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S-duality and Canonical Transformations in String Theory

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Abstract

The symmetries of the tree level string effective action are discussed. An appropriate effective action is constructed starting from the manifestly SL(2,R) invarint form of string effective action introduced by Schwarz and Sen. The conserved charges are derived and generators of infinitesimal transformations are obtained in the Hamiltonian formalism. Some interesting consequences of the canonical transformations are explored.

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It is now recognized that dualities play a crucial role in our understandings of string theory and field theory. The target space duality [1,2], termed as T-duality, is known to be a symmetry of the string effective action and holds good order by order in string perturbation theory. The S-duality allows us to relate the strong and weak coupling phases [3]. The Olive-Montonen conjecture [4] implies that the weak coupling and strong coupling regimes of gauge theories are connected through the duality transformation and these results were generalized to supersymmetric cases subsequently [5]. Recently, considerable attention has been focussed to understand various salient features of SUSY Yang-Mills theories in this context [6].

It is well known that dilaton has a special status in string theory. On the one hand it appears as a massless excitation of bosonic as well as superstring like other massless excitations such as graviton, antisymmetric tensor and gauge bosons and on the other hand the vacuum expectation value of this field is the loop expansion parameter of string theory and e^{ϕ} is identified as the string coupling constant. We recall that, in four space-time dimensions, the dual of the antisymmetric tensor field is a pseudoscalar field, λ_1, identified with axion. Thus we can define a complex field $\lambda = \lambda_1 + i\,\lambda_2,\ \ \lambda_2 = e^{-\phi}$, and under an SL(2,R) transformation

$$\lambda \rightarrow \lambda' = \frac{a\,\lambda + b}{c\,\lambda + d}, \qquad a\,d - b\,c = 1.$$

In fact SL(2,R) breaks to discrete S-duality subgroup SL(2,Z) and this is expected to be an exact symmetry of string theory. Indeed, this symmetry can only be tested nonperturbatively. It is well known that the equations of motion of the string effective action, with some constraints [1], are invariant under S-duality transformations whereas the action is invariant under T-duality and/or O(d,d) transformations.

There have been efforts to study the duality properties of σ-model string world sheet

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[1] The equations of motion do not remain invariant under S-duality in presence of cosmological constant term in the string effective action in certain cases as has been discussed in [7]

action through canonical transformations [8–11]. It is worth mentioning that the local symmetry properties of string theories were studied by implementing suitable canonical transformations in the frame work of BRST Hamiltonian path integral formalism [12,13]. The purpose of this article is to investigate the symmetries of the four-dimensional heterotic string effective action and construct generators of the S-duality transformations and explore some interesting properties of the action.

In what follows, we consider a four-dimensional string effective action derived by dimensionally reducing 10-dimensional heterotic string effective action on six-dimensional torus, T^6 [14]:

$$S = \int d^4x\sqrt{-G}\; e^{-\phi}(R_G + G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{8}Tr\partial_\mu M^{-1}\partial^\mu M \\ -\frac{1}{4}\mathcal{F}^i_{\mu\nu}\,(M^{-1})_{i\,j}\,\mathcal{F}^{j\,\mu\nu} - \frac{1}{12}H_{\mu\nu\lambda}\,H^{\mu\nu\lambda}). \tag{1}$$

M is defined in terms of scalar fields arising from dimensional reduction of metric, antisymmetric tensor and 16 gauge fields (belonging to the Cartan subalgebra of the gauge field sector of heterotic string theory). Note that M parametrises the coset $\frac{O(6,22)}{O(6)\times)(22)}$ and ϕ is the shifted dilaton. The field strengths are

$$H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} - \frac{1}{2}\mathcal{A}^i_\mu\mathcal{L}_{i\,j}\mathcal{F}^j_{\nu\lambda} + cyclic\ perm.$$
$$\mathcal{F}^i_{\mu\nu} = \partial_\mu\mathcal{A}^i_\nu - \partial_\nu\mathcal{A}^i_\mu\ , \quad i = 1, ..., 28,$$

$\mathcal{A}^i_\mu$ are 28 gauge fields which transform as vectors under O(6,22). The action (1) is manifestly invariant under O(6,22) global noncompact transformations given by

$$M\rightarrow \Omega\, M\, \Omega^T \ ,$$
$$\phi\rightarrow \phi,\ \ g_{\mu\nu} \rightarrow g_{\mu\nu},\ \ B_{\mu\nu} \rightarrow B_{\mu\nu},\ \ \mathcal{A}^i_\mu \rightarrow \Omega^i_j\,\mathcal{A}^j_\mu,$$
$$\Omega\,\mathcal{L}\,\Omega^T = \mathcal{L},\ \ \mathcal{L} = \begin{pmatrix} 0 & I_6 & 0 \\ I_6 & 0 & 0 \\ 0 & 0 & I_{16} \end{pmatrix}, \tag{2}$$

$\mathcal{L}$ is the O(d,d) metric, where I_d is d-dimensional identity matrix. Note that the metric appearing in (1) is the string σ-model metric and scalar curvature, R_G, is computed with respect to this metric. Our goal is to study S-duality properties of this four dimensional theory; for this purpose it is more convenient to go over to the Einstein-frame metric

$$G_{\mu\nu} \to g_{\mu\nu} = e^{-\phi} G_{\mu\nu}.$$

We mention in passing that equations of motion obtained from the action (derived from (1) after above rescaling of the metric) are invariant under S-duality and that action itself does not respect S-duality invariance. However, one can follow Schwarz and Sen [15] and reexpress the action in manifestly SL(2,R) invariant form by introducing an appropriate set of gauge fields. (From now on, we shall define the action in the Einstein frame and $g_{\mu\nu}$ will stand for the Einstein metric.)

The action (1) can be rewritten in manifestly SL(2,R) invariant form as

$$S = S_1 + S_2 + S_3 + S_4, \tag{3}$$

where S_a, $a = 1, ..., 4$ terms have following contents,

$$\begin{aligned} S_1 &= \int d^4x \sqrt{-g}\, (R + \frac{1}{8} Tr(\partial_\mu M^{-1} \partial^\mu M)), \\ S_2 &= \frac{1}{4} \int d^4x \sqrt{-g}\, tr(\partial_\mu \mathcal{M}^{-1} \partial^\mu \mathcal{M}), \\ S_3 &= -\frac{1}{4} \int d^4x \sqrt{-g}\, F^{(m,\alpha)}_{\mu\nu} \hat{G}_{mn}\, \mathcal{M}^{-1}_{\alpha\beta} F^{(n,\beta)\,\mu\nu}, \\ S_4 &= -\frac{1}{4} \int d^4x \sqrt{-g}\, F^{(m,\alpha)}_{\mu\nu} \hat{B}_{mn}\, \eta_{\alpha\beta} \tilde{F}^{(n,\beta)\,\mu\nu}. \end{aligned} \tag{4}$$

Note that here and every where $g_{\mu\nu}$ denotes four-dimensional Einstein metric; space-time indeces $\mu, \nu = 0, ..., 3$, internal indices $m, n = 1, ..., 6$ and SL(2,R) indices (α, β) run over (1, 2). The moduli matrix M parametrizes the coset $\frac{O(6,6)}{O(6)\times O(6)}$, matrix $\mathcal{M}$ parametrizes the coset $\frac{SL(2,R)}{U(1)}$. $F^{(m,\alpha)}_{\mu\nu}$ represents a SL(2,R) covariant set of gauge fields which come from the compactification of the heterotic string on the torus and the auxiliary $U(1)$ fields. These set of gauge fields transform as a vector under SL(2,R) transformations. We note that the 16 abelian gauge fields of 10-dimensional heterotic string effective action have been set to zero in the above action (3) for the sake of convenience. Now, the action (3) does not have manifest O(d,d) invariance; however, equations of motion still exhibit O(d,d) invariance.

We refer the reader to [15] for details of the construction of the action (3). The gauge field strength and the matrix $\mathcal{M}$ are defined below,

$$F^{(m,\alpha)}_{\mu\nu} = \partial_\mu A^{(m,\alpha)}_\nu - \partial_\nu A^{(m,\alpha)}_\mu, \quad \tilde{F}^{\mu\nu\,(m,\alpha)} = \frac{1}{2\sqrt{-g}}\epsilon^{\mu\nu\lambda\sigma}\, F^{(m,\alpha)}_{\lambda\sigma} \tag{5}$$

$$\mathcal{M} = \frac{1}{\lambda_2}\begin{pmatrix} 1 & \lambda_1 \\ \lambda_1 & |\lambda|^2 \end{pmatrix}, \quad \lambda = \lambda_1 + i\lambda_2, \tag{6}$$

where $\mathcal{M}$ is a SL(2,R) matrix satisfying the constraints,

$$\mathcal{M}^T = \mathcal{M}, \;\; \mathcal{M}^T\eta\mathcal{M} = \eta, \;\; \eta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{7}$$

η being the SL(2,R) metric. As was shown in [15] the above action has explicit invariance under the following SL(2,R) transformatioms,

$$\mathcal{M} \rightarrow \omega^T\,\mathcal{M}\,\omega, \;\; A_\mu \rightarrow \omega^T\,A_\mu, \tag{8}$$

where $\omega \in SL(2,R)$ matrix and satisfies $\omega^T\eta\,\omega = \eta$. It was noted earliar that the invariance of the action is achieved by doubling the number of gauge fields.

In what folllows, we present the transformation properties of the fields under infinitesimal SL(2,R) group. These transformations are

$$\begin{aligned} &\omega = 1 + \epsilon, \\ &\delta\mathcal{M} = \epsilon^T\,\mathcal{M} + \mathcal{M}\,\epsilon, \qquad \delta A_\mu = \epsilon^T\,A_\mu, \end{aligned} \tag{9}$$

where infinitesimal 2×2 matrix ϵ satisfies the constraint $\epsilon^T = \eta\,\epsilon\,\eta$. All other fields remain invariant under these transformations. If Σ^i are the generators of SL(2,R) then we can write,

$$\epsilon = \alpha^i\,\Sigma^i, \quad i = 1, ..., 3, \tag{10}$$

α^i's being infinitesimal constant parameters. Σ^i have following 2×2 matrix representation; one can use the combination ($\sigma_3, \sigma_1, i\sigma_2$);

$$\Sigma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \Sigma^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \Sigma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{11}$$

which satisfy the algebra:

$$[\Sigma^i, \Sigma^j] = 2 f_{ij}^k \, \Sigma^k, \quad (i = 1, 2, 3)$$

and

$$Tr(\Sigma^i \Sigma^j) = 2h^{ij}$$

where $h_{ij} = diag(1, 1, -1)$ and $(\Sigma^1)^2 = 1, (\Sigma^2)^2 = 1, (\Sigma^3)^2 = -1$. The f_{ij}^k's are the structure constants satisfying antisymmetric property $f_{ij}^k = -f_{ji}^k$. The nonvanishing ones are

$$f_{12}^3 = 1, f_{23}^1 = -1, f_{31}^2 = -1.$$

These matrices also satisfy following relation,

$$\Sigma^{iT} \eta \Sigma^j = -h^{ij} \, \eta.$$

The action (3) being invariant under finite SL(2,R) transformations (8), also respects invariance under infinitesimal ones. Consequently, we are in a position to reveal the underlying conservation laws. Let us proceed to construct "generating functions" of the infinitesimal $SL(2, R)$ transformations.

We note that S_1, appearing in (3), remains unaffected by SL(2,R) transformations since it involves the metric $g_{\mu\nu}$. Therefore from now on, we shall not explicitly mention the action S_1 in our discussions; although the full action contains contributions of S_1. We now examine the variations of S_2, S_3 and S_4 under (9);

Notice that any matrix P satisfying the constraint $P^T \, \eta \, P = \eta$ can be written in terms of the generators Σ^i as follows,

$$P = p^0 \, I_2 + p^i \, \Sigma^i, \quad i = 1, ...3. \tag{12}$$

Note that there are only three independent parameters in (12) due to the constraint on P. While an $\frac{SL(2,R)}{U(1)}$ matrix $\bar{\mathcal{M}} = \eta \, \mathcal{M}$ can be expressed as

$$\bar{\mathcal{M}} = m^i \, \Sigma^i \tag{13}$$

where only two of the m^i's are independent due to the constraint $m_3^2 - m_2^2 - m_1^2 = 1$ which follows from the constraints on $\mathcal{M}$ in (7). Thus we can rewrite the action S_2 as

$$S_2 = -\frac{1}{2}\int d^4x\sqrt{-g}\, h^{ij}\, \partial_\mu m_i\, \partial^\mu m_j, \tag{14}$$

where the metric $h = \mathrm{diag}(1, 1, -1)$, $m_1 = \frac{\lambda_1}{\lambda_2}$, $m_2 = \frac{|\lambda|^2-1}{2\lambda_2}$ and $m_3 = \frac{|\lambda|^2+1}{2\lambda_2}$ [2]. Correspondingly, one can deduce that the infinitesimal SL(2,R) transformation of m_i is

$$\delta m_k = f_{ij}^k\, \alpha_i\, m_j. \tag{15}$$

Explicitly

$$\begin{aligned}\delta\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} &= \begin{pmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ \alpha_2 & -\alpha_1 & 0 \end{pmatrix}\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} \\ &= -\sum_{i=1}^{3}\alpha_i\,\Gamma_i\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix},\end{aligned} \tag{16}$$

where the representation for $\{\Gamma^i\}$ is;

$$\Gamma^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \Gamma^2 = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \Gamma^3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{17}$$

which are traceless and satisfy the following algebra:

$$[\Gamma^i, \Gamma^j] = f_{ij}^k \Gamma^k$$

and

$$Tr[\Gamma^i\,\Gamma^j] = 2h^{ij},$$

$$\Gamma^{iT} = h\,\Gamma^i\, h.$$

Action in (14) is invariant under (16).

Let us now turn to the action S_3, we can write the action in the following form

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[2] we already know that m_i's depend on two independent fields λ_1 and λ_2. It is a reflection of the fact that the matrix $\mathcal{M}$ parametrizes the coset $\frac{SL(2,R)}{U(1)}$.

$$S_3 = \int d^4x \sqrt{-g}\, m_i\; K^i, \tag{18}$$

where

$$K^i = \frac{1}{4}\, F^{(m,\alpha)}_{\mu\nu} \hat{G}_{mn}\, (\Sigma^i\, \eta)_{\alpha\beta}\, F^{(n,\beta)\,\mu\nu}. \tag{19}$$

Using the SL(2,R) transformations for vector fields $A^{(m,\alpha)}_\mu$ we derive the transformation on K^i to be the following;

$$\delta K^i = -\{\sum_k \alpha_k\, (h\, \Gamma_k\, h)\}_{ij}\, K^j. \tag{20}$$

Note that $K^i = h^{ij} K_j$. Then the covariant K_i will transform as

$$\delta K_i = -(\sum_k \alpha_k\, \Gamma_k)_{ij}\, K_j. \tag{21}$$

Therefore, we can rewrite

$$S_2 + S_3 = \int d^4x \sqrt{-g} \left(-\frac{1}{2}\, h^{ij}\, \partial_\mu m_i\, \partial^\mu m_j + m_i\, K_j\, h^{ij} \right). \tag{22}$$

which is invariant under (16) and (21). It can be checked that S_4 also remain invariant. From here the canonical momentum conjugate to m_i is $\pi^i = \partial^0\, m^i$, which transforms as

$$\delta \pi^i = (\alpha\, .\Gamma^T)_{ij}\, \pi^j. \tag{23}$$

We remind the reader that a classical mechanical system in the Hamiltonian phase space formalism is described by a set of generalised coordinates $\{q_i\}$ and conjugate momenta $\{p^i\}$. Under infinitesimal canonical transformations they transform as

$$\begin{aligned} q'_i &= q_i -\, \alpha\, . \left(\frac{\partial \Phi_{(q)}(q,p)}{\partial p^i} \right), \\ p'^i &= p^i +\, \alpha\, . \left(\frac{\partial \Phi_{(q)}(q,p)}{\partial q_i} \right), \end{aligned} \tag{24}$$

where $\Phi_{(q)}$ is the generator of the transformation. Thus for the case of SL(2,R), we can construct the generators of infinitesimal transformations to be

$$\Phi^k_{(m)}(m,\pi) = \int \pi^j\, (\Gamma^k)_{ji}\, m_i. \tag{25}$$

Similarly, the canonical momentum conjugate to $A_\mu^{(m,\alpha)}$ is (here we suppress the indices for convenience)

$$\pi^\mu = \hat{G}\mathcal{M}^{-1}\, F^{0\mu} + \hat{B}\,\eta\,\tilde{F}^{0\mu},$$

and it transforms as

$$\delta\pi^\mu = -\alpha\,.\,\Sigma\,\pi^\mu. \tag{26}$$

Therefore, the corresponding generating function in the gauge field sector is given by

$$\Phi^i_{(A)}(A_\mu, \pi^\mu) = -\int \pi^{\mu\,(m,\alpha)}\,(\Sigma^i)_{\alpha\beta}\,A_\mu^{(m,\beta)}. \tag{27}$$

We can now write down the complete generating function for canonical transformation in the phase space of the full theory to be the sum of above two generating functions. That is

$$\Phi^i = \Phi^i_{(m)} + \Phi^i_{(A)}. \tag{28}$$

Now, we shall obtain conserved charges associated with the infinitesimal SL(2,R) transformations. We obtain the conserved current for the scalars m_i to be

$$J^{k,\mu}_{(m)} = m_i\,(\Gamma^k)_{ij}\,\partial^\mu m_j \tag{29}$$

and that for gauge fields A_μ is

$$J^{i,\mu}_{(A)} = A_\nu\,\Sigma^{iT}\,\hat{G}\,\mathcal{M}^{-1}\,F^{\mu\nu}. \tag{30}$$

Corresponding Noether charges are

$$\begin{aligned} Q^a_{(m)} &= \int d^3x\sqrt{-g}\,m_i\Gamma^a_{ij}\pi^j, \\ Q^a_{(A)} &= \int d^3x\sqrt{-g}\,A_\nu\,\Sigma^{aT}\,\pi^\nu, \end{aligned} \tag{31}$$

respectively. They satisfy the following algebra,

$$\begin{aligned} \{Q^a_{(m)}, m_i\} &= \Gamma^a_{ij}\,m_j \equiv (\delta m_i)^a, \\ \{Q^a_{(A)}, A_\mu^{(m,\alpha)}\} &= \Sigma^a_{\alpha\beta}\,A_\mu^{(m,\beta)} \equiv (\delta A_\mu^{(m,\alpha)})^a. \end{aligned} \tag{32}$$

At this point we can discuss some properties of the generating functional which follow from the invariance of the classical path integral under the canonical transformations in the Hamiltonian approach. One can write down the generating functional for correlation functions, $Z[\zeta_i, J^\mu]$, in the phase space,

$$Z[\zeta, J] = \int \mathcal{D}[m_i, \pi_i, A_\mu, \pi_\mu] \quad e^{\{i\, S_H[m,A,\pi] + (\text{source terms})\}} \tag{33}$$

where source terms are

$$\text{source terms} = i \int (m^i \zeta_i + A_\mu^{(m,\alpha)} J^\mu_{(m,\alpha)}), \tag{34}$$

with ζ_i and $J^\mu_{(m\alpha)}$ being the classical sources and summation over repeated indices is understood. $\mathcal{D}[m_i, \pi_i, A_\mu, \pi_\mu]$ collectively stands for the Hamiltonian phase space measure and S_H is the Hamiltonian action.

Now we adopt the procedure of Veneziano, further elaborated by Maharana and Veneziano, to exhibit some interesting properties of the partition function [12,13]. Note that when we implement infinitesimal canonical transformation, the variables in the Hamiltonian phase space change according to eq.(24). The Hamiltonian action S_H and field variables appearing in source terms (34) transform accordingly which can be compensated by a shift in the sources. On the other hand, the phase space measure remains invariant, modulo anomaly terms. In ref. [12] and [13], this property was exploited to derive Ward identities.

It is easy to see that for the problem at hand the following shifts of the sources

$$\delta\zeta_i = -\{\alpha_k\, \Gamma_k\}_{ij} \zeta_j, \qquad \delta J^\mu_{(m,\alpha)} = \{\alpha_k \Sigma_k\}_{\alpha\beta} J^\mu_{(m,\beta)}. \tag{35}$$

enable us to derive a relation which is satisfied by the generating functional

$$Z[\zeta, J] = Z[\zeta + \delta\zeta, J + \delta J]. \tag{36}$$

Now following the arguments of Maharana and Veneziano we are led to

$$\int d^4x \left(-\frac{\delta Z}{\delta \zeta_i(x)} (\Gamma^k)_{ij} \zeta_j(x) + \frac{\delta Z}{\delta J^\mu_{(m\alpha)}(x)} (\Sigma^k)_{\alpha\beta} J^\mu_{(m\beta)}(x) \right) \; \alpha^k = 0. \tag{37}$$

We mention in passing that the Hamiltonian approach is an elegant way to derive identities like (37) which can also be obtained in the Lagrangian formulation [16]. We recall that an equation like (37) was the starting point of the derivation of gravitational and gauge Ward identities in ref. [12] where the infinitesimal parameter was a local one. For the case at hand we are dealing with global symmetries and the infinitesimal parameters appearing in (37) are independent of spacetime. Notice, however, that the relation holds for arbitrary infinitesimal parameters and therefore, we can differentiate this equation with respect to α^k and then set $\alpha^k = 0$. Thus we arrive at

$$\int \left(-\frac{\delta Z}{\delta \zeta_i} (\Gamma^k)_{ij} \zeta_j + \frac{\delta Z}{\delta J^{\mu}_{(m\alpha)}} (\Sigma^k)_{\alpha\beta} J^{\mu}_{(m\beta)} \right) = 0. \tag{38}$$

Now we can functionally differentiate (38) with respect to the sources ζ_i and $J^{\mu}_{(m,\alpha)}$ several times and then set these sources to zero. In this process, we shall obtain interesting relations involving correlation functions which will be analogous to the Ward identities derived in [12]. We mention that these are not the Ward identities one derives for local symmetries; but our relations are obtained in the context of global symmetries.

To summarise, we have studied the SL(2,R), identified as the S-duality group, transformation properties of the string effective action. We start from an effective action introduced in [15] which contains an appropriate set of gauge fields to be manifestly SL(2,R) invariant. The actions S_2 and S_3 are reexpressed in a suitable form so that we can exploit the symmetry properties to derive eq.(38). The conserved currents are obtained and the generators of canonical transformation responsible for infinitesimal transformations are identified. We obtain a set of relations for correlation functions by exploiting the symmetry properties of the effective action under canonical transformations.

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