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The Neveu–Schwarz–Ramond String in Background Fields: Nilpotency of BRST Charge

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Abstract

The Neveu–Schwarz–Ramond type II closed superstring is quantized in the BRST Hamiltonian formalism in the presence of arbitrary background fields. Batalin, Fradkin and Vilkovisky technique is employed in order to construct the BRST charge. The background field equations of motion are derived from the requirement of the nilpotency of the quantum BRST charge.

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I. Introduction

This article is in sequel to our investigation[1] of the evolution of type II Neveu-Schwarz-Ramond superstring[2] in the background of the graviton and antisymmetric tensor fields[3] within the first quantized framework. We have obtained the constraints and their classical algebras in a systematic manner in I. Furthermore, we have presented the classical BRST charge which is nilpotent by construction. The purpose of the pr esent paper is to derive the consequences of the nilpotency of the quantum BRST charge. As is well known, all string theories are required to be conformally invariant and this condition imposes stringent constraints on the background field configurations when we consider evolution of strings in the presence of background fields.[4] One of the ways to impose conformal invariance is to demand the vanishing of the associated β-functions of the theory[5] which in turn corresponds to the equations of motion satisfied by the background fields. On the other hand, a more transparent and algebraic way of imposing conformal invariance on the theory is to demand the nilpotency of the quantum BRST charge.[6] Indeed, the nilpotency of the quantum BRST charge guarantees that all unphysical degrees of freedom are decoupled from the theory, ensuring the unitarity of the S-matrix. The critical dimension, $d = 10$, and the intercepts are derived in an elegant manner in the case of the free N-S-R superstring if we demand $Q^2_{BRST} = 0$ at the quantum level. As we shall demonstrate in Section IV, for the interacting case, the nilpotency condition leads to the equations of motion for the background fields in addition to providing the critical dimensions and the intercepts appropriate to the choice of the boundary condition for the world-sheet fermions.

We may observe here that, whereas the quantum superconformal algebra (nilpotency of the quantum BRST charge) for the free N-S-R string yields a c-number anomaly, the interacting N-S-R string (string propagating in nontrivial backgrounds) gives rise to q-number anomalies. The existence of such q-number anomalies have been noticed in the context of a bosonic string[7] propagating in arbitrary background fields and in the case of a type II superstring[8] in a curved background without torsion. However, these q-number anomalies disappear on-shell, i.e., when the backgrounds satisfy equations of motion. The present work generalizes the results of refs. 7 and 8.

At this point, some technical remarks are in order. We follow the Batalin, Fradkin and Vilkovisky[9] phase-space Hamiltonian formalism in order to obtain the expression for the BRST charge from the algebra of constraints. The details of the calculations can be found in I. We shall follow the normal ordering prescription similar to ref. 7 with suitable generalization to include the world-sheet fermions. Furthermore, we have adopted the weak field approximation scheme for the target manifold metric and the antisymmetric tensor field in order to facilitate the computation[10] of the nilpotency of the BRST charge.

The paper is organized as follows: In Section II, we briefly describe the Batalin-Fradkin-Vilkovisky (BFV) formalism for the construction of the BRST charge. In Section III, we identify the generators of the superconformal algebra and define the quantum operators in the weak field approximation. Section IV deals with BFV-BRST quantization of the free N-S-R string and the consequences of the nilpotency of BRST charge for the free superstring. We consider the case of an interacting string in Section V and obtain the equations of motion for the background fields from $Q^2_{BRST} = 0$. The summary and conclusions of our work is contained in Section VI. Our notational convention is given in Appendix A and Appendix B contains some steps of calculations of Section IV and Section V for elucidation.

II. The BFV–Formalism for Construction of Q_{BRST}

Batalin, Fradkin and Vilkovisky[9] have provided a prescription for constructing the BRST charge for a system with a set of first class constraints $\{\phi_a\}$ satisfying the closed algebra

$$\left\{\phi_a, \phi_b\right\} = U^c_{ab}\phi_c \tag{2.1}$$

$$\left\{H_0, \phi_a\right\} = V^b_a\phi_b \tag{2.2}$$

where ϕ_a's are fermionic or bosonic constraints. U^c_{ab} and V^b_a are structure constants and H_0 is the canonical Hamiltonian. The BRST charge is given by

$$Q_{BRST} = \phi_a\eta^a + \frac{1}{2}(-1)^{n_a}P_cU^c_{ab}\eta^a\eta^b \tag{2.3}$$

Here η^a and P_b are the phase–space variable and their Grassmann characters are opposite to that of ϕ_a. They satisfy

$$\left\{\eta^a, P_b\right\} = \delta^a_b \tag{2.4}$$

and $n_a = 0(1)$ for ϕ_a bosonic (fermionic). Q_{BRST} as defined in (2.3) is nilpotent by construction at the classical level, i.e., $Q^2_{BRST} = 0$.

The physical states are projected out through the following relation

$$\hat{Q}_{BRST}|\text{phys} >= 0 \tag{2.5}$$

provided, at the quantum level, we still have

$$\hat{Q}^2_{BRST}|\text{state} >= 0 \tag{2.6}$$

Here $\hat{Q}$ is the quantum charge. We have seen in I that the classical superconformal algebra closes for the N–S–R string without any anomaly.

III. Super Conformal Algebra for the N-S-R String and the Quantum Operators

The gauge fixed Lagrangian for the N–S–R string in arbitrary background is given by[1]

$$\begin{aligned}
L &= \frac{1}{2}\eta^{\mu\nu}\partial_\mu X^i\partial_\nu X^j G_{ij}(X) + \frac{k}{8\pi}\epsilon^{\mu\nu}B_{ij}(X)\partial_\mu X^i\partial_\nu X^j \\
&+ \frac{i}{2}\overline{\psi}^i\rho^\mu\left(\partial_\mu\psi^j + \Gamma^j_{k\ell}\partial_\mu X^k\psi^\ell\right)G_{ij}(X) \\
&- \frac{ik}{16\pi}T_{ijk}\overline{\psi}^i\rho^\mu\rho_5\psi^j\partial_\mu X^k - \frac{1}{12}R_{ijk\ell}\overline{\psi}^i\psi^k\overline{\psi}^j\psi^\ell \\
&- \frac{k}{64\pi}D_kT_{i\ell j}\overline{\psi}^i\psi^k\overline{\psi}^j\rho_5\psi^\ell - \frac{k^2}{512\pi^2}G^{mn}T_{ikm}T_{j\ell n}\overline{\psi}^i\rho_5\psi^k\overline{\psi}^j\rho_5\psi^\ell
\end{aligned} \tag{3.1}$$

T_{ijk} is the field strength associated with $B_{ij}(X)$. We have chosen the orthonormal gauge

$$\begin{aligned}
e_{a\mu} &= \eta_{a\mu} \\
\chi_\mu &= 0
\end{aligned} \tag{3.2}$$

$e_{a\mu}$ being the Zweibein and χ_μ is the world–sheet gravitino. The Lagrangian (3.1) is invariant under the 2-dimensional global supersymmetry transformations

$$\begin{aligned}
\delta X^i &= -\bar{\epsilon}\psi^i \\
\delta\psi^i &= i\rho^\mu\partial_\mu X^i\epsilon + \Gamma^i_{jk}\psi^k\left(\bar{\epsilon}\psi^j\right) + \frac{k}{16\pi}G^{i\ell}T_{jk\ell}\left(\overline{\psi}^j\rho_5\psi^k\right)\epsilon
\end{aligned} \tag{3.3}$$

Here ϵ is the infinitesimal fermionic parameter. The generators of the superconformal transformations are

$$T_{++} = \frac{1}{2}\Big[G_{ij}(X)\partial_+X^i\partial_+X^j + 2i\psi^A_+(\eta_{AB}\partial_\sigma\psi^B_+$$

$$
\begin{aligned}
&- \quad w^{(1)}_{i,AB}\partial_\sigma X^i\psi^B_+) - \frac{1}{2}R^{(1)}_{ijk\ell}\psi^i_-\psi^j_-\psi^k_+\psi^\ell_+\Big] \\
T_{--} &= \frac{1}{2}\Big[G_{ij}\partial_- X^i\partial_- X^j + 2i\psi^A_-(\eta_{AB}\partial_\sigma\psi^B_- \\
&- \quad w^{(2)}_{i,AB}\partial_\sigma X^i\psi^B) - \frac{1}{2}R^{(1)}_{ijk\ell}\psi^i_-\psi^j_-\psi^k_+\psi^\ell_+\Big]
\end{aligned} \tag{3.4}
$$

$$
\begin{aligned}
J_+ &= G_{ij}(X)\partial_+ X^j\psi^i_+ + \frac{ik}{24\pi}T_{ijk}\psi^i_+\psi^j_+\psi^k_+ \\
J_- &= G_{ij}(X)\partial_- X^j\psi^i_- + \frac{ik}{24\pi}T_{ijk}\psi^i_-\psi^j_-\psi^k_-
\end{aligned} \tag{3.5}
$$

and $\partial_\pm = \partial_\tau \pm \partial_\sigma, \psi^i_\pm = \frac{1}{2}(1 \mp \rho_5)\psi^i$ and $\psi^i = E^i_A(X)\psi^A$; E^i_A being the vielbein of the target manifold. The spin connections

$$
\begin{aligned}
w^{(1)}_{i,AB} &= -E_{jA}\frac{\partial}{\partial X^i}E^j_B - \Gamma^j_{i\ell}E_{jA}E^\ell_B + \frac{k}{8\pi}T^j_{i\ell}E_{jA}E^\ell_B \\
w^{(2)}_{i,AB} &= -E_{jA}\frac{\partial}{\partial X^i}E^j_B - \Gamma^j_{i\ell}E_{jA}E^\ell_B - \frac{k}{8\pi}T^j_{i\ell}E_{jA}E^\ell_B
\end{aligned} \tag{3.6}
$$

The generalized curvatures are defined as

$$
R^{(1)}_{ABk\ell} = \frac{\partial}{\partial X^k}w^{(1)}_{\ell,AB} - \frac{\partial}{\partial X^\ell}w^{(1)}_{k,AB} + w^{(1)}_{\ell,AC}w^{(1)C}_{k,\ \ B} - w^{(1)}_{k,AC}w^{(1)C}_{\ell,\ \ B} \tag{3.7}
$$

Similarly $R^{(2)}_{ABk\ell}$ is defined by replacing $w^{(1)}_{i,AB}$ with $w^{(2)}_{i,AB}$ in the above equation (3.7). The canonical momenta conjugate to X^i and $\psi^i_\pm$ are defined as

$$
\begin{aligned}
P_i &\equiv \frac{\partial L}{\partial \dot{X}^i} = G_{ij}(X)\dot{X}^j + \frac{k}{8\pi}B_{ij}(X)X'^j + \frac{i}{2}\psi^A_-\psi^B_- w^{(2)}_{i,AB} \\
&- \quad \frac{i}{2}\psi^A_+\psi^B_+ w^{(1)}_{i,AB} \\
\Pi_{\pm i} &\equiv \frac{\partial L}{\partial \dot{\psi}^i_\pm} = \pm\frac{i}{2}\psi^j_\pm G_{ij}(X)
\end{aligned} \tag{3.8}
$$

The fundamental brackets are

$$
\Big\{X^i(\sigma), P_j(\sigma')\Big\} = \delta^i_j\delta(\sigma - \sigma')
$$

$$\left\{\psi_\pm^A(\sigma), \psi_\pm^B(\sigma')\right\} = \pm i\eta^{AB}\delta(\sigma - \sigma') \tag{3.9}$$

The second bracket in (3.9) is actually the Dirac bracket where we have taken into account the nature of the second class constraint in the definition of the fermion canonical momentum. Note that the constraints defined in (3.4) and (3.5) satisfy the algebra (classical PB relations)

$$\begin{aligned}
\left\{J_\pm(\sigma,\tau), J_\pm(\sigma',\tau)\right\} &= \mp 2iT_{\pm\pm}\delta(\sigma-\sigma') \\
\left\{T_{\pm\pm}(\sigma,\tau), J_\pm(\sigma',\tau)\right\} &= \pm\left(2J_\pm(\sigma,\tau) + J_\pm(\sigma',\tau)\right)\partial_\sigma\delta(\sigma-\sigma') \\
\left\{T_{\pm\pm}(\sigma,\tau), T_{\pm\pm}(\sigma',\tau)\right\} &= \pm\left(T_{\pm\pm}(\sigma,\tau) + T_{\pm\pm}(\sigma',\tau)\right)\partial_\sigma\delta(\sigma-\sigma')
\end{aligned} \tag{3.10}$$

and all other brackets vanish.

In order to calculate the quantum algebra of the above generators, we go over to the interaction representation. In the interaction representation

$$\begin{aligned}
\dot{X}^i &= P_j\eta^{ij} \\
\Pi_{\pm i} &= \pm\frac{i}{2}\psi_\pm^j\eta_{ij}
\end{aligned} \tag{3.11}$$

Therefore,

$$\begin{aligned}
T_{++} &= \frac{1}{2}\Big[\dot{X}^i\dot{X}^jG_{ij} + 2\dot{X}_iX'^i + X'^iX'^jG_{ij} - \frac{k}{2\pi}G^{ij}B_{j\ell}\dot{X}_iX'^\ell \\
&+ \frac{k^2}{16\pi^2}B_{ij}G^{im}B_{mn}X'^jX'^n - i\psi_-^A\psi_-^B\left(\dot{X}_iG^{ij} + X'^j\right)w_{j,AB}^{(2)} \\
&+ i\psi_+^A\psi_+^Bw_{j,AB}^{(1)}\left(\dot{X}_iG^{ij} + X'^j\right) + \frac{ik}{4\pi}\psi_-^A\psi_-^Bw_{j,AB}^{(2)}B_{k\ell}G^{kj}X'^\ell \\
&- \frac{ik}{4\pi}\psi_+^A\psi_+^Bw_{j,AB}^{(1)}B_{k\ell}G^{kj}X'^\ell
\end{aligned}$$

$$
\begin{aligned}
&- \frac{1}{4}\Big(\psi_-^A\psi_-^B\psi_-^C\psi_-^D w^{(2)}_{i,AB} w^{(2)}_{j,CD} G^{ij} + \psi_+^A\psi_+^B\psi_+^C\psi_+^D w^{(1)}_{i,AB} w^{(1)}_{j,CD} G^{ij} \\
&- \psi_-^A\psi_-^B\psi_+^C\psi_+^D w^{(2)}_{i,AB} w^{(1)}_{j,CD} G^{ij} - \psi_+^A\psi_+^B\psi_-^C\psi_-^D w^{(1)}_{i,AB} w^{(2)}_{j,CD} G^{ij}\Big) \\
&+ i\psi_+^A\left(\eta_{AB}\partial_\sigma\psi_+^B - w^{(1)}_{i,AB}X'^i\psi_+^B\right) - \frac{1}{4}R^{(1)}_{ijk\ell}\psi_-^i\psi_-^j\psi_+^k\psi_+^\ell\Big] \qquad (3.12)
\end{aligned}
$$

$$
\begin{aligned}
T_{--} &= \frac{1}{2}\Big[\dot{X}^i\dot{X}^j G_{ij} - 2\dot{X}_i X'^i + G_{ij}X'^i X'^j - \frac{k}{2\pi}G^{ij}B_{j\ell}\dot{X}_i X'^\ell \\
&+ \frac{k^2}{16\pi^2}B_{ij}G^{im}X'^j X'^m - i\psi_-^A\psi_-^B\left(\dot{X}_i G^{ij} - X'^j\right)w^{(2)}_{j,AB} \\
&+ i\psi_+^A\psi_+^B w^{(1)}_{j,AB}\left(\dot{X}_i G^{ij} - X'^j\right) + \frac{ik}{4\pi}\psi_-^A\psi_-^B w^{(2)}_{j,AB}B_{k\ell}G^{kj}X'^\ell \\
&- \frac{ik}{4\pi}\psi_+^A\psi_+^B w^{(1)}_{j,AB}B_{k\ell}G^{kj}X'^\ell \\
&- \frac{1}{4}\Big(\psi_-^A\psi_-^B\psi_-^C\psi_-^D w^{(2)}_{i,AB} w^{(2)}_{j,CD} + \psi_+^A\psi_+^B\psi_+^C\psi_+^D w^{(1)}_{i,AB} w^{(1)}_{j,CD} \\
&- \psi_-^A\psi_-^B\psi_-^C\psi_-^D w^{(2)}_{i,AB} w^{(1)}_{j,CD} - \psi_+^A\psi_+^B\psi_+^C\psi_+^D w^{(1)}_{i,AB} w^{(2)}_{j,CD}\Big)G^{ij} \\
&+ i\psi_-^A\left(\eta_{AB}\partial_\sigma\psi_-^B - w^{(2)}_{i,AB}X'^i\psi_-^B\right) - \frac{1}{4}R^{(1)}_{ijk\ell}\psi_-^i\psi_-^j\psi_+^k\psi_+^\ell\Big] \qquad (3.13)
\end{aligned}
$$

$$
\begin{aligned}
J_+ &= \psi_+^A E_A^i\Big[\dot{X}^k G_{ki} + \left(G_{ij} - \frac{k}{4\pi}B_{ij}\right)X'^j \\
&- \frac{i}{2}\psi_-^B\psi_-^C w^{(2)}_{i,BC} + \frac{i}{2}\psi_+^B\psi_+^C w^{(1)}_{i,BC}\Big] + \frac{ik}{24\pi}T_{ijk}\psi_+^i\psi_+^j\psi_+^k \qquad (3.14)
\end{aligned}
$$

$$
\begin{aligned}
J_- &= \psi_-^A E_A^i\Big[\dot{X}^k G_{ki} - \left(G_{ij} + \frac{k}{4\pi}B_{ij}\right)X'^j - \frac{i}{2}\psi_-^B\psi_-^C w^{(2)}_{i,BC} \\
&+ \frac{i}{2}\psi_+^B\psi_+^C w^{(1)}_{i,BC}\Big] + \frac{ik}{24\pi}T_{ijk}\psi_-^i\psi_-^j\psi_-^k \qquad (3.15)
\end{aligned}
$$

Now we proceed to express the generators in the weak field approximation

$$
E_{iA} = \eta_{iA} + \frac{1}{2}h_{iA}(X)
$$

$$
\begin{aligned}
E^i_A &= \delta^i_A - \frac{1}{2}h^i_A(X) \\
G_{ij}(X) &= \eta_{ij} + h_{ij}(X) \\
B_{ij}(X) &= b_{ij}(X)
\end{aligned}
\tag{3.16}
$$

and keep terms linear in h_{ij} and b_{ij} in the expansions. Furthermore, all target manifold world indices are raised and lowered by the flat metric η^{ij} and η_{ij} in what follows. The spin connections and curvatures take the following form in the linearized approximation

$$w^{(1)lin.}_{i,AB} = \frac{1}{2}\left(\partial_\ell \rho_{mi} + \frac{k}{4\pi}\partial_i b_{\ell m} - \partial_m \rho_{\ell i}\right)\delta^m_A \delta^\ell_B \tag{3.17}$$

$$w^{(2)lin.}_{i,AB} = \frac{1}{2}\left(\partial_\ell \rho_{im} + \frac{k}{4\pi}\partial_i b_{m\ell} - \partial_m \rho_{i\ell}\right)\delta^m_A \delta^\ell_B \tag{3.18}$$

$$R^{(1)lin.}_{ijk\ell} = \left(\partial_i \partial_\ell \rho_{kj} - \partial_k \partial_j \rho_{ik} + \partial_k \partial_i \rho_{j\ell} - \partial_j \partial_\ell \rho_{ki}\right) \tag{3.19}$$

where

$$\rho_{ij}(X) = -h_{ij}(X) + \frac{k}{4\pi}b_{ij}(X) \tag{3.20}$$

The linearized form of the generators are

$$
\begin{aligned}
T_{++} &= \frac{1}{2}P^i_+ P^j_+ \eta_{ij} + i\psi^A_+ \partial_\sigma \psi^B_+ \eta_{AB} + \frac{1}{2}\rho_{ij} P^i_+ P^j_- \\
&- i w^{(2)lin.}_{i,m\ell} \psi^m_- \psi^\ell_- P^i_+ + \frac{i}{2} w^{(1)lin.}_{i,m\ell} \psi^m_+ \psi^\ell_+ (P_+ + P_-)^i \\
&- \frac{1}{4} R^{(1)lin.}_{ijk\ell} \psi^i_- \psi^j_- \psi^k_+ \psi^\ell_+
\end{aligned}
\tag{3.21}
$$

$$
\begin{aligned}
T_{--} &= \frac{1}{2}P^i_- P^j_- \eta_{ij} + i\psi^A_- \partial_\sigma \psi^B_- \eta_{AB} + \frac{1}{2}\rho_{ij} P^i_+ P^j_- \\
&+ i w^{(1)lin.}_{i,m\ell} \psi^m_+ \psi^\ell_+ P^i_- - \frac{i}{2} w^{(2)lin.}_{i,m\ell} \psi^m_- \psi^\ell_- (P_+ + P_-)^i \\
&- \frac{1}{4} R^{(1)lin.}_{ijk\ell} \psi^i_- \psi^j_- \psi^k_+ \psi^\ell_+
\end{aligned}
\tag{3.22}
$$

$$\begin{aligned}J_+ &= \psi_+^A P_+^j \eta_{Aj} + \frac{k}{8\pi} b_{ij} \psi_+^j P_+^i + \frac{1}{2}\rho_{ij}\psi_+^i P_-^j \\ &- \frac{i}{2} w_{i,m\ell}^{(2)lin.} \psi_+^i \psi_-^m \psi_-^\ell + \frac{i}{2} w_{i,m\ell}^{(1)lin.} \psi_+^i \psi_+^m \psi_+^\ell \\ &+ \frac{ik}{24\pi} T_{im\ell}^{lin.} \psi_+^i \psi_+^m \psi_+^\ell \end{aligned} \tag{3.23}$$

$$\begin{aligned}J_- &= \psi_-^A P_-^j \eta_{Aj} + \frac{k}{8\pi} b_{ij} \psi_-^j P_-^i + \frac{1}{2}\rho_{ij}\psi_-^i P_+^j \\ &- \frac{i}{2} w_{i,m\ell}^{(2)lin.} \psi_-^i \psi_-^m \psi_-^\ell + \frac{i}{2} w_{i,m\ell}^{(1)lin.} \psi_-^i \psi_+^m \psi_+^\ell \\ &+ \frac{ik}{24\pi} T_{ijk}^{lin.} \psi_-^i \psi_-^j \psi_-^k \end{aligned} \tag{3.24}$$

The generators can be expanded in Fourier series, the fourier modes are defined as follows

$$L_m = \frac{1}{2\pi}\int \frac{dz}{iz} : T_{++} : z^m \tag{3.25}$$

$$\overline{L}_m = -\frac{1}{2\pi}\int \frac{d\overline{z}}{i\overline{z}} : T_{--} : \overline{z}^m \tag{3.26}$$

$$G_m = \frac{1}{\sqrt{2}\pi}\int \frac{dz}{iz} : J_+ : z^m \tag{3.27}$$

$$\overline{G}_m = -\frac{1}{\sqrt{2}\pi}\int \frac{d\overline{z}}{i\overline{z}} : J_- : \overline{z}^m \tag{3.28}$$

with $z = e^{i\sigma}$ and $\overline{z}$ is its complex conjugate. The operators appearing in the definition of the generators are normal ordered. The normal ordering prescriptions for the Fubini–Veneziano fields [11] $P_\pm^i \equiv \left(\dot{X}^i \pm X'^i\right)$ and the fermions are given in Appendix B.

IV. Nilpotency of $\hat{Q}_{BRST}$ for the Free N–S–R String

In this section we present the consequence of the nilpotency of $\hat{Q}_{BRST}$ for the free N–S–R string and derive the well–known results on the critical dimensions and intercept.[12] The computation is carried out in terms of the Fubini–Veneziano fields and the world- -sheet fermions. This calculation sets the stage for the computation of $\hat{Q}^2_{BRST}$ for the superstring in a nontrivial background in Section V. Notice that the superconformal algebra (3.10) for the interacting string is the same as that of the free string (at the classical level). Therefore, the pure ghost sector of the BRST charge is the same in both the cases. Therefore, the computation of $\hat{Q}^2_{BRST} = 0$ provides us some insight for our next step.

The BRST charge is

$$\begin{aligned} Q_{BRST} &= \int d\sigma \Big[T_{++}\eta_+ + T_{--}\eta_- + J_+\lambda_+ + J_-\lambda_- \\ &+ \mathcal{P}_+\partial_\sigma\eta_+\eta_+ - \mathcal{P}_-\partial_\sigma\eta_-\eta_- + i\mathcal{P}_+\lambda_+\lambda_+ \\ &- i\mathcal{P}_-\lambda_-\lambda_- + \frac{1}{2}\zeta_+\lambda_+\partial_\sigma\eta_+ + \zeta_+\partial_\sigma\lambda_+\eta_+ \\ &- \zeta_-\partial_\sigma\lambda_-\eta_- - \frac{1}{2}\zeta_-\lambda_-\partial_\sigma\eta_- \Big] \end{aligned} \tag{4.1}$$

The form of Q_{BRST} in (4.1) is evident from the discussion of Section II on BFV formalism and the superconformal algebra (3.10). Here $\eta_\pm, \mathcal{P}_\pm$ are the fermionic ghosts and their conjugate momenta associated with the bosonic constraints $T_{\pm\pm}$. Similarly, $\lambda_\pm$ and $\zeta_\pm$ are the bosonic ghosts and their conjugate momenta corresponding to the supercharge densities $J_\pm$. The fundamental Poisson bracket relations are

$$\left\{\eta_+(z), \mathcal{P}_+(z')\right\} = \left\{\eta_-(z), \mathcal{P}_-(z')\right\} = \delta(z-z') \tag{4.2}$$

$$\left\{\lambda_+(z), \zeta_+(z')\right\} = \left\{\lambda_-(z), \zeta_-(z')\right\} = \delta(z-z') \tag{4.3}$$

and all other PB are zero. The mode expansion of the ghosts are

$$\eta_+(z) = \sum_{n=-\infty}^{+\infty} \eta_n z^{-n}$$

$$\eta_-(\bar{z}) = \sum_{n=-\infty}^{+\infty} \bar{\eta}_n \bar{z}^{-n}$$

$$\mathcal{P}_+(z) = \frac{i}{2\pi} \sum_{n=-\infty}^{+\infty} \mathcal{P}_n z^{-n} \tag{4.4}$$

$$\begin{aligned}
\mathcal{P}_-(\bar{z}) &= \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} \overline{\mathcal{P}}_n \bar{z}^{-n} \\
\lambda_+(z) &= \sum_{n=-\infty}^{+\infty} \lambda_n z^{-n} \\
\lambda_-(\bar{z}) &= \sum_{n=-\infty}^{+\infty} \bar{\lambda}_n \bar{z}^{-n} \\
\zeta_+(z) &= \frac{i}{2\pi} \sum_{n=-\infty}^{+\infty} \zeta_n z^{-n} \\
\zeta_-(\bar{z}) &= \frac{i}{2\pi} \sum_{n=-\infty}^{+\infty} \bar{\zeta}_n \bar{z}^{-n}
\end{aligned} \tag{4.5}$$

useful to write the BRST charge in a modified form which simplifies subsequent computations.

$$\begin{aligned}
:\hat{Q}_{BRST}: &= :\sum_{n=-\infty}^{+\infty} \left(L'_n \eta_{-n} + \overline{L}'_n \bar{\eta}_{-n} + G_n \lambda_{-n} + \overline{G}_n \bar{\lambda}_{-n} \right): \\
&+ :\sum_n \sum_m \Big[m\mathcal{P}_n \eta_m \eta_{-n-m} + m\overline{\mathcal{P}}_n \bar{\eta}_m \bar{\eta}_{-n-m} \\
&- \mathcal{P}_n \lambda_m \lambda_{-n-m} + \frac{1}{2}(m-n)\zeta_n \lambda_m \eta_{-n-m} \\
&- \frac{1}{2}(m-n)\bar{\zeta}_n \bar{\lambda}_m \bar{\eta}_{-n-m} \Big]:
\end{aligned} \tag{4.6}$$

where we have defined

$$\begin{aligned}
L'_n &= L_n - \alpha\delta_{n,0} \\
\overline{L}'_n &= \overline{L}_n - \beta\delta_{n,0}
\end{aligned} \tag{4.7}$$

in order to take into account the ambiguity in the definitions of L_0 and $\overline{L}_0$. The constants α and β (intercepts) will be eventually determined when we demand $\hat{Q}^2_{BRST} = 0$. The modes of the ghosts satisfy quantum relations

$$\begin{aligned}\left[\eta_n, \mathcal{P}_m\right]_+ &= \left[\overline{\eta}_n, \overline{\mathcal{P}}_m\right]_+ = \delta_{m,-n} \\ \left[\lambda_n, \zeta_m\right]_- &= \left[\overline{\lambda}_n, \overline{\zeta}_m\right]_- = \delta_{n,-m}\end{aligned} \tag{4.8}$$

The generators of the free N-S-R string are

$$\begin{aligned}L^{(0)}_m &= \frac{1}{2\pi}\oint \frac{dz}{iz} z^m : \left[\frac{1}{2}P^i_+ P^j_+ \eta_{ij} + i\psi^A_+ \partial_\sigma \psi^B_+ \eta_{AB}\right] : \\ \overline{L}^{(0)}_m &= -\frac{1}{2\pi}\oint \frac{d\overline{z}}{i\overline{z}} \overline{z}^m : \left[\frac{1}{2}P^i_- P^j_- \eta_{ij} + i\psi^A_- \partial_\sigma \psi^B_- \eta_{AB}\right] : \\ G^{(0)}_m &= \frac{1}{\sqrt{2}\pi}\oint \frac{dz}{iz} z^m : \psi^A_+ P^j_+ \eta_{Aj} : \\ \overline{G}^{(0)} &= -\frac{1}{\sqrt{2}\pi}\oint \frac{d\overline{z}}{i\overline{z}} \overline{z}^m : \psi^A_- P^j_- \eta_{Aj} :\end{aligned} \tag{4.9}$$

and they satisfy the quantum super-Virasoro algebra

$$\left[G^{(0)}_m, G^{(0)}_n\right]_+ = 2L^{(0)}_{m+n} + \frac{1}{2} dm^2 \delta_{m+n,0} \tag{4.10}$$

$$\left[L^{(0)}_m, G^{(0)}_n\right]_- = \left(\frac{m}{2} - n\right) G^{(0)}_{m+n} \tag{4.11}$$

and using the Jacobi identity

$$\begin{aligned}\left[G^{(0)}_m, \left[G^{(0)}_n, L^{(0)}_p\right]_-\right]_+ &+ \left[L^{(0)}_p, \left[G^{(0)}_m, G^{(0)}_n\right]_+\right]_- \\ &+ \left[G^{(0)}_n, \left[L^{(0)}_p, G^{(0)}_m\right]_-\right]_+ = 0\end{aligned} \tag{4.12}$$

we get

$$\left[L^{(0)}_m, L^{(0)}_n\right] = (m-n)\, L^{(0)}_{m+n} + \frac{dm^3}{8}\delta_{m+n} \tag{4.13}$$

Similar algebra holds for $\overline{L}_m^{(0)}$ and $\overline{G}_m^{(0)}$. Note that all other brackets vanish. Now it is easy to compute : $\hat{Q}^2_{BRST}$: given the commutation/anticommutation relations among the generators.

$$
\begin{aligned}
\Big[: \hat{Q}_{BRST} : \quad , \quad : \hat{Q}_{BRST} : \Big]_+ & \\
&= 2 : \hat{Q}^2 :_{BRST} \\
&= \frac{1}{8}(d-10)\sum_n n^3 \eta_{-n}\eta_n + \frac{1}{8}(d-10+16\alpha)\sum n\eta_{-n}\eta_n \\
&+ \frac{1}{2}(d-10)\sum n^2 \lambda_{-n}\lambda_n + 2\alpha \sum \lambda_{-n}\lambda_n \\
&+ \frac{1}{8}(d-10)\sum_n n^3 \overline{\eta}_{-n}\overline{\eta}_n + \frac{1}{8}(d-10+16\beta)\sum_n n\overline{\eta}_{-n}\overline{\eta}_n \\
&+ \frac{1}{2}(d-10)\sum n^2 \overline{\lambda}_{-n}\overline{\lambda}_n + 2\beta \sum \overline{\lambda}_{-n}\overline{\lambda}_n
\end{aligned}
\qquad (4.14)
$$

It is evident from (4.14) that $\hat{Q}_{BRST}$ is nilpotent only if $d = 10$ and $\alpha = \beta = 0$. Notice that we have taken the Ramond boundary condition for the world-sheet fermions. The same procedure could be followed for the Neveu-Schwarz fermions satisfying antiperiodic condition and we get $d = 10$ and $\alpha = \beta = 1/2$ in this case from : $\hat{Q}^2_{BRST} := 0$.

V. The Interacting Neveu–Schwartz–Ramond String

Now we are in a position to investigate the consequences of the nilpotency of the BRST charge for the N-S-R string in a nontrivial background. The generators of the superconformal transformation defined in eqs. (3.24)-(3.27) can be decomposed as a sum of two terms

$$G_m = G_m^{(0)} + G_m^{(1)} \qquad (5.1)$$

$$\overline{G}_m = \overline{G}_m^{(0)} + \overline{G}_m^{(1)} \tag{5.2}$$

$$L_m = L_m^{(0)} + L_m^{(1)} \tag{5.3}$$

$$\overline{L}_m = \overline{L}_m^{(0)} + \overline{L}_m^{(1)} \tag{5.4}$$

where $G^{(0)}, \overline{G}^{(0)}, L^{(0)}$ and $\overline{L}^{(0)}$ correspond to the generators defined in eq. (4.9) and

$$\begin{aligned} G_m^{(1)} &= \frac{1}{\sqrt{2}\pi} \oint \frac{dz}{iz} z^m : \Big[\frac{k}{8\pi} b_{ij}(X) \psi_+^j P_+^i + \frac{1}{2}\rho_{ij}(X)\psi_+^i P_-^j \\ &- \frac{i}{2} w_{i,n\ell}^{(2)lin.} \psi_+^i \psi_-^n \psi_-^\ell + \frac{i}{2} w_{i,n\ell}^{(1)lin.} \psi_+^i \psi_+^n \psi_+^\ell \\ &+ \frac{ik}{24\pi} T_{in\ell}^{lin.} \psi_+^i \psi_+^n \psi_+^\ell \Big] : \end{aligned} \tag{5.5}$$

$$\begin{aligned} \overline{G}_m^{(1)} &= -\frac{1}{\sqrt{2}\pi} \oint \frac{d\overline{z}}{i\overline{z}} \overline{z}_m : \Big[\frac{k}{8\pi} b_{ij}(X) \psi_-^j P_-^i + \frac{1}{2}\rho_{ij}(X)\psi_-^i P_+^j \\ &- \frac{i}{2} w_{i,n\ell}^{(2)lin.} \psi_-^i \psi_-^n \psi_-^\ell + \frac{i}{2} w_{i,n\ell}^{(1)lin.} \psi_-^i \psi_+^n \psi_+^\ell \\ &+ \frac{ik}{24\pi} T_{in\ell}^{lin.} \psi_-^i \psi_-^n \psi_-^\ell \Big] : \end{aligned} \tag{5.6}$$

$$\begin{aligned} L_m^{(1)} &= \frac{1}{2\pi} \oint \frac{dz}{iz} z^m : \Big[\frac{1}{2}\rho_{ij}(X) P_+^i P_-^j - i w_{i,n\ell}^{(2)lin.} \psi_-^n \psi_-^\ell P_+^i \\ &+ \frac{i}{2} w_{i,n\ell}^{(1)lin.} \psi_+^n \psi_+^\ell \left(P_+^i + P_-^i \right) \\ &- \frac{1}{4} R_{ijk\ell}^{(1)lin.} \psi_-^i \psi_-^k \psi_+^k \psi_+^\ell \Big] : \end{aligned} \tag{5.7}$$

$$\begin{aligned} \overline{L}_m^{(1)} &= -\frac{1}{2\pi} \oint \frac{d\overline{z}}{i\overline{z}} \overline{z}^m : \Big[\frac{1}{2}\rho_{ij} P_+^i P_-^i + i w_{i,n\ell}^{(1)lin.} \psi_+^n \psi_+^\ell P_-^i \\ &- \frac{i}{2} w_{i,n\ell}^{(2)lin.} \psi_-^n \psi_-^\ell \left(P_+^i + P_-^i \right) \\ &- \frac{1}{4} R_{ijk\ell}^{(1)lin.} \psi_-^i \psi_-^j \psi_+^k \psi_+^\ell \Big] : \end{aligned} \tag{5.8}$$

Now on we choose Ramond (periodic) boundary condition for the world–sheet fermions. The Fourier expansion of the backgrounds are

$$b_{ij}(X) = \int \frac{d^D p}{(2\pi)^D} b_{ij}(p) : e^{ip\cdot X} : \tag{5.9}$$

$$\rho_{ij}(X) = \int \frac{d^D p}{(2\pi)^D} \rho_{ij}(p) : e^{ip\cdot X} : \tag{5.10}$$

The Fourier expansions of connections, antisymmetric field strength and the curvature are

$$w^{(1)lin.}_{i,m\ell} = \frac{1}{2}\int \frac{d^D p}{(2\pi)^D}\Big[ip_\ell \rho_{mi}(p) + \frac{ik}{4\pi} p_i b_{\ell m}(p) - ip_m \rho_{\ell i}(p)\Big] : e^{ip\cdot X} : \tag{5.11}$$

$$w^{(2)lin.}_{i,m\ell} = \frac{1}{2}\int \frac{d^D p}{(2\pi)^D}\Big[ip_\ell \rho_{im}(p) + \frac{ik}{4\pi} p_i b_{\ell m}(b) - ip_m \rho_{i\ell}(p)\Big] : e^{ip\cdot X} : \tag{5.12}$$

$$T^{lin.}_{im\ell} = \int \frac{d^D p}{(2\pi)^D}\Big[ip_i b_{m\ell}(p) + ip_m b_{\ell i}(p) + ip_\ell b_{im}(p)\Big] : e^{ip\cdot X} : \tag{5.13}$$

$$R^{(1)lin.}_{ijk\ell} = \int \frac{d^D p}{(2\pi)^D}\Big[-p_i p_\ell \rho_{kj}(p) + p_k p_j \rho_{ik}(p) - p_k p_i \rho_{j\ell}(p) + p_j p_\ell \rho_{ki}(p)\Big] : e^{ip\cdot X} \tag{5.14}$$

Now we are in a position to compute the algebra of $G_m, \overline{G}_m, L_m$ and $\overline{L}_m$. Note that the brackets involving G_m, $\overline{G}_m$; $G_m, \overline{L}_m$ and $L_m, \overline{L}_m$ vanish at the classical level and such is the case for the free N–S–R string. However, the quantum brackets do not vanish in the interacting case giving rise to anomalies. We need to compute the following set of quantum brackets

$$\Big[G_m, G_n\Big]_+ = \Big[G^{(0)}_m, G^{(0)}_n\Big]_+ + \Big[G^{(0)}_m, G^{(1)}_n\Big]_+ + \Big[G^{(1)}_m, G^{(0)}_m\Big]_+ \tag{5.15}$$

$$\Big[L_m, G_n\Big]_- = \Big[L_m^{(0)}, G_n^{(0)}\Big]_- + \Big[L_m^{(0)}, G_n^{(1)}\Big]_- + \Big[L_m^{(1)}, G_n^{(0)}\Big]_- \tag{5.16}$$

$$\Big[G_m, \overline{G}_n\Big]_+ = \Big[G_m^{(0)}, \overline{G}_n^{(0)}\Big]_+ + \Big[G_m^{(0)}, \overline{G}_n^{(1)}\Big]_+ + \Big[G_m^{(1)}, \overline{G}_n^{(0)}\Big]_+ \tag{5.17}$$

$$\Big[L_m, \overline{G}_n\Big]_- = \Big[L_m^{(0)}, \overline{G}_n^{(0)}\Big]_- + \Big[L_m^{(0)}, \overline{G}^{(1)}\Big]_- + \Big[L_m^{(1)}, \overline{G}_n^{(0)}\Big]_- \tag{5.18}$$

We have retained only linear terms in the background fields on the right hand side of eqs. (5.15)–(5.18). Notice that the first term of eqs. (5.15) and (5.16) are given by eqs. (4.10) and (4.11) and the first term in each of the equations (5.17) and (5.18) vanish. However, terms such as $\Big[G_m^{(0)}, \overline{G}_n^{(1)}\Big]_+$, $\Big[G_m^{(1)}, \overline{G}_n^{(0)}\Big]_+$, $\Big[L_m^{(0)}, \overline{G}_n^{(1)}\Big]_-$ and $\Big[L_m^{(1)}, \overline{G}_n^{(0)}\Big]_-$ do not vanish automatically. Note that in the computation of the anticommutator of $\hat{Q}_{BRST}$ with itself we will have to compute the commutators/anticommutators involving $L_m, \overline{L}_m, G_m$and $\overline{G}_m$ and the ghost fields. As mentioned earlier, the structure of the BRST charge in the ghost sector is the same for free as well as interacting N–S–R string. Therefore, it follows from the algebra of the "zeroth" order generators and the pure ghost sectors that critical dimension is ten, $d = 10$, and $\alpha = \beta = 0$ for the Ramond boundary conditions. Now we are left with the task of computing the commutators/anticommutators of the generators involving zeroeth order and the first order terms in the background fields. It is a tedious but straightforward calculation. The computation involving the last two terms in equations (5.15) through (5.18) are presented below in the final form. Some of the steps involved in these computations are given in Appendix B.

$$\begin{aligned}
&\Big[G_m^{(0)}, G_n^{(1)}\Big]_+ + \Big[G_m^{(1)}, G_n^{(0)}\Big]_+ \\
= \;& 2L_{m+n}^{(1)} + \frac{1}{2}\bigg(\frac{k}{32\pi^2}\oint \frac{dz'}{iz'}\frac{d}{dz}\Big[: -\partial^2 b_{\ell m}\left(X\left(z\right)\right)\psi_+^m(z)\psi_+^\ell(z) : z^m z'^n \left(z + z'\right) \\
- \;& : \partial^2 b_{\ell m}\left(X\left(z'\right)\right)\psi_+^m\left(z'\right)\psi_+^\ell\left(z'\right) : z^{m-1} z'^{n+1}\left(z + z'\right)\Big]_{z=z'}
\end{aligned}$$

$$+ \quad \frac{k}{64\pi^2}\oint\frac{dz'}{iz'}\Big[: -\partial^2 b_{\ell m}\left(X\left(z'\right)\right)\psi_-^m\left(\overline{z}\right)\psi_-^\ell\left(\overline{z}\right) : z^m z'^n\left(z+z'\right)$$

$$- \quad : \partial^2 b_{\ell m}\left(X\left(z'\right)\right)\psi_-^m\left(\overline{z}'\right)\psi_-^\ell\left(\overline{z}'\right) : z^{m-1}z'^{n+1}\left(z+z'\right)\Big]_{z=z'}\Big) \tag{5.19}$$

$$\left[L_m^{(0)}, G_n^{(1)}\right]_- + \left[L_m^{(1)}, G_n^{(0)}\right]_-$$

$$= \left(\frac{m}{2}-n\right)G_{m+n}^{(1)} + \frac{1}{2\sqrt{2}}\Big(\frac{k}{64\pi^2}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[- : \partial^2 b_{\ell m}\left(X\left(z'\right)\right)\psi_+^m\left(z'\right)P_+^\ell\left(z'\right) : z^{m-1}z'^{n+2}\Big]_{z=z'}$$

$$+ \quad \frac{1}{16\pi}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[: -\partial^2\rho_{\ell m}\left(X\left(z'\right)\right)\psi_+^\ell\left(z'\right)P_-^\ell\left(z'\right) : z^{m-1}z'^{n+2}\Big]_{z=z'}$$

$$+ \quad \frac{1}{32\pi}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[: \Big(-i\partial_\ell\partial^2\rho_{qm} - \frac{ik}{4\pi}\partial_q\partial^2 b_{m\ell} + i\partial_m\partial^2\rho_{q\ell}\Big)\left(X\left(z'\right)\right)$$

$$\psi_+^q\left(z'\right)\psi_-^m\left(z'\right)\psi_-^\ell\left(z'\right) : z^{m+1}z'^{n+2}\Big]_{z=z'}$$

$$+ \quad \frac{1}{96\pi^2}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[: \Big(-i\partial_\ell\partial^2 b_{qm} - i\partial_q\partial^2 b_{m\ell} + i\partial^2 b_{q\ell}\Big)\left(X\left(z'\right)\right)$$

$$\psi_+^q\left(z'\right)\psi_+^m\left(z'\right)\psi_+^\ell\left(z'\right) : z^{m-1}z'^{n+2}\Big]_{z=z'}$$

$$+ \quad \frac{1}{32\pi}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[: -\partial^2\rho_{m\ell}\left(X\left(z'\right)\right)\psi_+^m\left(z'\right)\Big(P_+^\ell\left(z'\right) + p_-^\ell\left(z'\right)\Big) : z^m z'^n\left(z+z'\right)\Big]_{z=z'}$$

$$- \quad \frac{1}{32\pi}\oint\frac{dz'}{iz'}\frac{d}{dz}\Big[: \Big(-i\partial_m\partial^2\rho_{k\ell}\left(X\left(z'\right)\right) - i\partial_\ell\partial^2\rho_{km}\left(X\left(z'\right)\right)\Big)$$

$$\psi_-^m\left(\overline{z}'\right)\psi_-^\ell\left(\overline{z}'\right)\psi_+^k\left(z'\right) : z^m z'^n\left(z+z'\right)\Big]_{z=z'}\Big) \tag{5.20}$$

$$\left[G_m^{(0)}, \overline{G}_n^{(1)}\right]_+ + \left[G_m^{(1)}, \overline{G}_n^{(0)}\right]_+$$

$$= -\frac{1}{2}\Big(\frac{1}{16\pi}\oint\frac{d\overline{z}'}{i\overline{z}'}\overline{z}'^n\frac{d}{dz}\Big[: -\partial^2\rho_{\ell m}\left(X\left(z'\right)\right)\psi_-^m\left(\overline{z}'\right)\psi_+^\ell\left(z'\right) : z^{m+1}z'\left(z+z'\right)\Big]$$

$$- \quad \frac{1}{16\pi}\oint\frac{dz}{iz}z^m\frac{d}{d\overline{z}'}\Big[: -\partial^2\rho_{\ell m}\left(X\left(z'\right)\right)\psi_-^m\left(\overline{z}'\right)\psi_+^\ell\left(z'\right) : \overline{z}'^n\left(\overline{z}+\overline{z}'\right)\Big]_{\overline{z}=\overline{z}}\Big) \tag{5.21}$$

$$\left[L_m^{(1)}, \overline{G}_n^{(0)}\right]_- + \left[L_m^{(0)}, \overline{G}_n^{(1)}\right]_-$$

$$= -\frac{1}{2\sqrt{2}}\Big(\frac{k}{16\pi^2}\oint\frac{d\overline{z}'}{i\overline{z}'}\overline{z}'^n\frac{d}{dz}\Big[: \Big(-i\partial_\ell\partial^2 b_{qm}$$

$$
\begin{aligned}
&- \; i\partial_q\partial^2 b_{m\ell} - i\partial_m\partial^2 b_{q\ell}\Big)\left(X\left(z'\right)\right)\psi_-^q\left(\overline{z}'\right)\psi_-^m\left(\overline{z}'\right)\psi_-^\ell\left(\overline{z}'\right) : z^{m-1}z'^2\Big]_{z=z'} \\
&- \; \frac{1}{16\pi}\oint\frac{d\overline{z}'}{i\overline{z}'}\overline{z}'^n\frac{d}{dz}\Big[: \Big(- i\partial_\ell\partial^2\rho_{mq} - \frac{ik}{4\pi}\partial_q\partial^2 b_{\ell m} \\
&- \; i\partial_m\partial^2\rho_{\ell q}\Big)\left(X\left(z'\right)\right)\psi_-^q\left(\overline{z}'\right)\psi_+^m\left(z'\right)\psi_+^\ell\left(z'\right) : z^{m-1}z'^2\Big]_{z=z'} \\
&+ \; \frac{1}{8\pi}\oint\frac{dz}{iz}z^m\frac{d}{d\overline{z}'}\Big[: \Big(- \partial^2\rho_{\ell m}\left(X\left(z'\right)\right)\psi_-^m\left(\overline{z}'\right)P_+^\ell\left(z'\right)\Big) : \overline{z}'^{n-1}\overline{z}\left(\overline{z}+\overline{z}'\right)\Big]_{z=z'} \\
&+ \; \frac{1}{32\pi}\oint\frac{dz}{iz}z^m\frac{d}{dz'}\Big[: \Big(- i\partial_k\partial^2\rho_{i\ell}\left(X\left(z'\right)\right) - i\partial_\ell\partial^2\rho_{ki}\left(X\left(z'\right)\right)\Big) \\
&\quad \psi_-^i\left(\overline{z}'\right)\psi^k\left(z'\right)\psi_+^\ell\left(z'\right) : \overline{z}'^{n-1}\overline{z}\left(\overline{z}+\overline{z}'\right)\Big]_{z=z'}\Big)
\end{aligned}
\tag{5.22}
$$

Notice that X^i is a function of both z and $\overline{z}$ and we have not exhibited the explicit $\overline{z}$ dependence for notational simplicity. The derivatives, acting on the linearized background fields, such as ∂_ℓ and ∂^2 are to be understood as $\frac{\partial}{\partial X^\ell}$ and $\frac{\partial^2}{\partial X^i \partial X^j}\eta^{ij}$ respectively. We have not explicitly displayed the terms whose coefficients are $\partial_\ell\rho^{\ell m}$ and $\partial_m\rho^{\ell m}$ in equations (5.19)–(5.22) since these terms disappear when we impose the transversality conditions on the massless backgrounds.

$$\partial_\ell\rho^{\ell m} = \partial_m\rho^{\ell m} = 0 \tag{5.23}$$

The computation of : $\hat{Q}^2_{BRST}$: essentially involves the quantum superconformal algebra. It is worthwhile to point out that the algebras presented in (5.19)–(5.22) demonstrate the existence of q–number anomalies. These anomalies vanish when

$$\partial^2\rho^{ij} = 0 \tag{5.24}$$

together with the transversality condition (5.23) satisfied by the linearized background fields. In other words, the quantum BRST charge is nilpotent when the massless excitations of the string are transverse and they satisfy the equations of motion along with the requirements of critical dimensions and intercepts, i.e. $d = 10$ and $\alpha = \beta = 0$.

VI. Summary and Conclusions

We have studied the propagation of the N–S–R string (closed type II) propagating in an arbitrary background and have quantized the theory following the formalism of BFV. We have only considered graviton and antisymmetric tensor background; however, our procedure can be extended easily to account for the dilaton background. As was demonstrated explicitly in I, the N-S-R string satisfies classical superconformal algebra in the presence of arbitrary massless background fields. However, when we compute the quantum superconformal algebra in the weak field linearized approximation, we encounter both c–number and q–number anomalies. The nilpotency of the BRST charge forces us to have a ten dimensional space time with zero intercept for the Ramond boundary conditions in the case of free NSR string. In the process of this computation we find that the super–Virasoro algebras give rise to new q–number anomalies mentioned earlier. These anomalies disappear when the massless backgrounds satisfy transversality condition and the equations of motion, besides the requirement of critical dimension and zero intercept condition which remove the c–number anomalies. Thus, the background encitations are not allowed to propagate in any arbitrary configurations and the mass–shell and transversality conditions are imposed as a consequence of superconformal invariance, i.e. : $\hat{Q}^2_{BRST} := 0$. We have carried out the calculation with Ramond fermions; however, the same technique is applicable to the Neveu–Schwarz fermions and computation simplifies considerably due to the absence of zero modes for the fermion sector.

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Appendix A

Notations and conventions:

Greek alphabets μ, ν, etc. are world sheet indices. The indices $i, j, \cdots$ are the target manifold world indices and $A, B, \cdots$ denote thetangent space indices in the target manifold. The two–dimensional Dirac matrices are

$$\rho^0 = \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\rho^1 = -i\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\rho_5 = \rho^0\rho^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.1}$$

The charge conjugation matrix C satisfies the relation $C = -C^T = -C^{-1}$ and we have $C = \rho^1$. The connection

$$\Gamma^i_{jk} = \frac{1}{2} G^{i\ell} \left(\partial_k G_{\ell j} + \partial_j G_{\ell k} - \partial_\ell G_{jk} \right) \tag{A.2}$$

and G_{ij} being the metric in the target manifold. The antisymmetric field strength tensor is

$$T_{ijk} = \left(\partial_k B_{ij} + \partial_i B_{jk} + \partial_j B_{ki} \right) \tag{A.3}$$

The Riemann curvature tensor is

$$R^i_{jk\ell} = \partial_\ell \Gamma^i_{jk} - \partial_k \Gamma^i_{j\ell} + \Gamma^m_{jk}\Gamma^i_{m\ell} - \Gamma^m_{j\ell}\Gamma^i_{km} \tag{A.4}$$

Our metric convention is $(+,-)$ for the world-sheet and $(+,-,-,\cdots)$ for the space time and $\epsilon^{01} = 1$.

Appendix B

Here we summarize the essential results that allow us computations in the interaction representation. Bosons:

$$X^i(\sigma,\tau) = q^i + \frac{1}{2}P^i\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\left(\alpha_n^i e^{-in(\tau+\sigma)} + \overline{\alpha}_n^i e^{-in(\tau-\sigma)}\right) \qquad (B.1)$$

$$P_+^i(\tau=0,\sigma) \equiv \left(\dot{X}^i + X'^i\right)(\tau=0,\sigma) = \frac{1}{2}P^i + \sum_{n\neq 0}\alpha_n^i z^{-n} \qquad (B.2)$$

$$P_-(\tau=0,\sigma) \equiv \left(\dot{X}^i - X'^i\right)(\tau=0,\sigma) = \frac{1}{2}P^i + \sum_{n\neq 0}\overline{\alpha}_n^i \overline{z}^{-n} \qquad (B.3)$$

with $z = e^{i\sigma}$. The canonical commutation relations lead to

$$\left[q^i, p_j\right] = \delta_j^i$$
$$\left[\alpha_n^i, \alpha_m^j\right] = \left[\overline{\alpha}_n^i, \overline{\alpha}_m^j\right] = \delta_{m+n,0}\eta^{ij} \qquad (B.4)$$

World–sheet fermions: i) Ramond sector (periodic boundary condition).

$$\psi_+^A = \frac{1}{2}\left(\Gamma^A + \sqrt{2}\sum_{n\neq 0} d_n^A z^{-n}\right) \qquad (B.5)$$

$$\psi_-^A = \frac{i}{2}\left(\Gamma^A + \sqrt{2}\sum_{n\neq 0} \overline{d}_n^A \overline{z}^{-n}\right) \qquad (B.6)$$

with

$$\left[d_n^A, d_m^B\right]_+ = \left[\overline{d}_n^A, \overline{d}_m^B\right]_+ = \eta^{AB}\delta_{m+n,0} \qquad (B.7)$$

Γ^A's are the zero modes satisfying $\left[\Gamma^A, \Gamma^B\right]_+ = 2\eta^{AB}$.

ii) Neveu–Schwarz sector.

$$\psi_+^A = \frac{1}{\sqrt{2}}\sum_{n=z+\frac{1}{2}} b_n^A z^{-n} \qquad (B.8)$$

$$\psi_-^A = \frac{i}{\sqrt{2}}\sum_{n=Z+\frac{1}{2}} \overline{b}_n^A z^{-n} \qquad (B.9)$$

$$\left[b^A_m, b^B_n\right]_+ = \left[\bar{b}^A_n, \bar{b}^B_m\right]_+ = \delta_{m+n,0}\eta^{AB} \tag{B.10}$$

Normal ordering prescriptions.

$$P^i_+(z)P^j_+(z') =: P^i_+(z)P^j_+(z') : +\frac{zz'}{(z-z')^2}\eta^{ij}, |z| > |z'| \tag{B.11}$$

$$P^i_-(\bar{z}') =: P^i_-(\bar{z}') : +\frac{\overline{zz'}}{(\bar{z}-\bar{z}')^2}\eta^{ij}, |\bar{z}| > |z'| \tag{B.12}$$

$$P^i_+(z) : e^{ik\cdot X(z',\bar{z}')} :=: P^i_+(z)e^{ik\cdot X(z',\bar{z}')} : +\frac{k^i z'}{2(z-z')} : e^{ik\cdot X(z',\bar{z}')} :, |z| > |z'| \tag{B.13}$$

$$: e^{ik\cdot X(z',\bar{z}')} : P^i_+(z) =: e^{ik\cdot X(z',\bar{z}')}P^i_+(z) : -\frac{z'k^i}{2(z'-z)} : e^{ik\cdot X(z',\bar{z}')} :, |z'| > |z| \tag{B.14}$$

$$P^i_-(\bar{z}) : e^{ik\cdot X(z',\bar{z}')} := P^i_-(\bar{z})e^{ik\cdot X(z',\bar{z}')} : +\frac{k^i\bar{z}'}{2(\bar{z}-\bar{z}')} : e^{ik\cdot X(z',\bar{z}')} :, |\bar{z}| > |\bar{z}'| \tag{B.15}$$

$$: e^{ik\cdot X(z',\bar{z}')} : P^i_-(\bar{z}) =: e^{ik\cdot X(z',\bar{z}')}P^i_-(\bar{z}) : -\frac{k^i\bar{z}'}{2(\bar{z}-\bar{z}')} : e^{ik\cdot X(z',\bar{z}')} :, |\bar{z}'| > |\bar{z}| \tag{B.16}$$

Above relations hold good for bosons. Now for fermions.

a) Ramond sector

$$\psi^A_+(z)\psi^B_+(z) =: \psi^A_+(z)\psi^B_+(z') : +\frac{(z+z')}{4(z-z')}\eta^{AB}, |z| > |z'| \tag{B.17}$$

$$\psi^A_-(\bar{z})\psi^B_-(\bar{z}') =: \psi^A_-(\bar{z})\psi^B_-(\bar{z}') : -\frac{(\bar{z}+\bar{z}')}{4(\bar{z}-\bar{z}')}\eta^{AB}, |\bar{z}| > |\bar{z}'| \tag{B.18}$$

b) Neveu–Schwarz sector

$$\psi^A_+(z)\psi^B_+(z') =: \psi^A_+(z)\psi^B_+(z') : +\frac{(zz')^{\frac{1}{2}}}{2(z-z')}\eta^{AB}, |z| > |z'| \tag{B.19}$$

$$\psi^A_-(\bar{z})\psi^B_-(\bar{z}') =: \psi^A_-(\bar{z})\psi^B_-(\bar{z}') : -\frac{(\overline{zz'})^{\frac{1}{2}}}{2(\bar{z}-\bar{z}')}\eta^{AB}, |\bar{z}| > |z'| \tag{B.20}$$

Now we can compute the commutation/anticommutation relations involving any generator with the aid of the normal ordering prescriptions (B.11) - (B.20). Let us look at

$$\Big[G_m, G_n\Big]_+ = \Big[G^{(0)}_m, G^{(0)}_n\Big]_+ + \Big[G^{(0)}_m, G^{(1)}_n\Big]_+ + \Big[G^{(1)}_m, G^{(0)}_n\Big]_+$$

as an example.

$$G^{(0)}_m = \frac{1}{\sqrt{2}\pi}\oint \frac{dz}{iz} z^m : \psi^A_+(z) P^i_+(z) : \eta_{Ai}$$

$$\begin{aligned} G^{(0)}_m G^{(0)}_n = \frac{1}{2\pi^2}\oint \frac{dz}{iz} z^m \oint \frac{dz'}{iz'} z'^n \Big[&: \psi^A_+(z) P^i_+(z)\eta_{Ai}\psi^B_+(z')P^i_+\eta_{Bj} : \\ &+ \frac{zz'}{(z-z')^2}\eta^{ij} : \psi^A_+(z)\psi^B_+(z')\eta_{Ai}\eta_{Bj} : \\ &+ \frac{(z+z')}{4(z-z')}\eta^{AB} : P^i_+(z)P^j_+(z') : \eta_{Ai}\eta_{Bj} \\ &+ \frac{zz'(z+z')}{4(z-z')^3} d\Big] \quad for \ |z| > |z'| \end{aligned} \tag{B.21}$$

where d is the space–time dimensions. Similarly, we can obtain an expression for $G^{(0)}_n G^{(0)}_m$. Then it is straightforward to calculate the $\Big[G^{(0)}_m, G^{(0)}_n\Big]_+$ using the method of residues which gives (4.10). In order to compute $\Big[G^{(0)}_m, G^{(1)}_n\Big]_+$, we need to define first the product

$$\begin{aligned} G^{(0)}_m G^{(1)}_n = \frac{1}{2\pi^2}\oint \frac{dz}{iz}\oint \frac{dz}{iz} z^m \oint \frac{dz'}{iz'} z'^n \int \frac{d^D p}{(2\pi)^D}\Big[&: \psi^A_+(z)P^i_+(z) : \eta_{Ai} \\ + \frac{k}{8\pi} b_{qj}(p) : e^{ip\cdot X(z',\bar{z}')}\psi^j_+(z')P^q_+(z') : &+ \frac{1}{2}\rho_{qj}(p) : e^{ip\cdot X(z',\bar{z}')}\psi^q_+(z')P^j_-(\bar{z}') : \\ -\frac{i}{2}\Big\{ ip_\ell \rho_{jm}(p) + \frac{ik}{4\pi}p_j b_{m\ell}(p) - ip_m\rho_{j\ell}(p)\Big\} &: e^{ip\cdot X(z',\bar{z}')}\psi^j_+(z')\psi^m_-(\bar{z}')\psi^\ell_-(\bar{z}') : \\ +\frac{i}{2}\left(ip_\ell\rho_{mj}(p) + \frac{ik}{4\pi}p_j b_{\ell m}(p) - ip_m\rho_{\ell j}(p)\right) &: e^{ip\cdot X(z',\bar{z}')}\psi^j_+(z')\psi^m(z')\psi^\ell_+(z') : \\ +\frac{ik}{24\pi}\{ ip_j b_{m\ell}(p) + ip_m b_{\ell j}(p) + ip_\ell b_{jm}(p)\} : e^{ip\cdot X(z',\bar{z}')} &\psi^j_+(z')\psi^m_+(z')\psi^\ell_+(z') : \Big)\Big], \quad |z| > |z'| \end{aligned} \tag{B.22}$$

We shall demonstrate how the contraction of a single term takes place in (B.22) as an example.

$$
\begin{aligned}
&: \psi_+^A(z) P_+^i(z) : \eta_{Ai} : \frac{k}{8\pi} b_{qj}(p) : e^{ip\cdot X(z',\bar{z}')} \psi_+^j(z') P_+^q(z') : \\
&= \frac{k}{8\pi} b_{qj} \Big(: \psi_+^A(z) P_+^i(z) \eta_{Ai} e^{ip\cdot X(z,z')} \psi_+^j(z') P_+^q(z') : \\
&\quad + : P_+^i(z) e^{ip\cdot X(z',\bar{z}')} P_+^q(z') : \eta_{Ai} \eta^{Aj} \frac{(z+z')}{4(z-z')} \\
&\quad + : \psi_+^A(z) e^{ip\cdot X(z',\bar{z}')} \psi_+^j(z') : \frac{\eta_{Ai}\eta^{iq} z z'}{(z-z')^2} \\
&\quad + : \psi_+^A(z) e^{ip\cdot X(z',\bar{z}')} \psi_+^j(z' P_+^q(z') : \frac{P_+^i z'}{2(z-z')} \\
&\quad + : e^{ip\cdot X(z',\bar{z}')} : \eta_{Ai} \eta^{Aj} \eta^{iq} \frac{z z'(z+z')}{(z-z')^3} \\
&\quad + : e^{ip\cdot X(z',\bar{z}')} P_+^q(z') : \eta_{Ai} \eta^{Aj} \frac{P_+^i z'(z+z')}{8(z-z')^3} \Big)
\end{aligned} \tag{B.23}
$$

Similar contraction is carried out for all other terms in (B.22) and finally $\left[G_m^{(0)}, G_n^{(1)}\right]_+$ and $\left[G_m^{(1)}, G_n^{(0)}\right]_+$ are computed.

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