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Intensity fluctuations in thermal light with orthogonally polarised multiple-peak spectrum

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Abstract. The moment generating function of the integrated light intensity of thermal radiation having multiple peak spectrum is obtained. Cases of two-peak and three-peak spectra where different peaks are in orthogonal states of polarisation are considered. The moment generating function is shown to be the product of two simpler generating functions.

Keywords. Photon counting distribution; polarisation; coherency matrix; spectral profile; intensity fluctuation; multiple-peak spectrum.

1. Introduction

Studies in the photoelectron counting statistics provide one of the most convenient methods of studying the statistical properties of light beams. We study the distribution of electron pulses produced by a detector during a specified interval of time. Consider a plane, quasimonochromatic, stationary light wave incident normally on an ideal photo-detector. If $I(t)$ represents the light intensity at time t, then under usual experimental conditions, the probability $p(n, T)$ that n photoelectrons will be released in a time interval T is given by Mandel's formula (Mandel 1958, 1959)

$$p(n, T) = \int_0^\alpha \frac{(\alpha W)^n}{n!} \exp(-\alpha w)\, P(W)\, dw, \tag{1}$$

where
$$W = \int_t^{t+T} I(t')\, dt'. \tag{2}$$

α is the quantum efficiency of the detector and $P(W)$ is the probability density of the fluctuating time integrated intensity W. We note that $p(n, T)$ is the Poisson transform of $P(W)$.

Although the photoelectron counting statistics with thermal light has been studied extensively no general formulae are available for $P(W)$ except in some special cases, such as polarised thermal light with a single peak Lorentzian spectrum or that with a rectangular spectral profile. For polarised thermal light with an arbitrary spectral profile one can obtain an expression for $P(W)$ for counting times either very small or very large as compared to the coherence time (Rice 1945). However, this expression is only approximate and is not valid when time T becomes of the order of the coherence time.

The problem of determining $P(W)$ may be reduced to solving an associated integral equation (Kac and Siegert 1947; Slepian 1958). For Gaussian processes the wave field $V(t)$ is a complex analytic signal. The correlation function is written in the following form

$$\Gamma(\tau) = \langle V^*(t)\, V(t+\tau)\rangle. \tag{3}$$

Angular brackets denote the ensemble average. Let us assume that $\lambda_1, \lambda_2, \ldots$ are the eigenvalues of integral equation

$$\int_0^T \Gamma(t-t')\, \phi^{(k)}(t')dt' = \lambda_k \phi^{(k)}(t). \tag{4}$$

The generating function $G(s)$ defined as

$$G(s) = \langle \exp(-\alpha s W)\rangle, \tag{5}$$

is then given by

$$G(s) = \prod_k (1+\alpha s\lambda_k)^{-1}, \tag{6}$$

and $P(W)$ is the inverse Laplace transform of $G(s)$.

In the case of partially polarised radiation, the wave field $V(t)$ is a vector analytic signal and instead of $\Gamma(\tau)$ one now considers the coherence tensor $\Gamma_{ij}(\tau)$. The integral equation is now replaced by a matrix integral equation

$$\int_0^T \sum_{i=x,y} \Gamma_{ij}(t-t')\, \phi_i^{(k)}(t')\, dt' = \lambda_k\, \phi_j^{(k)}(t). \tag{7}$$

Intensity fluctuations in partially polarized light were first studied by Mandel (1963). Later on Helstrom (1964) followed the method first given by Kac and Siegert (1947) for the calculation of $P(W)$ in the long and short counting time-interval limits. Jaiswal and Mehta (1969) diagonalised the coherency matrix for partially polarised radiation with special reference to Gaussian Lorentzian light.

In an earlier paper, Mehta and Gupta (1975) have considered the problem of determining the moment generating function of W when the incident plane polarised thermal light has a multiple peak spectrum. In this paper we consider the problem of intensity fluctuations in thermal light for the cases when the different peaks of the spectrum are orthogonally polarised. In § 2 we consider the case of a light beam with two peak spectral profile when the two peaks are orthogonally polarised. In § 3 we consider the case of three peak spectrum where one of the peaks is orthogonally polarised to the other two peaks which are Lorentzian with same height and line width. This latter case is applicable, for example, to the light resulting from Zeeman splitting of a spectral line.

2. Orthogonally polarised two peak spectral profile

We consider the incoherent superposition of two peaks which are orthogonally polarised to each other, the line shapes and their separation being arbitrary. The plane wave field $\mathbf{V}(t)$ incident on the photodetector may then be expressed in the form

$$\mathbf{V}(t) = V_1(t)\hat{e}_1 + V_2(t)\hat{e}_2, \tag{8}$$

where $\hat{e}_1$ and $\hat{e}_2$ are the two orthogonal unit vectors perpendicular to the direction of propagation. These vectors are in general complex and represent the state of polarisation of each spectral peak separately. If these peaks are linearly polarised in x and y directions then $\hat{e}_1$ and $\hat{e}_2$ are real unit vectors in these directions. Orthogonality of $\hat{e}_1$ and $\hat{e}_2$ implies

$$\hat{e}_i^* \cdot \hat{e}_j = \delta_{ij}. \tag{9}$$

The instantaneous intensity $I(t)$ is now given by

$$\begin{aligned} I(t) &= |V(t)|^2 = |V_1(t)|^2 + |V_2(t)|^2, \\ &= I_1(t) + I_2(t). \end{aligned} \tag{10}$$

The integrated intensity W may then be written as

$$W = W_1 + W_2 \tag{11}$$

where
$$W_i = \int_t^{t+T} I_i(t')\, dt'. \tag{12}$$

Since we assume $V_1(t)$ and $V_2(t)$ to be statistically independent, W_1 and W_2 are also statistically independent random variables and $P(W)$ is therefore given by

$$P(W) = \int P_1(W_1)\, P_2(W-W_1) dW_1, \tag{13}$$

where $P_1(W_1)$ and $P_2(W_2)$ are the probability densities of the variables W_1 and W_2 respectively. The generating function as defined in equation (6)

$$G(s) = \int P(W) \exp(-\alpha s W)\, dW,$$

is then a product of two generating functions

$$G(s) = G_1(s) \cdot G_2(s), \tag{14}$$

where $G_1(s)$ and $G_2(s)$ are the generating functions corresponding to the intensity fluctuations in the two peaks individually. Using (5), we therefore obtain

$$G(s) = \prod_k \frac{1}{(1+\alpha s\lambda_k)} \prod_{k'} \frac{1}{(1+\alpha s\lambda_{k'})}. \tag{15}$$

λ_k and $\lambda_{k'}$ are the eigenvalues of the following integral equations

$$\int_0^T \Gamma_1(t-t')\,\phi_1^{(k)}(t')dt' = \lambda_k\,\phi_1^{(k)}(t), \tag{16}$$

and

$$\int_0^T \Gamma_2(t-t')\,\phi_2^{(k')}(t')dt' = \lambda_{k'}\phi_2^{(k')}(t), \tag{17}$$

where

$$\Gamma_i(t-t') = \langle V_i^*(t')V_i(t)\rangle. \tag{18}$$

Thus we see that from the knowledge of the generating function for radiation from the individual peaks of the spectrum, one can obtain information about the fluctuations in the time integrated light intensity. For example when the spectral profiles of the two peaks are Lorentzian with half widths σ_1 and σ_2 we obtain (Mehta 1970, Bedard 1966)

$$G(s) = G_1(s)\,G_2(s), \tag{19}$$

where

$$G_i(s) = \exp(\sigma_i T)\left[\cosh Z_i + \tfrac{1}{2}\left(\frac{\sigma_i T}{Z_i} + \frac{Z_i}{\sigma_i T}\right)\sinh Z_i\right]^{-1} \tag{20}$$

and

$$Z_i^2 = \sigma_i^2 T^2 + 2\sigma_i\,\alpha\,\langle I_i\rangle\, sT^2 \tag{20a}$$

Similarly one can calculate $G(s)$ for the case when the peaks are rectangular in shape using the calculations done by Mehta and Mehta (1973).

3. Three peak spectral profile

There are many situations in which we observe light having a symmetric spectrum with three spectral lines. We may have, for example, the light resulting from Brillouin scattering or that encountered as a result of Zeeman splitting. The spectrum of the light resulting from Zeeman splitting has three spectral lines. The middle peak of the spectrum is completely polarised orthogonally to the other two peaks which are symmetrically placed on the either side. In our calculations, however, we only require that the two peaks having similar polarisation should be symmetric about some central frequency. The third peak having polarisation orthogonal to the other two can have a different shape. Proceeding in a strictly analogous manner as before one can prove that in this case also

$$G(s) = G_1(s)\cdot G_2(s)$$

$G_1(s)$ and $G_2(s)$ are the generating functions corresponding to the light of the two polarisations. In a particular case when the three peaks are Lorentzian one can obtain the generating function $G(s)$ by using the generating function for the two Lorentzian peak profile (Mehta and Gupta 1975) and that for a single Lorentzian peak (equations (20) and (20a)).

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