where Φ is the dilaton field, $g_{\mu\nu}$ is the string $\sigma-$model metric and m is the mass parameter. This action can be identified as the low energy limit of the type IIA string theory with m^2 playing the role of cosmological constant. The NS-NS and R-R field strengths are defined as follows:

$$\begin{aligned} F_{\mu\nu} &= \partial_{[\mu}A_{\nu]} + mf_{\mu\nu}, \\ H_{\mu\nu\lambda} &= \partial_\mu B_{\nu\lambda} + \text{cyclic permutations}, \\ G_{\mu\nu\lambda\rho} &= \partial_{[\mu}C_{\nu\lambda\rho]} + 2A_{[\mu}H_{\nu\lambda\rho]} + 2m\ g_{\mu\nu\lambda\rho}, \end{aligned} \tag{2}$$

where coefficients of the mass parameter terms are $f_{\mu\nu} = B_{\mu\nu}$ and $g_{\mu\nu\lambda\rho} = B_{[\mu\nu}B_{\lambda\rho]}$. The notation $[\mu\nu\cdots]$ implies the antisymmetrization of the indices. Note that the field strengths have mass dependent terms and are the generalisations of their massless counterparts. The advantage of writing massive type IIA action as in (1) is that the action for the massless theory can be obtained by taking the limit $m \to 0$. The action has the invariance under massive 'Stückelberg' gauge transformations

$$\begin{aligned} \delta A_\mu &= -m\Lambda_\mu \\ \delta B_{\mu\nu} &= \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu \\ \delta C_{\mu\nu\lambda} &= -2m(\Lambda_\mu B_{\nu\lambda} + \text{cyclic perms.}). \end{aligned} \tag{3}$$

The above action has $N = 2$ supersymmetry even though it involves mass terms. The constant mass term in the R-R sector of the theory which has the interpretation of the cosmological constant can also be envisaged as the dual of 10-form filed strength alluded to earlier. Therefore, in ten dimensions, the appearance of m^2 terms provides a clue for the presence of an 8-brane in type IIA theory with the hindsight.

Let us consider compactification of the ten dimensional effective action, in presence of the cosmological constant term, on a d-dimensional torus. We adopt the prescription of Schwarz and one of the authors (JM)[25]. The coordinates of D-dimensional spacetime are denoted by x^μ, whereas the rest which make the internal dimensions, the d-dimensional torus, are denoted as x^α. In our notational conventions, we denote ten-dimensional fields with hats over the fields as well as over the tensor indices ($\hat{\Phi},\ \hat{g}_{\hat{\mu}\hat{\nu}},\ etc.$), while reserve the quantities without hats for D-dimensional ones. Furthermore, we assume that the fields are independent of the "internal" coordinates, x^α. The ten dimensional vielbein can be expressed in the following form $\hat{e}_{\hat{\mu}}^{\hat{r}} = \begin{pmatrix} e_\mu^r & A_\mu^{(1)\beta} E_\beta^a \\ 0 & E_\alpha^a \end{pmatrix}$ and "spacetime" metric $g_{\mu\nu} = e_\mu^r \eta_{rs} e_\nu^s$, "internal" metric $G_{\alpha\beta} = E_\alpha^a \delta_{ab} E_\beta^b$.

Thus, the ten dimensional metric components will be expressed in terms of the D-dimensional metric, gauge fields and scalars.

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + A_\mu^{(1)\alpha} A_\nu^{(1)\beta} G_{\alpha\beta}, \quad A_{\mu\alpha}^{(1)} = \hat{g}_{\mu\alpha}, \quad G_{\alpha\beta} = \hat{g}_{\alpha\beta}, \tag{4}$$

Similarly for the antisymmetric tensor field, coming from the NS-NS sector, the decompositions are

$$\begin{aligned} A_{\mu\alpha}^{(2)} &= \hat{B}_{\mu\alpha} - A_\mu^{(1)\beta} b_{\alpha\beta}, \quad b_{\alpha\beta} = \hat{B}_{\alpha\beta}, \\ B_{\mu\nu}^{(1)} &= \hat{B}_{\mu\nu} - A_{[\mu}^{(1)\alpha} A_{\nu]\alpha}^{(2)} - A_\mu^{(1)\alpha} A_\nu^{(1)\beta} b_{\alpha\beta}, \end{aligned} \tag{5}$$

and the R-R fields can be decomposed as follows:

$$\begin{aligned} c_{\alpha\beta\gamma} &= \hat{C}_{\alpha\beta\gamma}, \quad a_\alpha = \hat{A}_\alpha \\ A_{\mu\alpha\beta}^{(3)} &= \hat{C}_{\mu\alpha\beta} - A_\mu^{(1)\delta} c_{\alpha\beta\delta}, \\ B_{\mu\nu\alpha}^{(2)} &= \hat{C}_{\mu\nu\alpha} + A_{[\mu}^{(1)\delta} \hat{C}_{\nu]\delta\alpha} + A_\mu^{(1)\delta_1} A_\nu^{(1)\delta_2} c_{\delta_1\delta_2\alpha} \\ C_{\mu\nu\lambda} &= \hat{C}_{\mu\nu\lambda} - (A_\mu^{(1)\delta} \hat{C}_{\delta\nu\lambda} + \text{cyclic perms. in } \mu, \nu, \lambda), \\ &\quad + (A_\mu^{(1)\delta_1} A_\nu^{(1)\delta_2} \hat{C}_{\delta_1\delta_2\lambda} + \text{cyclic perms in } \mu, \nu, \lambda), \\ &\quad - A_\mu^{(1)\delta_1} A_\nu^{(1)\delta_2} A_\lambda^{(1)\delta_3} c_{\delta_1\delta_2\delta_3} \\ A_{\mu\alpha}^{(4)} &= \hat{A}_\mu - A_\mu^{(1)\delta} a_\delta. \end{aligned} \tag{6}$$

Recall that the scalars are constructed in the ten dimensional theory by contracting the hat indices of various tensors. In order to obtain tensors with unhatted indices, i.e. tensors in D-dimensions we adopt the following prescription:

$$\mathcal{H}_{\mu\nu..\alpha\beta..} = \mathcal{O}_\mu^{\hat{\mu}} \mathcal{O}_\nu^{\hat{\nu}} \cdots \hat{\mathcal{H}}_{\hat{\mu}\hat{\nu}..\alpha\beta..} \tag{7}$$

where, $\mathcal{O}_\mu^{\hat{\mu}} = e_\mu^r \; \hat{e}_r^{\hat{\mu}}$ and $\hat{\mathcal{H}}_{\hat{\mu}\hat{\nu}..\alpha\beta..}$ is a tensor in ten dimensions. Thus scalars constructed out of contraction of ten dimensional indices, as is the case with kinetic energy terms in the action, can be expressed in the following form in terms of scalars constructed out of various tensors in D-dimensions, obtained through the dimensional reduction procedure,

$$\hat{\mathcal{H}}_{\hat{\mu}\hat{\nu}..} \hat{\mathcal{H}}^{\hat{\mu}\hat{\nu}..} = \mathcal{H}_{\mu\nu..} \mathcal{H}^{\mu\nu..} + n \mathcal{H}_{\mu\nu..\alpha} \mathcal{H}^{\mu\nu..\alpha} + \frac{n(n-1)}{2!} \mathcal{H}_{\mu\nu..\alpha\beta} \mathcal{H}^{\mu\nu..\alpha\beta} + \cdots + \mathcal{H}_{\alpha\beta..} \mathcal{H}^{\alpha\beta..}, \tag{8}$$

for the $n-$form field strength. Following (7) NS-NS field strengths are obtained as below,

$$\begin{aligned} H_{\mu\nu\rho}^{(1)} &= \partial_{[\mu} B_{\nu\rho]} - F_{[\mu\nu}^{(1)\delta} A_{\rho]\delta}^{(2)}, \\ H_{\mu\nu\alpha} &= F_{\mu\nu\alpha}^{(2)} - F_{\mu\nu}^{(1)\delta} b_{\alpha\delta} \\ H_{\mu\alpha\beta} &= \partial_\mu b_{\alpha\beta}. \end{aligned} \tag{9}$$

where $F^{(i)}_{\mu\nu} = \partial_{[\mu}\ A^{(i)}_{\nu]}$. The Chern-Simon ($CS$) term in eq.(2), $\hat{A} \wedge \hat{H}$, will give

$$
\begin{aligned}
[CS]_{\mu\alpha\beta\gamma} &= -\left(a_\alpha \partial_\mu b_{\beta\gamma} + \text{cyclic perms. of } \alpha, \beta, \gamma\right), \\
[CS]_{\mu\nu\alpha\beta} &= A^{(4)}_{[\mu}\partial_{\nu]} b_{\alpha\beta} + \left\{a_\alpha (F^{(2)}_{\mu\nu\beta} - b_{\beta\delta} F^{(1)\delta}_{\mu\nu}) - (\alpha \leftrightarrow \beta)\right\} \\
[CS]_{\mu\nu\rho\alpha} &= A^{(4)}_{[\mu}\left(F^{(2)}_{\nu\rho]\alpha} - F^{(1)\delta}_{\nu\rho]} b_{\alpha\delta}\right) - a_\alpha H^{(1)}_{\mu\nu\rho} \\
[CS]_{\mu\nu\rho\sigma} &= A^{(4)}_{[\mu} H^{(1)}_{\nu\rho\sigma]}.
\end{aligned}
\tag{10}
$$

Then RR field strengths reduce as given below,

$$
\begin{aligned}
G_{\alpha\beta\gamma\delta} &= 2m\ b_{[\alpha\beta} b_{\gamma\delta]} \\
G_{\mu\alpha\beta\gamma} &= \partial_\mu c_{\alpha\beta\gamma} + 2[CS]_{\mu\alpha\beta\gamma} + 2m\ A^{(2)}_{\mu[\alpha} b_{\beta\gamma]} \\
G_{\mu\nu\alpha\beta} &= \partial_{[\mu} A^{(3)}_{\nu]\alpha\beta} + F^{(1)\delta}_{\mu\nu} c_{\delta\alpha\beta} + 2[CS]_{\mu\nu\alpha\beta} + 2m\ \left(B^{(1)}_{\mu\nu} b_{\alpha\beta} - (A^{(2)}_{\mu\alpha} A^{(2)}_{\nu\beta} - \{\alpha \leftrightarrow \beta)\}\right) \\
G_{\mu\nu\rho\alpha} &= \partial_{[\mu} B^{(2)}_{\nu\rho]\alpha} - F^{(1)\delta}_{[\mu\nu} A^{(3)}_{\rho]\delta\alpha} + 2[CS]_{\mu\nu\rho\alpha} + 2m B^{(1)}_{[\mu\nu} A^{(2)}_{\rho]\alpha} \\
G_{\mu\nu\rho\sigma} &= \partial_{[\mu} C_{\nu\rho\sigma]} + F^{(1)\delta}_{[\mu\nu} B^{(2)}_{\rho\sigma]\delta} + 2[CS]_{\mu\nu\rho\sigma} + 2m B^{(1)}_{[\mu\nu} B^{(1)}_{\rho\sigma]} \\
F_{\alpha\beta} &= m b_{\alpha\beta} \\
F_{\mu\alpha} &= \partial_\mu a_\alpha + m A^{(2)}_{\mu\alpha} \\
F_{\mu\nu} &= F^{(4)}_{\mu\nu} + F^{(1)\delta}_{\mu\nu} a_\delta + m B^{(1)}_{\mu\nu}.
\end{aligned}
\tag{11}
$$

Now, to consider a specific case, let us look at the reduced effective action in six spacetime dimensions. We utilise the identity (8) and use various definitions from eqs.(9) and (11) to write down the six-dimensional massive IIA action,

$$
\begin{aligned}
\int d^6x\sqrt{-g}\ \ & \Big[e^{-2\phi}[\ R + 4\partial_\mu\phi\partial^\mu\phi - \frac{1}{12} H^{(1)}_{\mu\nu\lambda} H^{(1)\mu\nu\lambda} + \frac{1}{8} Tr\partial_\mu M^{-1}\partial^\mu M - \frac{1}{4} F^{(i)}_{\mu\nu} M^{-1}_{ij} F^{(j)\mu\nu}\ ] \\
& -\sqrt{G}\{\frac{1}{2\cdot 2!}\Big((F^{(4)}_{\mu\nu} + F^{(1)\delta}_{\mu\nu} a_\delta + m B^{(1)}_{\mu\nu})^2 + 2(\partial_\mu a_\alpha + m A^{(2)}_{\mu\alpha})^2 + (m\ b_{\alpha\beta})^2\Big) \\
& \quad + \frac{1}{2\cdot 4!}[(\partial_{[\mu} C_{\nu\rho\sigma]} + F^{(1)\delta}_{[\mu\nu} B^{(2)}_{\rho\sigma]\delta} + 2A^{(4)}_{[\mu} H^{(1)}_{\nu\rho\sigma]} + 2m B^{(1)}_{[\mu\nu} B^{(1)}_{\rho\sigma]})^2 \\
& \quad + 4(\partial_{[\mu} B^{(2)}_{\nu\rho]\alpha} - F^{(1)\delta}_{[\mu\nu} A^{(3)}_{\rho]\delta\alpha} + 2A^{(4)}_{[\mu}\left(F^{(2)}_{\nu\rho]\alpha} - F^{(1)\delta}_{\nu\rho]} b_{\alpha\delta}\right) - 2a_\alpha H^{(1)}_{\mu\nu\rho} \\
& \qquad\qquad + 2m B^{(1)}_{[\mu\nu} A^{(2)}_{\rho]\alpha})^2 \\
& \quad + 6(F^{(3)}_{\mu\nu\alpha\beta} + F^{(1)\delta}_{\mu\nu} c_{\delta\alpha\beta} + 2A^{(4)}_{[\mu}\partial_{\nu]} b_{\alpha\beta} + 2\left\{a_\alpha (F^{(2)}_{\mu\nu\beta} - b_{\beta\delta} F^{(1)\delta}_{\mu\nu}) - (\alpha \leftrightarrow \beta)\right\} \\
& \qquad\qquad + 2m\left\{B^{(1)}_{\mu\nu} b_{\alpha\beta} + A^{(2)}_{\mu[\alpha} A^{(2)}_{\beta]\nu}\right\})^2 \\
& \quad + 4(\partial_\mu c_{\alpha\beta\gamma} - 2\left(a_\alpha \partial_\mu b_{\beta\gamma} + \text{cyclic perms. of } \alpha, \beta, \gamma\right) \\
& \qquad\qquad + 2m\ A^{(2)}_{\mu[\alpha} b_{\beta\gamma]})^2 + (2m\ b_{[\alpha\beta} b_{\gamma\delta]})^2] + \frac{1}{2} m^2\}\Big],
\end{aligned}
\tag{12}
$$

where $\phi = \hat{\Phi} - \frac{1}{2}\ln G$ is shifted dilaton field and the scalars coming from $G_{\alpha\beta}$ and $b_{\alpha\beta}$ have been combined to form the symmetric 8×8 matrix

$$M = \eta M^{-1}\eta = \begin{pmatrix} G^{-1} & -G^{-1}b \\ bG^{-1} & G - bG^{-1}b \end{pmatrix}, \qquad \eta = \begin{pmatrix} 0 & I_4 \\ I_4 & 0 \end{pmatrix} \tag{13}$$

where η is $O(4,4)$ metric and I_4 is 4-dimensional identity matrix. We mention in passing that, if we had chosen to consider compactification on the d-dimensional torus, T^d, then the corresponding $2d \times 2d$ symmetric M-matrix will appear, defined in terms of scalars coming from the NS-NS sector and the metric η for $O(d,d)$ group with off diagonal identity matrix I_d has to be introduced. Let us recall how various fields appear in the six dimensional action (12), after the compactification. In the NS-NS sector we have dilaton field, ϕ, graviton, $g_{\mu\nu}$, tensor field, $B^{(1)}_{\mu\nu}$, eight vector fields, coming from ten dimensional metric and two index antisymmetric tensor fields after compactification and sixteen scalar fields, $\hat{g}_{\alpha\beta}$ and $\hat{B}_{\alpha\beta}$, appearing in matrix M which parameterize the coset $\frac{O(4,4)}{O(4)\times O(4)}$. On the other hand in the R-R sector there are eight scalars from $\hat{A}_\alpha$ and $\hat{C}_{\alpha\beta\gamma}$, seven vectors from $\hat{A}_\mu$ and $\hat{C}_{\mu\alpha\beta}$, four 2-rank potentials from $\hat{C}_{\mu\nu\alpha}$ and one 3-rank potential $C_{\mu\nu\lambda}$. Let us recapitulate the symmetry of the six dimensional effective action for the case when $m = 0$ following the works of Sen and Vafa [26]. It was shown in ref.26 that the action is invariant under $SO(4,4)$ symmetry after the transformation properties of scalar, vector, and tensor fields were defined. In fact the equations of motion are invariant under a larger noncompact symmetry group $SO(5,5)$. On this occasion, the massless case, one can combine dual of 3-rank tensor field $C_{\mu\nu\lambda}$ with seven other RR vector fields to form 8-dimensional spinorial representation, $\psi^a_\mu (1 \leq a \leq 8)$, of $SO(4,4)$. Similarly, 3-form field strengths $G_{\mu\nu\lambda\alpha}$ can be taken to be (anti)self-dual to form another 8-dimensional spinorial representation, $\psi^a_{\mu\nu\lambda}$. Note that eight RR-scalars do also transform under one of these spinor representation. The afore mentioned symmetry of massless six-dimensional theory was exploited in [26] to generate type II dual pairs. However, one can explicitly check that in the case of dimensionally reduced massive theory, the action is no longer invariant under the above noncompact symmetry group.

The above action (12) is invariant under the following set of Stückelberg transformations (although it is tedius calculation),

$$\begin{aligned}
&\delta A^{(4)}_\mu = -m\ \Lambda_\mu, \\
&\delta a_\alpha = -m\ \lambda_\alpha, \\
&\delta b_{\alpha\beta} = 0, \\
&\delta A^{(1)\delta}_\mu = 0, \quad \delta A^{(2)}_{\mu\alpha} = \partial_\mu \lambda_\alpha, \\
&\delta B^{(1)}_{\mu\nu} = \partial_{[\mu}\Lambda_{\nu]} + F^{(1)\delta}_{\mu\nu}\lambda_\delta, \\
&\delta c_{\alpha\beta\gamma} = -2m\ \lambda_{[\alpha} b_{\beta\gamma]},
\end{aligned}$$

$$\delta A^{(3)}_{\mu\alpha\beta} = -2m\ (\Lambda_\mu b_{\alpha\beta} + A^{(2)}_{\mu[\alpha}\lambda_{\beta]}),$$
$$\delta B^{(2)}_{\mu\nu\alpha} = -2m\ (\Lambda_{[\mu} A^{(2)}_{\nu]\alpha} + \lambda_\alpha B^{(1)}_{\mu\nu}),$$
$$\delta C_{\mu\nu\rho} = -2m\ \Lambda_{[\mu} B^{(1)}_{\nu\rho]}. \tag{14}$$

Here, Λ_μ and λ_α are vector and scalar gauge functions respectively. Note that in (14) RR-scalars do also transform under Stückelberg transformations in lower dimensions.

Next, we present black hole solutions in five and four dimensions. While looking for black hole solutions, we keep only dilaton and the two form field strengths in the action (12) and set the other scalar and tensor fields to zero. First we consider, the following D-dimensional action (in Einstein frame)

$$S_m = \int d^Dx\sqrt{-g}\ \Big[(R_g - \frac{4}{D-2}\ \partial_\mu\phi\partial^\mu\phi - \frac{1}{2\cdot 2!}e^{\frac{-4}{D-2}\phi}F_{\mu\nu}F^{\mu\nu} - 2\lambda e^{\frac{4}{D-2}\phi}) \\ -\frac{1}{2\cdot 2!}e^{\frac{2(D-4)}{D-2}\phi}F_{R\mu\nu}F^{\mu\nu}_R - \frac{1}{2}m^2 e^{\frac{2D}{D-2}\phi}\Big], \tag{15}$$

We have added the term, $\lambda e^{\frac{4}{(D-2)}\phi}$, to the action and the presence of this term can be interpreted as a dilatonic potential which owes its origin from the NS-NS sector and might appear due to some nonperturbative effects. The m^2 piece comes from the massive ten dimensional action (1) after compactification. In eq.(15) the gauge field strengths $F_{\mu\nu}$ and $F_{R\mu\nu}$ come from the NS-NS and RR sectors respectively.

The equations of motion are

$$\nabla_\mu \nabla^\mu \phi + \frac{1}{8}e^{-\frac{4}{D-2}\phi}F^2 - \frac{D-4}{16}e^{\frac{2(D-4)}{D-2}\phi}F_R^2 - \lambda e^{\frac{4}{D-2}\phi} - \frac{m^2 D}{8}e^{\frac{2D}{D-2}\phi} = 0,$$
$$(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) - \frac{4}{D-2}(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2) - \frac{1}{4}e^{-\frac{4}{D-2}\phi}(2F_{\mu\lambda}F^\lambda_\nu - \frac{1}{2}g_{\mu\nu}F^2)$$
$$-\frac{1}{4}e^{\frac{2(D-4)}{D-2}\phi}(2F_{R\mu\lambda}F^\lambda_{R\nu} - \frac{1}{2}g_{\mu\nu}F_R^2) + \frac{1}{2}g_{\mu\nu}(2\lambda e^{\frac{4}{D-2}\phi} + \frac{1}{2}m^2 e^{\frac{2D}{D-2}\phi}) = 0,$$
$$\partial_\mu e^{\frac{-4}{D-2}}F^{\mu\nu} = 0,$$
$$\partial_\mu e^{\frac{2(D-4)}{D-2}\phi}F^{\mu\nu}_R = 0. \tag{16}$$

We seek maximally symmetric black holes solutions [28] and we choose constant dilaton backgrounds $\phi = \phi_c$ with the metric ansatz

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2_{D-2}, \tag{17}$$

where ϵ_{D-2} and $d\Omega^2_{D-2}$ are the volume element and metric on unit S^{D-2}, respectively. Our first example is a black hole with the following choice of backgrounds: $F = 0,\ F_R = 0, with\ \lambda < 0$; the solution (17) is Schwarzschild-Anti-deSitter(SAdS) space with

$$e^{2\phi_c} = \frac{8}{5}\ \frac{|\lambda|}{m^2}$$

$$f(r) = 1 - \frac{2M}{r^2} + \frac{|\lambda|}{10}\left[\frac{8|\lambda|}{5m^2}\right]^{\frac{2}{3}} r^2 \tag{18}$$

Note that the black hole solution is asymptotically an AdS space with effective cosmological constant $\Lambda = \frac{2|\lambda|}{5}\left[\frac{8|\lambda|}{5m^2}\right]^{\frac{2}{3}}$.

The second example corresponds to the backgrounds: $F \neq 0,\ F_R \neq 0$ *and* $\lambda < 0$, with the constraint

$$Q_R^2 \frac{m^2}{2} = \frac{8}{5} Q^2 |\lambda|, \tag{19}$$

is satisfied and the charges are defined as

$$Q = \frac{1}{2\pi^2}\int_{S^3} * e^{-\frac{4}{3}\phi} F, \qquad Q_R = \frac{1}{2\pi^2}\int_{S^3} * e^{\frac{2}{3}\phi} F_R. \tag{20}$$

The solution is the Reissner-Nordstrom-AntideSitter(R-N-AdS) black hole in 5-dimensions

$$\begin{aligned}
e^{2\phi_h} &= \frac{1}{2}\frac{Q_R^2}{Q^2} \\
f(r) &= [1 - (\frac{r_+}{r})^2][1 - (\frac{r_-}{r})^2] + \frac{|\lambda|}{10}\left[\frac{8|\lambda|}{5m^2}\right]^{\frac{2}{3}} r^2 \\
*e^{-\frac{4}{3}\phi} F &= Q\epsilon_3, \qquad *e^{\frac{2}{3}\phi} F_R = Q_R \epsilon_3, \\
r_\pm^2 &= M \pm (M^2 - \frac{e^2}{2})^{\frac{1}{2}},
\end{aligned} \tag{21}$$

where M is a parameter, analog of mass (notice that the space is not asymptotically flat) and $e = \frac{1}{2}\left[\frac{Q_R^2}{2} Q^2\right]^{\frac{1}{3}}$ is related to the product of the two charges Q_R and Q defined through eq.(20). It follows from eqs. (19) and (21) the string coupling at the black hole horizon is given by the ratio of the two charges Q_R and Q, and thus can be adjusted to be small through the judicious choice of the ratio of the two charges. Note that the spacetime in (21) is not asymptotically flat but has the curvature equal to 5Λ. We see from eq.(21) that near extremal blackhole solution can be obtained in the limit when λ goes to zero and the two horizons come very close to each other, *i.e.*, $r_+ \sim r_-$. Moreover, for $\lambda = 0, m = 0$ above solution in (21) reduces to the Strominger-Vafa's five-dimensional extremal black hole solution [27] as expected.

We find a black hole solution in four dimensions for the case when only the 2-form RR-field strength is nonzero and as is well known for $D = 4$, the gauge field couples to gravity conformally. The black hole solution of the type (17) in four dimensions exists when $F = 0,\ F_R \neq 0,\ \lambda < 0$. The solution is R-N-AdS with

$$e^{2\phi_h} = \frac{2|\lambda|}{m^2},$$

$$
\begin{aligned}
f(r) &= (1 - \frac{2M}{r} + \frac{Q_R^2}{r^2}) + \left[\frac{\lambda^2}{3m^2}\right] r^2, \\
*F_R &= Q_R \epsilon_2,
\end{aligned} \tag{22}
$$

where $Q_R = \frac{1}{4\pi}\int_{S^2} *F_R$.

Next we turn our attentions to obtain 4-brane solutions in six-dimensional model with cosmological constant term in the RR sector, i.e. we set $\lambda = 0$ in this case. We choose the background field configurations in (12) so that except dilaton and moduli matrix M are nonvanishing and the resulting action takes the following form,

$$
\begin{aligned}
S_m = \int d^6x\sqrt{-g}\ \Big[& e^{-2\phi}\left(R + 4\partial_\mu\phi\partial^\mu\phi + \frac{1}{8}Tr\partial_\mu M^{-1}\partial^\mu M\right) \\
& -\sqrt{G}\left(\frac{1}{2\cdot 2!}(m\ b_{\alpha\beta})^2 + \frac{1}{2\cdot 3!}(mb_{[\alpha\beta}b_{\gamma\delta]})^2 + \frac{1}{2}m^2\right)\Big],
\end{aligned} \tag{23}
$$

Note that with the introduction of mass term, m, in the ten dimensional effective action (1), the reduced action, with the specific choice of the backgrounds, gets a piece which amounts to introducing a potential term involving the moduli $G_{\alpha\beta}$ and $b_{\alpha\beta}$, $\alpha,\beta = 6,7,8,9$. We seek for a four-brane solution around $b_{\alpha\beta} = 0$ and, in the Einstein frame, ($g^E_{\mu\nu} = e^{-\phi}g_{\mu\nu}$) the 4-brane solution is

$$
\begin{aligned}
& ds_E^2 = U^{\frac{1}{4}}[-dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2] \ + \ U^{\frac{5}{4}}dy^2, \\
& e^{-\frac{2}{3}\phi} = U^{\frac{1}{2}}, \qquad U = \pm m|y - y_0|, \\
& G_{\alpha\beta} = \delta_{\alpha\beta}\ e^{\frac{2}{3}\phi}, \quad b_{\alpha\beta} = 0,
\end{aligned} \tag{24}
$$

these background configurations satisfy all the equations of motion derived from (23). This is a domain wall solution with a kink singularity (delta-function) at $y = y_0$. The solution is not asymptotically flat, however, for the choice, $U = |y - y_0|$ at large distances, e^ϕ vanishes. This solution can be oxidised to obtain 8-brane solution in ten dimensions.

To summarize our results: We considered ten dimensional effective action of type IIA theory in the presence of cosmological constant term which arises as the dual of the ten dimensional field strength coming from the RR sector. The action is dimensionally reduced on a d-dimensional torus with the assumption that the fields do not depend on internal coordinates. The gauge invariance properties of the reduced action is investigated and the transformation properties of the fields in the NS-NS and RR sectors are derived in the presence of the cosmological constant term. One of the interesting result is noticed in the six dimensional theory. It is found, that in the case of the massive theory, in the presence of this cosmological constant term, the $SO(4,4)$ invariance is lost; whereas the massless theory respects this symmetry. Thus, the cosmological constant coming from the RR sector, in this case, breaks the T-duality symmetry: $SO(4,4)$. Moreover, it is quite evident that, for the six dimensional massive theory, the equations of motion do not respect the $SO(5,5)$ symmetry unlike the masless case [26]. We recall that when one considers a four dimensional heterotic string theory and introduces a cosmological constant (in this case assumed to come from NS-NS sector as central charge deficeit), the equations of motion do not respect the S-duality invariance, as was discussed in ref.16. We presented, in this note, classical solutions of the effective action. In five dimensions we find black hole solutions in the presence of cosmological constants. It is possible to get near extremal solutions for the choice of small values of cosmological constant parameter, λ. In this context, we would like to point out that our black holes are anti-de Sitter type and these solutions do not correspond to asymptotically flat case. Therefore, one has to define the Hawking temperature with some care. There have been attempts to understand thermodynamic properties of black holes with (negative) cosmological constant term [29]. Brown, Creighton and Mann [30] identify the thermodynamic internal energy of such a black hole and equate the entropy to $\frac{1}{4}$ of the area of the black hole event horizon. The temperature on the boundary can be defined through thermodynamic relation between these two quantities, such that the black hole temperature, T_H, is $\frac{\kappa_H}{2\pi}$ times the redshift factor [31] for temperature in stationary gravitational field. The desired relation is

$$2\pi T(R) = \frac{\kappa_H}{\mathcal{N}(R)} \tag{25}$$

where κ_H is the surface gravity at the horizon of the black hole and $\mathcal{N}(R) = \sqrt{-g_{tt}}$, is the lapse function. The temperature, accordingly, depends on the location of the boundary. We have mentioned earlier that a massive type IIB effective action can be obtained in nine dimensions from the ten dimensional type IIB theory through generalised dimensional reduction due to Scherk and Schwarz. One can adopt the toroidal compactification for that nine dimensional theory to obtain reduced effective action in a way similar to the one presented recently [32] and explore various symmetries in the massive theory.

We conclude this note with some speculations about the cosmological constant problem and how the string symmetries might resolve it. We recall that for the four dimensional string effective action, the equations of motion are not invariant under S-duality when the cosmological constant is nonzero. In the present case, we find that starting from the ten dimensional type IIA theory, with the cosmological constant, when we consider the six dimensional theory after dimensional reduction, the $SO(4,4)$ symmetry is broken. If we turn the argument around, the $SO(4,4)$, a T-duality symmetry, if required to be a good symmetry, will force us to set the cosmological constant to zero. Of course, we are talking of toroidal compactification of type IIA to six dimension here and in the case of four dimensional theory it was the heterotic string effective action [16]. Nevertheless, it is quite amusing that in the two different cases, the constant is required to vanish (the symmetry requirements are different too). Therefore, it is quite tempting to conjecture that the web of string dualities will impose strong constraints on the four dimensional theory to tell us why the cosmological constant is vanishingly small.

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