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SYMMETRIES OF FOUR-DIMENSIONAL STRING EFFECTIVE ACTION WITH COSMOLOGICAL CONSTANT

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Abstract

Classical solutions for a four-dimensional Minkowskian string effective action and an Euclidean one with cosmological constant term are derived. The former corresponds to electrovac solutions whereas the later solutions are identified as gravitational instanton solutions for Fubini-Study metric. The symmetries of the effective actions are identified and new classical solutions are generated by implementing appropriate noncompact transformations. The S-duality transformations on the equations of motion are discussed and it is found that they are S-duality noninvariant due to the presence of cosmological constant term.

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The target space symmetries of string theory such as T-duality, $O(d, d)$ transformations and S-duality have attracted considerable attention in the recent past. Let us recall some salient aspects of T-duality: if we consider a compactified string on a circle of radius R , the spectrum of the string remains invariant under the transformation $R \rightarrow \frac{1}{R}$. In a more general scenario, when we envisage evolution of a string in the background of its massless excitations and compactify d of its coordinates on a torus, a richer symmetry structure emerges if all these backgrounds are independent of those compactified coordinates. The reduced string effective action has an invariance under global symmetry transformations, $O(d, d)$. The T-duality belongs to discrete subgroup of $O(d, d)$ transformations. The target space duality (T-duality) is the generalisation of $R \rightarrow \frac{1}{R}$ transformation. The metric that appears in the reduced string effective action is the σ -model metric. Under T-duality [1] and $O(d, d)$ transformations [2], the space-time metric and the (shifted) dilaton remain invariant. It is well known that under T-duality and $O(d, d)$ transformations, one can generate new background configurations starting from a known solution of the string effective action. On the other hand, study of S-duality [3,4] in string theory holds the prospect of revealing the nonperturbative features of string theory. It is possible to relate strong and weak coupling regimes of string theory under this transformation. In four dimensions, the axion and dilaton can be combined to define a complex scalar field which transforms under $SL(2, \mathbf{Z})$ group while leaving equations of motion unaltered. However, it is more convenient to go over to the Einstein metric while discussing S-duality transformations and their consequences.

The purpose of this note is to derive classical solutions of four-dimensional string effective actions. Then we generate new solutions through $O(d, d)$ transformations. First we consider a Minkowskian action where solution was derived for $N = 2$ supergravity action which was designated as “electrovac” solution [5]. However, the dilaton field appears in the massless multiplet of the string theory and solutions of the string effective action are required to satisfy equations of motion associated with each massless mode of the string. Therefore, the presence of the dilaton field in the action imposes additional constraints on the background fields. We shall present solutions in the presence of constant dilaton background. However,

this effective action has an invariance under noncompact symmetries. Thus we shall obtain new background fields from the known classical solution by suitably implementing noncompact symmetry transformations. We recall that T-duality and $O(d, d)$ transformations have been applied to obtain new solutions in the context of string cosmological solutions [6,7], black holes [8–14] and topology changing processes [15,16].

The second problem we consider is the Euclidean effective action in four-dimension in the presence of Abelian gauge fields. This action admits classical solutions such that Weyl curvature tensor and the gauge field strength satisfy self-duality condition and the solution has the interpretation of gravitational instanton [17]. Furthermore, we show that new metric and gauge field configurations can be generated by a global noncompact transformation. However, the new backgrounds are such that they do not satisfy self-duality condition.

We would like to mention that the two effective actions that are considered in obtaining the classical solutions have cosmological constant terms. We shall see later that the equations of motion do not remain invariant under S-duality transformations due to the presence of cosmological constant term (we go over to Einstein frame when we examine this issue).

Let us recall the most salient aspects of dimensional reduction of the string effective action. We write the low energy heterotic string effective action in D dimensions with Abelian gauge fields following ref. [2],

$$S = \int d^D X^M \sqrt{\det \hat{G}_{MN}} e^{-\hat{\Phi}} \left(R_{\hat{G}} + \hat{G}^{MN} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{4} \hat{F}^{I MN} \hat{F}_{MN}^I - \frac{1}{12} \hat{H}_{MNP} \hat{H}^{MNP} - 2\Lambda \right) \quad (1)$$

where

$$\begin{aligned} \hat{H}_{MNP} &= \partial_M \hat{B}_{NP} + \text{cyclic permutations}, \\ \hat{F}_{MN}^I &= \partial_M \hat{A}_N^I - \partial_N \hat{A}_M^I \\ M, N &= 1, \dots, D; \quad I = 1, \dots, n. \end{aligned} \quad (2)$$

Here \hat{G}_{MN} , $\hat{\Phi}$, \hat{A}_M^I and \hat{B}_{MN} denote the graviton, dilaton, n-component Abelian vector field and antisymmetric tensor fields, respectively. $R_{\hat{G}}$ denotes D dimensional scalar curvature and

Λ is the deficit in central charge which plays the role of cosmological constant. In dimensional reduction scheme, for backgrounds independent of d coordinates (say, X^α , $1 \leq \alpha \leq d$), with toroidal compactification on T^d , the action (1) can be rewritten as

$$S = \int d^{D-d} X^\mu \sqrt{\det g_{\mu\nu}} e^{-\Phi} \left(R_g + g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{8} \text{Tr} \partial_\mu M^{-1} \partial^\mu M - \frac{1}{4} \mathcal{F}_{\mu\nu}^i (M^{-1})_{ij} \mathcal{F}^{j\mu\nu} - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - 2\Lambda \right) \quad (3)$$

where

$$\hat{G}_{MN} = \begin{pmatrix} g_{\mu\nu} + A_{\mu\alpha}^{(1)} A_{\nu}^{(1)\alpha} & A_{\mu\beta}^{(1)} \\ A_{\nu\alpha}^{(1)} & G_{\alpha\beta} \end{pmatrix}$$

$$\Phi = \hat{\Phi} - \frac{1}{2} \ln \det G_{\alpha\beta}$$

$$M = \mathcal{L} M^{-1} \mathcal{L} = \begin{pmatrix} G^{-1} & -G^{-1}C & -G^{-1}A^T \\ -C^T G^{-1} & G + C^T G^{-1}C + A^T A & C^T G^{-1}A^T + A^T \\ -AG^{-1} & AG^{-1}C + A & 1 + AG^{-1}A^T \end{pmatrix}$$

$$C_{\alpha\beta} = \frac{1}{2} A_\alpha^I A_\beta^I + B_{\alpha\beta}, \quad (4)$$

with the space-time dependent background fields ($G_{\alpha\beta}$, $A_\alpha^I \equiv \hat{A}_\alpha^I$, $B_{\alpha\beta} \equiv \hat{B}_{\alpha\beta}$) defining a generic point in moduli-space in the toroidal compactification of the heterotic string theory. Note that moduli M satisfies the condition $M\mathcal{L}M\mathcal{L} = 1$, where \mathcal{L} is the $O(d, d+n)$ metric,

$$\mathcal{L} = \begin{pmatrix} 0 & I_d & 0 \\ I_d & 0 & 0 \\ 0 & 0 & I_n \end{pmatrix} \quad \Omega^T \mathcal{L} \Omega = \mathcal{L}. \quad (5)$$

Here I_d is d -dimensional identity matrix and Ω is an element of the group $O(d, d+n)$. The definitions of other two fields in (3) are

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{2} \mathcal{A}^i \mathcal{L}_{ij} \mathcal{F}_{\nu\lambda}^j + \text{cyclic permutations}$$

$$\mathcal{F}_{\mu\nu}^i = \partial_\mu \mathcal{A}_\nu^i - \partial_\nu \mathcal{A}_\mu^i \quad (6)$$

where i, j are $O(d, d)$ matrix indices. $\mathcal{A}_\mu^i = (A_\mu^{(1)\alpha}, A_{\mu\alpha}^{(2)}, A_\mu^{(3)I})$ is a $(2d + n)$ component vector field with the following definition of its components,

$$\begin{aligned} A_{\mu\alpha}^{(1)} &= \hat{G}_{\mu\alpha} \\ A_{\mu\alpha}^{(2)} &= \hat{B}_{\mu\alpha} + \hat{B}_{\alpha\beta} A_\mu^{(1)\beta} + \frac{1}{2} \hat{A}_\alpha^I A_\mu^{(3)I} \\ A_\mu^{(3)I} &= \hat{A}_\mu^I - \hat{A}_\alpha^I A_\mu^{(1)\alpha}. \end{aligned} \quad (7)$$

Now it is straightforward to check that the action (3) is manifestly invariant under global $O(d, d + n)$ transformations

$$\begin{aligned} M &\rightarrow \Omega M \Omega^T \\ \Phi &\rightarrow \Phi, \quad g_{\mu\nu} \rightarrow g_{\mu\nu}, \quad B_{\mu\nu} \rightarrow B_{\mu\nu}, \\ \mathcal{A}_\mu^i &\rightarrow \Omega_j^i \mathcal{A}_\mu^j \end{aligned} \quad (8)$$

where Ω is an $O(d, d + n)$ matrix such that $\Omega \mathcal{L} \Omega^T = \mathcal{L}$. Note that \mathcal{A}_μ^i transforms as a vector multiplet under $O(d, d + n)$ transformations. We mention in passing that $M \rightarrow M^{-1}$ is the analog of $R \rightarrow \frac{1}{R}$ duality for the general case. In general, an $O(d) \times O(d + n)$ transformation, which is a subgroup of $O(d, d + n)$ generates new solutions which cannot be obtained from the old ones through general coordinate transformations or gauge deformations. There are specific examples where an $O(d) \times O(d)$ transformations can "boost away" curvature singularities and in fact can turn flat string backgrounds into nonflat ones and vice versa. It has been shown explicitly in reference [18] that Nappi-Witten backgrounds in four-dimension can be obtained from $O(2, 2)$ boosting of the direct product of a pair of two-dimensional models.

We consider the following four-dimensional string effective action

$$S = \int d^4 X^M \sqrt{\det \hat{G}_{MN}} e^{-\hat{\Phi}} \left(R_{\hat{G}} + \hat{G}^{MN} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{4} \hat{F}^{MN} \hat{F}_{MN} + 2g_B^2 \right) \quad (9)$$

The equations of motion are satisfied for following background field configurations,

$$\hat{ds}^2 = dx^2 + dy^2 + d\xi^2 - \cosh^2[\sqrt{2}g_B \xi] dt^2$$

$$\begin{aligned}\hat{A}_M &= (0 \quad 0 \quad 0 \quad \sqrt{2} \sinh[\sqrt{2}g_B\xi]) \\ \hat{\Phi} &= \text{constant} .\end{aligned}\tag{10}$$

This effective action is similar to the $N = 2$ supergravity action considered by Freedman and Gibbons [5] with appropriate choice of backgrounds. The solution given by (10) are analog of the “electrovac” solutions of ref. [5] . Notice, however, that massless dilaton appears in the action (9). Now the dilaton equation of motion is required to be satisfied in addition to Einstein and Maxwell field equation. It is worthwhile to point out that this equation, even for constant dilaton background, imposes constraints on the background field equations. The topology of the spacetime is $R^2 \times AdS_2$ with the curvature scalar $R = -4g_B^2$. The Maxwell field strength is covariantly constant, *i.e.* $\nabla_M \hat{F}^{MN} = 0$, and is of purely electric-type

$$\hat{F}_{\xi 0} = 2g_B \cosh[\sqrt{2}g_B\xi], \quad \hat{F}_{ij} = 0, \quad i, j = x, y, \xi ,$$

and the field strength satisfies the constraint $\hat{F}_{MN}\hat{F}^{MN} = -8g_B^2$.

Note that backgrounds in (10) are independent of the coordinates x, y and t . Therefore action (9) whose contents are graviton, dilaton and an Abelian gauge field is invariant under $O(3, 4)$ symmetry transformations. Thus one can perform $O(3, 4)$ transformations on this background to obtain new classical solutions which can generate a nontrivial antisymmetric tensor field \bar{B}_{MN} . For sake of definiteness, we shall choose a specific transformations restricted to $t - y$ plane only [19]. This amounts to restricting ourselves to $O(2, 3)$ which is a subgroup of $O(3, 4)$. Let us first rewrite action (9) isolating y - and t -isometries ,

$$\begin{aligned}S &= \int dydt \int dx d\xi \sqrt{\det g_{\mu\nu}} e^{-\Phi} \\ &\quad \left(R_g + \partial_\mu \Phi \partial^\mu \Phi + \frac{1}{8} Tr(\partial_\mu M^{-1} \partial^\mu M) - \frac{1}{4} \mathcal{F}_{\mu\nu}^{(i)} (M^{-1})_{ij} \mathcal{F}^{(j)\mu\nu} + 2g_B^2 \right)\end{aligned}\tag{11}$$

where,

$$\begin{aligned}g_{\mu\nu} &= \delta_{\mu\nu}; \quad \mu, \nu = x, \xi \\ \Phi &= \hat{\Phi} - \frac{1}{2} \ln \det G_{\alpha\beta}\end{aligned}$$

$$\begin{aligned}
G_{\alpha\beta} &= \begin{pmatrix} -\cosh^2[\sqrt{2}g_B\xi] & 0 \\ 0 & 1 \end{pmatrix}; \quad \alpha, \beta = t, y \\
A_\alpha &= (\sqrt{2}\sinh[\sqrt{2}g_B\xi] , 0) \\
B_{\alpha\beta} &= 0 \\
\mathcal{F}_{\mu\nu}^{(i)} &= 0.
\end{aligned} \tag{12}$$

The moduli matrix M can be constructed using the definition given in eq. (4). To obtain new background we choose Ω to be of the following form

$$\Omega = \Omega_1 \times \Omega_2 \tag{13}$$

where

$$\begin{aligned}
\Omega_1 &= \frac{1}{2} \begin{pmatrix} c+1 & -s & 1-c & -s & 0 \\ -s & c+1 & s & c-1 & 0 \\ 1-c & s & 1+c & s & 0 \\ -s & c-1 & s & c+1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} \\
\Omega_2 &= \frac{1}{2} \begin{pmatrix} \hat{c}+1 & 0 & 1-\hat{c} & 0 & -\sqrt{2}\hat{s} \\ 0 & 1 & 0 & 0 & 0 \\ 1-\hat{c} & 0 & \hat{c}+1 & 0 & \sqrt{2}\hat{s} \\ 0 & 0 & 0 & 1 & 0 \\ -\sqrt{2}\hat{s} & 0 & \sqrt{2}\hat{s} & 0 & 2\hat{c} \end{pmatrix}
\end{aligned} \tag{14}$$

and $c = \cosh \theta$, $s = \sinh \theta$, $\hat{c} = \cosh \gamma$, and $\hat{s} = \sinh \gamma$ with $0 < \theta < \infty$, $0 < \gamma < \infty$ being the boost parameters. Notice that our rotations involve the $t-y$ plane and acts on $G_{\alpha\beta}$ and the gauge field A_α . These two noncompact rotations, generically, called “boosts”, appear because the t -coordinate is involved. The “boosts” Ω_1 and Ω_2 form the elements of the group $O(1, 1) \times O(1, 2)$ which is a subgroup of $O(2, 3)$. The backgrounds thus generated satisfy

the equations of motion and are not connected to the original background configuration by general coordinate transformations and/or gauge transformations.

We note that the metric and the gauge potential have the following form after $O(1, 1) \times O(1, 2)$ transformations summarised in (8),

$$\begin{aligned} \bar{d}\bar{s}^2 &= dx^2 + d\xi^2 - \frac{1 + c^2 a(\xi)^2}{(1 + c \hat{s} a(\xi))^2} dt^2 + \frac{2sa(\xi)(ca(\xi) - \hat{s})}{(1 + c \hat{s} a(\xi))^2} dy dt \\ &\quad + \frac{1 + 2c \hat{s} a(\xi) + (\hat{c}^2 - c^2) a(\xi)^2}{(1 + c \hat{s} a(\xi))^2} dy^2, \\ \bar{A}_M &= (0, 0, \frac{\sqrt{2}c \hat{c} a(\xi)}{1 + c \hat{s} a(\xi)}, -\frac{\sqrt{2}s \hat{c} a(\xi)}{1 + c \hat{s} a(\xi)}). \end{aligned} \quad (15)$$

where $a(\xi) = \sinh[\sqrt{2}g_B \xi]$. Furthermore, the antisymmetric tensor field strength is nonzero after the $O(1, 1) \times O(1, 2)$ transformations (although, the original background had vanishing \hat{H}_{MNP}),

$$\bar{B}_{ty} = \frac{s \hat{s} a(\xi)}{1 + c \hat{s} a(\xi)}. \quad (16)$$

Moreover, we find that the transformed dilaton depends on coordinate ξ nontrivially and is given by

$$\bar{\Phi} = \hat{\Phi} - \ln(1 + c \hat{s} a(\xi)). \quad (17)$$

The new Maxwell field strength, \bar{F}_{MN} , has both electric and magnetic components and $\bar{F}_{MN} \bar{F}^{MN} = -8g_B^2 \frac{\hat{c}^2}{(1 + c \hat{s} a(\xi))^2}$.

The curvature scalar is

$$\bar{R} = -g_B^2 \frac{\mathcal{R}(\xi)}{(1 + c \hat{s} a(\xi))^6} \quad (18)$$

where the numerator

$$\begin{aligned} \mathcal{R}(\xi) &= 4 + \hat{s}^2(1 + 7c^2) + (12 + 4\hat{s}^2(1 + 7c^2)) c \hat{s} a(\xi) \\ &\quad + (8 + 6\hat{s}^2(1 + 7c^2)) c^2 \hat{s}^2 a(\xi)^2 - (8 - 4\hat{s}^2(1 + 7c^2)) c^3 \hat{s}^3 a(\xi)^3 \\ &\quad - (12 - \hat{s}^2(1 + 7c^2)) c^4 \hat{s}^4 a(\xi)^4 - 4c^5 \hat{s}^5 a(\xi)^5. \end{aligned}$$

The new background is singular when $(1 + c \hat{s} a(\xi)) = 0$ while the original background had a constant curvature, $-4g_B^2$.

Now we turn to discuss S-duality transformations. The dilaton and axion χ (dual to the antisymmetric tensor field in four dimensions) can be combined to define a complex scalar field, $\Psi = \chi + i \eta$, where $\eta = e^{-\Phi}$. The S-duality transformations [4] correspond to

$$\begin{aligned} \Psi &\rightarrow \frac{a \Psi + b}{c \Psi + d}, \quad a, b, \dots \in \mathbf{Z} \\ \hat{F}_{MN} &\rightarrow c \eta \tilde{\hat{F}}_{MN} + (c \chi + d) \hat{F}_{MN} \\ a d - b c &= 1 \end{aligned} \tag{19}$$

where

$$\tilde{\hat{F}}_{MN} = \frac{1}{2} \sqrt{\det \hat{G}_{MN}} \epsilon_{MNR S} \hat{F}^{RS}. \tag{20}$$

While we consider S-duality transformations in four dimensions, it is more convenient to work in the Einstein frame rather than working in the string (σ -model) metric. Therefore it is to be understood that whenever we discuss S-duality in this paper we are in the Einstein frame. It is worthwhile to consider the above $SL(2, \mathbf{Z})$ transformations on the equations of motion. We find that the equations of motion are not invariant under these transformations due to the presence of the cosmological constant term. We may remark in passing that if cosmological constant were to have vanishing value the string equations of motion would have been invariant under S-duality.

Next we consider the Euclidean string effective action in four-dimension

$$S_E = \int d^4 X^M \sqrt{\det \hat{G}_{MN}^E} e^{-\hat{\Phi}} \left(R_{\hat{G}} + \hat{G}^{E M N} \partial_M \hat{\Phi} \partial_N \hat{\Phi} - \frac{1}{4} \hat{F}^{M N} \hat{F}_{M N} - 2\Lambda \right) \tag{21}$$

where $M = 1, \dots, 4$ and \hat{G}_{MN}^E is an Euclidean metric. As mentioned earlier, the presence of dilaton imposes constraints on background. In this case the Weyl tensor is anti-self dual. This is as close as one can get to self-dual Ricci tensor. Strictly speaking one demands self-duality or anti-self-duality while looking for instanton like solutions. It is necessary to introduce a self-dual Abelian field strength in (21) in order to satisfy Einstein as well as matter field equations. The background fields are given by

$$\begin{aligned}
\hat{ds}^2 &= \frac{dr^2}{f(r)^2} + \frac{r^2}{4f(r)^2}(d\psi + \cos\theta d\phi)^2 + \frac{r^2}{4f(r)}(d\theta^2 + \sin^2\theta d\phi^2) \\
\hat{A}_M &= (0, 0, \sqrt{\frac{\Lambda}{2}} \frac{r^2 \cos\theta}{2f(r)}, \sqrt{\frac{\Lambda}{2}} \frac{r^2}{2f(r)}) \\
\hat{\Phi} &= \text{constant}
\end{aligned} \tag{22}$$

where $f(r) = 1 + \frac{\Lambda}{6}r^2$. This is the Fubini-Study metric for CP^2 manifold and the solution has the interpretation of gravitational instantanton [17]. The Weyl tensor satisfies the anti-self-duality condition

$$C_{MNPQ} = -{}^*C_{MNPQ} = -\frac{\sqrt{\det G^E}}{2} \epsilon_{MNR S} C_{PQ}^{RS}. \tag{23}$$

Note, however, Gibbons-Pope instantanton solution [17] was obtained for pure gravity in presence of cosmological constant, Λ , whereas action (21) contains a gauge field and the dilaton.

The solution (22) is a stringy background with vanishing antisymmetric tensor field strength. All background fields are independent of two periodic coordinates, $0 \leq \phi \leq 2\pi$ and $0 \leq \psi \leq 4\pi$. It is obvious that the action (21) is invariant under $O(2, 3)$ transformations since the backgrounds are independent of ϕ and ψ . We can exploit this symmetry of the action to obtain new classical backgrounds with nonvanishing antisymmetric tensor field, by adopting a procedure similar to those discussed above. The first step is to construct the corresponding moduli matrix M for the problem at hand. The various moduli fields which are used to construct M are

$$\begin{aligned}
G_{\alpha\beta} &= \begin{pmatrix} \frac{r^2}{4f(r)^2}(\cos^2\theta + f(r)\sin^2\theta) & \frac{r^2}{4f(r)^2}\cos\theta \\ \frac{r^2}{4f(r)^2}\cos\theta & \frac{r^2}{4f(r)^2} \end{pmatrix} \\
A_\alpha &= \left(\sqrt{\frac{\Lambda}{2}} \frac{r^2 \cos\theta}{2f(r)}, \sqrt{\frac{\Lambda}{2}} \frac{r^2}{2f(r)} \right) \\
B_{\alpha\beta} &= 0,
\end{aligned} \tag{24}$$

where indices α, β run over coordinates (ϕ, ψ) in the given order. The next step is to choose an $O(2, 3)$ matrix Ω . To generate inequivalent backgrounds from the one given in (22) we choose Ω to be a special element of $O(2) \times O(3)$, which is a subgroup of $O(2, 3)$, given by

$$\Omega = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \quad (25)$$

Now one can perform the transformations in (8) using eqs.(25) and (24) above. We get the new field configuration

$$\begin{aligned} \bar{G}_{\alpha\beta} &= \begin{pmatrix} \frac{r^2 \sin^2 \theta}{4f(r)} & 0 \\ 0 & \frac{4f(r)^2}{r^2(1+\frac{\Lambda}{4}r^2)^2} \end{pmatrix} \\ \bar{A}_\alpha &= \left(0, -\sqrt{2\Lambda} \frac{f(r)}{(1+\frac{\Lambda}{4}r^2)} \right) \\ \bar{B}_{\alpha\beta} &= \begin{pmatrix} 0 & \cos \theta \\ -\cos \theta & 0 \end{pmatrix} \\ \bar{\Phi}(r) &= \hat{\Phi} - \ln \left(\frac{r^2(1+\frac{\Lambda}{4}r^2)}{4(1+\frac{\Lambda}{6}r^2)^2} \right). \end{aligned} \quad (26)$$

Interestingly, we have got a new field configuration in which the metric is diagonal. Also, dilaton and antisymmetric tensor fields are nontrivial. It should be noted that both Weyl tensor as well as vector field strength have lost their respective self-duality properties after this transformation. Asymptotically (i.e., as $r \rightarrow \infty$) the new four-metric has the form

$$\bar{d}s^2 = \omega(r)^2(dr^2 + r^2d\Theta^2) + \frac{6}{4\Lambda}(d\theta^2 + \sin^2 \theta d\phi^2) \quad (27)$$

where $\omega(r) = \frac{6}{\Lambda r^2}$ and $\Theta = \frac{2\Lambda}{9}\psi$. Let us for the sake of simplicity take $\hat{\Phi} = 0$ and discuss the behaviour of $\bar{\Phi}$ as a function of r . Asymptotic value of the dilaton, $\bar{\Phi}_{asym} = -\ln \frac{9}{4\Lambda}$, depends on the cosmological constant Λ . Notice that for $r \sim 0$, $\bar{\Phi}_{r \sim 0} \sim -\ln \frac{r^2}{4}$. We recall that $e^{\bar{\Phi}}$ is the string coupling constant. It is interesting to note that string coupling has $\frac{1}{r^2}$ behaviour near the origin while attains a constant value of $\frac{4\Lambda}{9}$ at $r \rightarrow \infty$. We observe that for large r , $e^{\bar{\Phi}}$ is a constant, if $\Lambda \rightarrow 0$ the string coupling tends to vanishing value. Thus for this simple model under consideration, there is a connection between string coupling constant and the cosmological constant.

To summarize, in this work we have studied the solutions of four-dimensional heterotic string effective actions in presence of cosmological constant, Λ . The solutions of equations of motion corresponding to the Minkowskian string effective action are analogous to “electrovac” solution of Freedman and Gibbons [5]. This action has an invariance under $O(2, 3)$ transformations. Using this property, we could generate new backgrounds with nontrivial B_{MN} . However, the curvature scalar corresponding to the new metric is singular while initial geometry had constant curvature.

The Euclidean string action in four dimensions with CP^2 geometry and an Abelian self-dual gauge field strength admits “gravitational instanton” solution. This action also possesses an $O(2, 3)$ invariance. We have generated new backgrounds such that the metric is diagonal and dilaton as well as antisymmetric fields acquire coordinate dependence. Furthermore, Weyl tensor and F_{MN} , corresponding to new background, are no longer (anti) self-dual.

We note that, due to the presence of the cosmological constant term in the action the equations of motion are not invariant under the $SL(2, \mathbf{Z})$ transformations. This is an interesting feature that S-duality is not respected in presence of Λ for both the problems we have considered.

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