

Periodic energy switching of bright solitons in mixed coupled nonlinear Schrödinger equations with linear self and cross coupling terms

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Abstract

The bright soliton solutions of the mixed 2-coupled nonlinear Schrödinger (CNLS) equations with linear self and cross coupling terms have been obtained by identifying a transformation that transforms the corresponding equation to the integrable mixed 2-CNLS equations. The study on the collision dynamics of bright solitons shows that there exists periodic energy switching, due to the coupling terms. This periodic energy switching can be controlled by the new type of shape changing collisions of bright solitons arising in mixed 2-CNLS system, characterized by intensity redistribution, amplitude dependent phase shift and relative separation distance. We also point out that this system exhibits large periodic intensity switching even with very small linear self coupling strengths.

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I. INTRODUCTION

The study on soliton collisions has been receiving sustained attention since the identification of their particle like collision behaviour by Zabusky and Kruskal [1] in 1965. Due to their intriguing collision properties and their robustness against external perturbations solitons find applications in diverse areas of science which encompass the current thrust research areas including nonlinear optics, Bose-Einstein condensates, plasma physics, biophysics [2, 3, 4, 5]. Particularly, it has been shown that soliton propagation in systems such as birefringent fibers [6, 7], multimode fibers [2], fiber couplers [2], photorefractive medium [8], left handed materials [9], Bose-Einstein condensates [10] and continuum limit of Hubbard model in 1D [11] is governed by multicomponent nonlinear Schrödinger type equations which become integrable for specific choices of system parameters [12, 13, 14].

Recent theoretical and experimental studies show that the integrable CNLS system with focusing type nonlinearity exhibits fascinating shape-changing (energy/intensity redistributing) collision of bright solitons characterized by intensity redistribution among the colliding solitons in all the components, as well as amplitude dependent phase-shifts and change in relative separations distances [15, 16, 17]. In such a two soliton collision process there occurs suppression/enhancement of intensity in few components and enhancement/suppression of intensity in the remaining components after interaction. We call such a collision scenario as type-I shape changing collisions (type-I SCC). This interesting collision behaviour has also been experimentally verified in birefringent fibers [18] and in photorefractive media [19]. Further investigations on this type-I SCC has led to exciting applications in collision based optical computing [17, 20, 21, 22]. However the collision scenario is different if the nonlinearities are of mixed type [23, 24], which includes both focusing and defocusing types. The corresponding mixed CNLS equations admit shape changing collision of bright solitons in a quite different manner from the collision scenario of type-I SCC. Very recently, it has been shown that in mixed CNLS equations during a two soliton collision process there is a possibility of either enhancement or suppression of intensity in a given soliton in all the components [24]. Here also the collision process is characterized by intensity redistribution, amplitude dependent phase-shift and relative separation distances. We denote this kind of collision scenario as type-II shape changing collision (type-II SCC). The most important consequence of type-II SCC is the possibility of *soliton amplification* in all the components.

Mixed CNLS equations are not only of mathematical interest but also of considerable physical significance. In particular, mixed 2-CNLS equations arise as governing equations for electromagnetic pulse propagation in left handed materials with Kerr-type nonlinearity [9] and in the modified Hubbard model in the long-wavelength approximation [11]. It can also be noticed that in the mixed 2-CNLS system, if we assume the fields q_1 and q_2^* (see Eq. (1) below) propagate in the anomalous and normal dispersion regimes, respectively, the self phase modulation (SPM) coefficients are positive and cross phase modulation (XPM) coefficients are negative in both the components. This kind of nonlinearities can be realized in the quadratic nonlinear materials with large phase-matching [25]. The main aim of this paper is to analyse type-II SCC behaviour of bright solitons in a possible integrable extension of mixed 2-CNLS equations involving the linear self and cross coupling terms which can be of physical interest. This study reveals the fact that in this system during the type-II SCC process there also occurs periodic energy switching due to the linear coupling terms which can be suppressed/enhanced by type-II SCC. The distinct feature is that the system exhibits large periodic energy switching between the components with very small linear self coupling strength.

The plan of the paper is as follows. In Sec. II we present the statement of the problem. Bright soliton solutions of the mixed 2-CNLS equations with linear self and cross coupling terms are obtained in Sec. III. Section IV is devoted to a detailed analysis of the role of the coupling terms on type-II SCC. The final section is allotted for conclusion.

II. STATEMENT OF THE PROBLEM

Lazarides and Tsironis [9] have obtained the following set of governing equations for electromagnetic pulse propagation in isotropic and homogeneous nonlinear left handed materials,

$$iq_{1,z} + q_{1,tt} + 2\mu (\sigma_1 |q_1|^2 + \sigma_2 |q_2|^2) q_1 = 0, \quad (1a)$$

$$iq_{2,z} + q_{2,tt} + 2\mu (\sigma_1 |q_1|^2 + \sigma_2 |q_2|^2) q_2 = 0, \quad (1b)$$

by taking the effective permittivity and effective permeability to be intensity dependent and following a reductive perturbational approach. Here q_1 and q_2 are the electric and magnetic field components of the electromagnetic pulse, respectively, the subscripts z and t denote the

partial derivatives with respect to normalized distance and retarded time respectively, while μ is the measure of nonlinearity, σ_1 and σ_2 can be either +1 or -1. From a mathematical point of view the above equation reduces to the integrable mixed CNLS equation presented in Ref. [13] when $\sigma_1 = 1$ and $\sigma_2 = -1$. In Ref. [24] singular and nonsingular bright soliton solutions of Eq. (1) have been obtained. There it has been shown that even though system (1) admits SCC as in Manakov system, the intensity redistribution occurs in a completely different way which is not possible in the Manakov system [6]. As pointed out in the introduction we denote such a collision picture as type-II SCC. A typical type-II SCC where enhancement (suppression) of intensity occurs in both the components of soliton $S_1(S_2)$ after collision is shown in Fig. 1 (All the quantities in this and rest of the figures are dimensionless). Note that the reverse scenario is also possible.

The next natural step is to study how the linear self coupling resulting from the same component and the cross coupling arising from the other component influence the type-II SCC process. In this regard we consider the following non-dimensional mixed CNLS equations with linear self and cross couplings.

$$iq_{1z} + q_{1tt} + \rho q_1 - \chi q_2 + 2\mu (|q_1|^2 - |q_2|^2) q_1 = 0, \quad (2a)$$

$$iq_{2z} + q_{2tt} - \rho q_2 + \chi q_1 + 2\mu (|q_1|^2 - |q_2|^2) q_2 = 0, \quad (2b)$$

where ρ and χ are the self and cross coupling coefficients, respectively. The above system reduces to the Lindner-Fedyanin system [11] with $\chi = 0$, which is a 1D continuum limit of 2D Hubbard model. We also believe that this mathematical model can be experimentally realized in fiber couplers fabricated from suitably engineered left handed materials. Another physical model associated with Eq. (2) is two component BECs coupled with two photon microwave field where the signs of co-efficients of SPM and XPM can be tuned suitably through Feshbach resonance. In fact, this type of linear couplings can be introduced in BECs by external microwave or radio frequency that induces Rabi or Josephson oscillation between population of two states [26]. Also at this point it is worth to point out that a similar set of equations with focusing nonlinearity ($|q_1|^2 + |q_2|^2$) arises in pulse propagation in twisted birefringent fibers [2, 27].

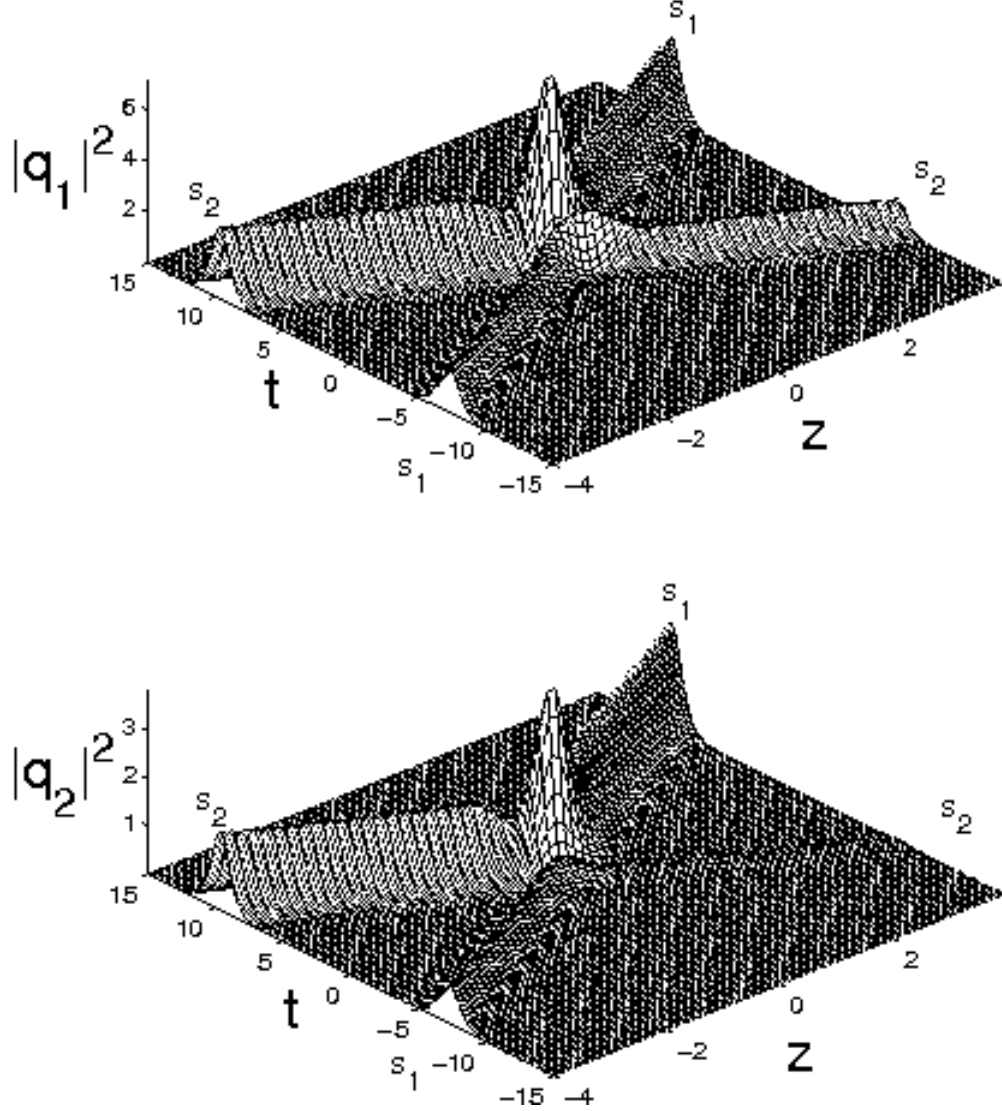


FIG. 1: Shape changing collision of bright solitons in mixed 2-CNLS system.

III. BRIGHT SOLITON SOLUTIONS

To obtain the bright soliton solutions of Eq. (2), we identify a transformation which reduces Eq. (2) to the well known integrable mixed CNLS equations (1) with $\sigma_1 = 1$ and $\sigma_2 = -1$. The transformation can be written as

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \cosh(\frac{\theta}{2})e^{i\Gamma z} & \sinh(\frac{\theta}{2})e^{-i\Gamma z} \\ \sinh(\frac{\theta}{2})e^{i\Gamma z} & \cosh(\frac{\theta}{2})e^{-i\Gamma z} \end{pmatrix} \begin{pmatrix} q_{1m} \\ q_{2m} \end{pmatrix}, \quad (3a)$$

where the real parameters θ and Γ are expressed in terms of the coupling parameters ρ and χ as

$$\theta = \tanh^{-1} \left(\frac{\chi}{\rho} \right), \quad \Gamma = \sqrt{\rho^2 - \chi^2}, \quad \chi \leq \rho. \quad (3b)$$

In Eq. (3a), q_1 and q_2 satisfy Eq. (2) with linear self and cross coupling terms included while q_{1m} and q_{2m} satisfy Eq. (1) in the absence of linear couplings ($\rho = \chi = 0$). It is obvious that if the cross coupling term χ becomes zero the above solution (q_1, q_2) is the same as that of the mixed CNLS equations (1), with $\sigma_1 = -\sigma_2 = 1$, except for a multiplicative phase factor $e^{i\Gamma z}$ in the q_1 component and $e^{-i\Gamma z}$ in the q_2 component. We now confine our analysis to the cases where (q_{1m}, q_{2m}) correspond to bright soliton solutions and analyse the nature of (q_1, q_2) satisfying Eq. (2) through the relation (3).

A. Bright one-soliton solution

With the knowledge of the bright one soliton solution of the integrable mixed 2-CNLS equations for (q_{1m}, q_{2m}) , given in Eq. (6) of Ref. [24], we write down the one soliton solution of Eq. (2) by using the transformation (3) as

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} \cosh(\frac{\theta}{2})e^{i\Gamma z} & \sinh(\frac{\theta}{2})e^{-i\Gamma z} \\ \sinh(\frac{\theta}{2})e^{i\Gamma z} & \cosh(\frac{\theta}{2})e^{-i\Gamma z} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} k_{1R} \text{sech} \left(\eta_{1R} + \frac{R}{2} \right) e^{i\eta_{1I}}, \quad (4a)$$

where

$$\eta_1 = k_1(t + ik_1 z) = k_{1R}(t - 2k_{1I}z) + i(k_{1I}t + (k_{1R}^2 - k_{1I}^2)z) \equiv \eta_{1R} + i\eta_{1I}, \quad (4b)$$

$$A_j = \frac{\alpha_1^{(j)}}{\left[\mu \left(\sigma_1 |\alpha_1^{(1)}|^2 + \sigma_2 |\alpha_1^{(2)}|^2 \right) \right]^{1/2}}, \quad j = 1, 2, \quad (4c)$$

$$e^R = \frac{\mu \left(\sigma_1 |\alpha_1^{(1)}|^2 + \sigma_2 |\alpha_1^{(2)}|^2 \right)}{(k_1 + k_1^*)^2}, \quad \sigma_1 = -\sigma_2 = 1. \quad (4d)$$

In the above expressions the suffices R and I denote the real and imaginary parts, respectively. The one soliton solution (4) for (q_1, q_2) is characterized by three arbitrary complex parameters $\alpha_1^{(1)}$, $\alpha_1^{(2)}$, and k_1 , in addition to the real coupling parameters ρ and χ . Also note that the value of χ is restricted by Eq. (3b) as $|\chi| \leq |\rho|$ since $\tanh \theta = \chi/\rho$. As in the case of mixed CNLS equations, solution (4) can be both singular and nonsingular. The condition for non-singular solution is given by $|\alpha_1^{(1)}| > |\alpha_1^{(2)}|$ [24]. In this work we deal with nonsingular solutions only as they are of specific physical interest.

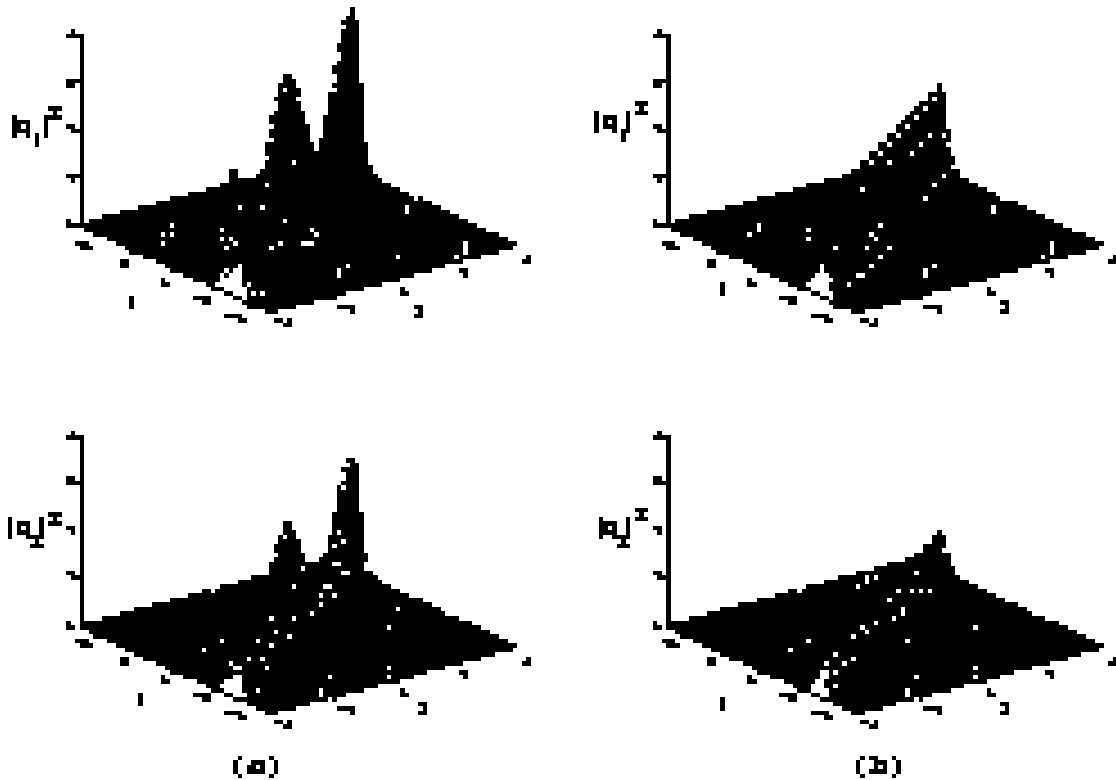


FIG. 2: Intensity plots of bright one soliton solution (4): (a) in the presence of coupling ($\rho, \chi > 0$), (b) in the absence of coupling ($\rho = \chi = 0$).

1. Analysis on bright one-soliton solution

Typical plot of non-singular bright one soliton solution (4) of Eq. (2) with the condition $|\chi| \leq |\rho|$ is shown in Fig. 2. From the figure we observe that the role of the linear coupling terms in (2) is to induce spatially periodic intensity switching between the two components q_1 and q_2 . The periodic oscillations Fig. 2(a) during the intensity switching depends particularly on the difference between the self and cross coupling terms (ρ and χ) in addition to the soliton parameters k_1 , $\alpha_1^{(1)}$ and $\alpha_1^{(2)}$. For comparison we have plotted the corresponding one-soliton solution in the absence of coupling terms in Fig. 2(b). It is interesting to note that this periodic intensity switching can be completely suppressed by suitably choosing A_2 or $\alpha_1^{(2)}$. To see this, we compute the intensity of the soliton in the two components and write

them as

$$\begin{aligned} \left| \frac{q_1}{P} \right|^2 &= k_{1R}^2 \left(|A_1|^2 \cosh^2 \left(\frac{\theta}{2} \right) + |A_2|^2 \sinh^2 \left(\frac{\theta}{2} \right) \right. \\ &\quad \left. + 2|A_1||A_2| \cosh \left(\frac{\theta}{2} \right) \sinh \left(\frac{\theta}{2} \right) \cos(2\Gamma z + Q) \right), \end{aligned} \quad (5a)$$

$$\begin{aligned} \left| \frac{q_2}{P} \right|^2 &= k_{1R}^2 \left(|A_1|^2 \sinh^2 \left(\frac{\theta}{2} \right) + |A_2|^2 \cosh^2 \left(\frac{\theta}{2} \right) \right. \\ &\quad \left. + 2|A_1||A_2| \cosh \left(\frac{\theta}{2} \right) \sinh \left(\frac{\theta}{2} \right) \cos(2\Gamma z + Q) \right), \end{aligned} \quad (5b)$$

where

$$P = \operatorname{sech} \left(k_{1R}(t - 2k_{1I}z) + \frac{R}{2} \right), \quad (5c)$$

$$Q = \tan^{-1} \left(\frac{A_{1I}}{A_{1R}} \right) - \tan^{-1} \left(\frac{A_{2I}}{A_{2R}} \right). \quad (5d)$$

It is clear from the above expressions that the oscillatory term $\cos(2\Gamma z + Q)$ appearing in (5a) and (5b) leads to the periodic oscillations during energy switching. One can also verify that the spatial period of oscillation is $Z = \frac{\pi}{\Gamma}$. Thus for larger Γ the width of spatial oscillations is smaller. Also the amplitude of oscillation ($2|A_1||A_2| \cosh \left(\frac{\theta}{2} \right) \sinh \left(\frac{\theta}{2} \right)$) increases with decreasing Γ due to the dependence of θ on χ and ρ (see Eq. (3b)). We also note from Eqs. (5a) and (5b) that the oscillatory term (third term on the right hand side) vanishes when $|A_2| = 0$, that is $\alpha_1^{(2)} = 0$, or $|A_1| = 0$, that is $\alpha_1^{(1)} = 0$. At a first glance, it seems that the periodic energy switching scenario is similar to that of the Manakov system [6] with linear coupling terms arising in the context of twisted birefringent fibers [27]. But the way in which the switching occurs is different due to the hyperbolic terms in Eqs. (5a) and (5b), since $0 \leq \sinh^2 \theta/2 < \infty, 0 \leq \cosh^2 \theta/2 < \infty$. To be more precise, the amplitude of periodic oscillations and periodic switching of energy between the two components can vary exponentially but restricted by the condition (3b). In this connection, we would like to add that a quite different kind of multi-scale periodic beating of intensities without switching and of different physical origin as compared with the present mixed CNLS case, has been observed during the propagation of multisoliton complexes in the integrable N-CNLS system with focusing type nonlinearities [28].

B. Two soliton solution

The bright two soliton solution of Eq. (2) can be obtained by applying the transformation (3) to the two soliton solution of the integrable mixed CNLS equation given by Eq. (10) in Ref. [24]. The explicit form of the solution is

$$\begin{aligned}
q_1 = & \frac{1}{D} \left(\left(\alpha_1^{(1)} e^{i\Gamma z} \cosh\left(\frac{\theta}{2}\right) + \alpha_1^{(2)} e^{-i\Gamma z} \sinh\left(\frac{\theta}{2}\right) \right) e^{\eta_1} \right. \\
& + \left(\alpha_2^{(1)} e^{i\Gamma z} \cosh\left(\frac{\theta}{2}\right) + \alpha_2^{(2)} e^{-i\Gamma z} \sinh\left(\frac{\theta}{2}\right) \right) e^{\eta_2} \\
& + \left(e^{\delta_{11}} \cosh\left(\frac{\theta}{2}\right) e^{i\Gamma z} + e^{\delta_{12}} \sinh\left(\frac{\theta}{2}\right) e^{-i\Gamma z} \right) e^{\eta_1 + \eta_1^* + \eta_2} \\
& \left. + \left(e^{\delta_{21}} \cosh\left(\frac{\theta}{2}\right) e^{i\Gamma z} + e^{\delta_{22}} \sinh\left(\frac{\theta}{2}\right) e^{-i\Gamma z} \right) e^{\eta_2 + \eta_2^* + \eta_1} \right), \tag{6a}
\end{aligned}$$

$$\begin{aligned}
q_2 = & \frac{1}{D} \left(\left(\alpha_1^{(1)} e^{i\Gamma z} \sinh\left(\frac{\theta}{2}\right) + \alpha_1^{(2)} e^{-i\Gamma z} \cosh\left(\frac{\theta}{2}\right) \right) e^{\eta_1} \right. \\
& + \left(\alpha_2^{(1)} e^{i\Gamma z} \sinh\left(\frac{\theta}{2}\right) + \alpha_2^{(2)} e^{-i\Gamma z} \cosh\left(\frac{\theta}{2}\right) \right) e^{\eta_2} \\
& + \left(e^{\delta_{11}} \sinh\left(\frac{\theta}{2}\right) e^{i\Gamma z} + e^{\delta_{12}} \cosh\left(\frac{\theta}{2}\right) e^{-i\Gamma z} \right) e^{\eta_1 + \eta_1^* + \eta_2} \\
& \left. + \left(e^{\delta_{21}} \sinh\left(\frac{\theta}{2}\right) e^{i\Gamma z} + e^{\delta_{22}} \cosh\left(\frac{\theta}{2}\right) e^{-i\Gamma z} \right) e^{\eta_2 + \eta_2^* + \eta_1} \right), \tag{6b}
\end{aligned}$$

where $\alpha_l^{(j)}$'s are complex parameters and the denominator D is given by

$$D = 1 + e^{\eta_1 + \eta_1^* + R_1} + e^{\eta_1 + \eta_2^* + \delta_0} + e^{\eta_1^* + \eta_2 + \delta_0^*} + e^{\eta_2 + \eta_2^* + R_2} + e^{\eta_1 + \eta_1^* + \eta_2 + \eta_2^* + R_3}. \tag{6c}$$

Various quantities found in Eq. (6) are defined as below following Ref. [24]:

$$\begin{aligned}
\eta_i &= k_i(t + ik_i z), \quad e^{\delta_0} = \frac{\kappa_{12}}{k_1 + k_2^*}, \quad e^{R_1} = \frac{\kappa_{11}}{k_1 + k_1^*}, \quad e^{R_2} = \frac{\kappa_{22}}{k_2 + k_2^*}, \\
e^{\delta_{1j}} &= \frac{(k_1 - k_2)(\alpha_1^{(j)} \kappa_{21} - \alpha_2^{(j)} \kappa_{11})}{(k_1 + k_1^*)(k_1^* + k_2)}, \quad e^{\delta_{2j}} = \frac{(k_2 - k_1)(\alpha_2^{(j)} \kappa_{12} - \alpha_1^{(j)} \kappa_{22})}{(k_2 + k_2^*)(k_1 + k_2^*)}, \\
e^{R_3} &= \frac{|k_1 - k_2|^2}{(k_1 + k_1^*)(k_2 + k_2^*)|k_1 + k_2^*|^2} (\kappa_{11} \kappa_{22} - \kappa_{12} \kappa_{21}), \tag{6d}
\end{aligned}$$

and

$$\kappa_{ij} = \frac{\mu \left(\sigma_1 \alpha_i^{(1)} \alpha_j^{(1)*} + \sigma_2 \alpha_i^{(2)} \alpha_j^{(2)*} \right)}{(k_i + k_j^*)}, \quad i, j = 1, 2, \tag{6e}$$

where $\sigma_1 = 1$ and $\sigma_2 = -1$. This solution represents the interaction of two bright solitons in the presence of self and cross coupling terms. Although the above solution features both

singular and nonsingular solutions in the following we consider only the nonsingular soliton solution which results for the choice [24]

$$\kappa_{11} \geq 0, \quad \kappa_{22} \geq 0, \quad \kappa_{11}\kappa_{22} - |\kappa_{12}|^2 > 0, \quad (7a)$$

$$\frac{1}{2} \sqrt{\frac{\kappa_{11}\kappa_{22}}{k_{1R}k_{2R}}} + \frac{|k_1 - k_2|}{2|k_1 + k_2^*|} \sqrt{\frac{\kappa_{11}\kappa_{22} - |\kappa_{12}|^2}{k_{1R}k_{2R}}} > \frac{|\kappa_{12}|}{|k_1 + k_2^*|}. \quad (7b)$$

and analyse their collision behaviour. In a similar way the multi-soliton solution of Eq. (2) can be obtained by applying the transformation to the multi-soliton solution given in the appendix of [24] with $N = 2$.

IV. SHAPE CHANGING COLLISION OF SOLITONS WITH PERIODIC ENERGY SWITCHING

We have already explained the nature of type-II SCC of bright solitons in Sec. II. In this section, we analyse the influence of linear cross coupling terms on the above mentioned type-II SCC. We perform an asymptotic analysis [24] for the choice $k_{1R}, k_{2R} > 0$ and $k_{1I} > k_{2I}$. To facilitate the understanding of the collision dynamics we consider the intensities of the two colliding solitons in the asymptotic limits at $z \rightarrow -\infty$ (before collision) and $z \rightarrow \infty$ (after collision). In their explicit forms the intensities of solitons as $z \rightarrow \pm\infty$ read as

$$\begin{aligned} \left| \frac{q_j^{n\pm}}{P_n} \right|^2 &= k_{nR}^2 \left(|A_1^{n\pm}|^2 \cosh^2 \left(\frac{\theta}{2} \right) + |A_2^{n\pm}|^2 \sinh^2 \left(\frac{\theta}{2} \right) \right. \\ &\quad \left. + 2|A_1^{n\pm}| |A_2^{n\pm}| \cosh \left(\frac{\theta}{2} \right) \sinh \left(\frac{\theta}{2} \right) \cos(2\Gamma z + Q_n) \right), \end{aligned} \quad (8a)$$

where

$$P_n = \operatorname{sech}(k_{nR}(t - 2k_{nI}z + R_n)), \quad (8b)$$

$$Q_n = \tan^{-1} \left(\frac{A_{1I}^{n\pm}}{A_{1R}^{n\pm}} \right) - \tan^{-1} \left(\frac{A_{2I}^{n\pm}}{A_{2R}^{n\pm}} \right), \quad n = 1, 2, j = 1, 2. \quad (8c)$$

Here the quantities $A_j^- k_{nR}$ and $A_j^+ k_{nR}$, $j, n = 1, 2$, are the amplitudes of the n th soliton in j th component before and after interaction respectively in the absence of linear coupling

($\chi = \rho = 0$), where A_j^n 's take the following forms before and after interaction [24]:

$$\begin{pmatrix} A_1^{1-} \\ A_2^{1-} \end{pmatrix} = \begin{pmatrix} \alpha_1^{(1)} \\ \alpha_1^{(2)} \end{pmatrix} \frac{e^{-R_1/2}}{(k_1 + k_1^*)}, \quad (9a)$$

$$\begin{pmatrix} A_1^{2-} \\ A_2^{2-} \end{pmatrix} = \begin{pmatrix} e^{\delta_{11}} \\ e^{\delta_{12}} \end{pmatrix} \frac{e^{-(R_1+R_3)/2}}{(k_2 + k_2^*)}, \quad (9b)$$

$$\begin{pmatrix} A_1^{1+} \\ A_2^{1+} \end{pmatrix} = \begin{pmatrix} e^{\delta_{21}} \\ e^{\delta_{22}} \end{pmatrix} \frac{e^{-(R_2+R_3)/2}}{(k_1 + k_1^*)}, \quad (9c)$$

$$\begin{pmatrix} A_1^{2+} \\ A_2^{2+} \end{pmatrix} = \begin{pmatrix} \alpha_2^{(1)} \\ \alpha_2^{(2)} \end{pmatrix} \frac{e^{-R_2/2}}{(k_2 + k_2^*)}. \quad (9d)$$

Various quantities occurring in Eqs. (9) are defined in Eq. (6). From Eqs. (8) it can also be verified that

$$\left| \frac{q_1^{n\pm}}{P_n} \right|^2 - \left| \frac{q_2^{n\pm}}{P_n} \right|^2 = k_{nR}^2 (|A_1^{n\pm}|^2 - |A_2^{n\pm}|^2) = \frac{k_{nR}^2}{\mu}, \quad n = 1, 2, \quad (10)$$

which is a consequence of the conservation law of intensities in the mixed CNLS system. The role of linear coupling parameters on type-II SCC and vice-versa can be well understood by analysing the asymptotic expressions (8) which clearly shows that these terms induce periodic switching of intensity between the two colliding solitons in both the components q_1 and q_2 . At a first sight it seems that the periodic intensity switching in a given soliton (say soliton S_1) is influenced only by the same soliton present in the other component. But a careful analysis shows that the presence of the second soliton (say soliton S_2) plays an indirect but predominant role in controlling the switching process through type-II SCC and vice-versa. Various possibilities of such collision scenario are given below:

1. The coupling results in periodic oscillations in the energy switching process throughout the collision process due to the oscillatory term $\cos(2\Gamma z + Q_n)$ appearing in Eq. (8a). As in the case of one soliton solution here also the amplitude and width of the periodic oscillations increase with decreasing Γ . Thus the important feature of such collision process is that the amplitude of periodic energy switching can be large depending upon the relative signs of linear coupling terms ρ and χ . This periodic energy switching behaviour, in the presence of coupling, depends on the α and k -parameters and also

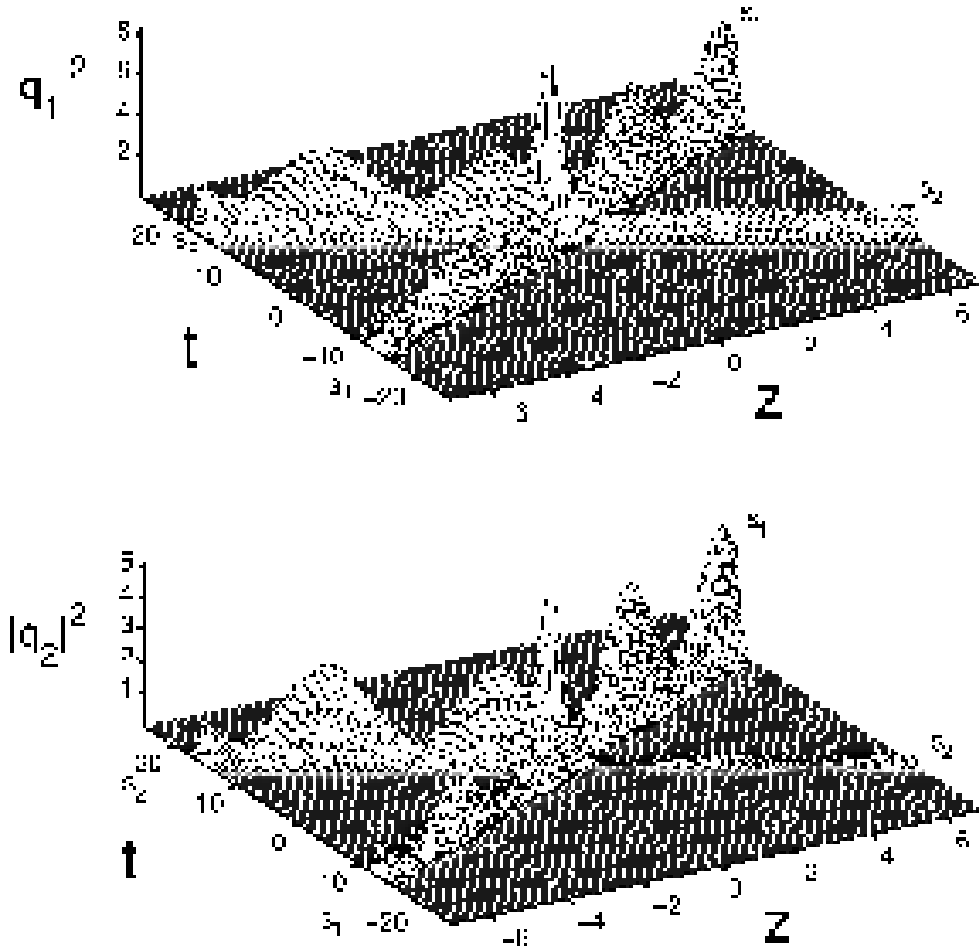


FIG. 3: Type-II shape changing collisions with periodic intensity switching.

on the linear coupling coefficients. Thus the oscillating energy switching process coexists with type-II SCC for $\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} \neq \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}$. Such a two soliton collision process with periodic intensity switching is shown in Fig. 3 for $\alpha_1^{(1)} = 0.7226 + 1.1254i$, $\alpha_1^{(2)} = 0.8484 + 0.2625i$, $\alpha_2^{(1)} = 0.5511 + 0.8584i$, $\alpha_2^{(2)} = 0.1923 + 0.0595i$, $\rho = 1$, $\chi = 0.5$, $k_1 = 1+i$, $k_2 = 1.1-i$. Eq. (8) also shows that the coupling enhances the amplitude of the soliton in a given component before and after interaction due to the contribution from the other component as compared with the bright soliton collision case in the absence of coupling.

2. The distinct feature of this collision process is that the intensity redistribution can

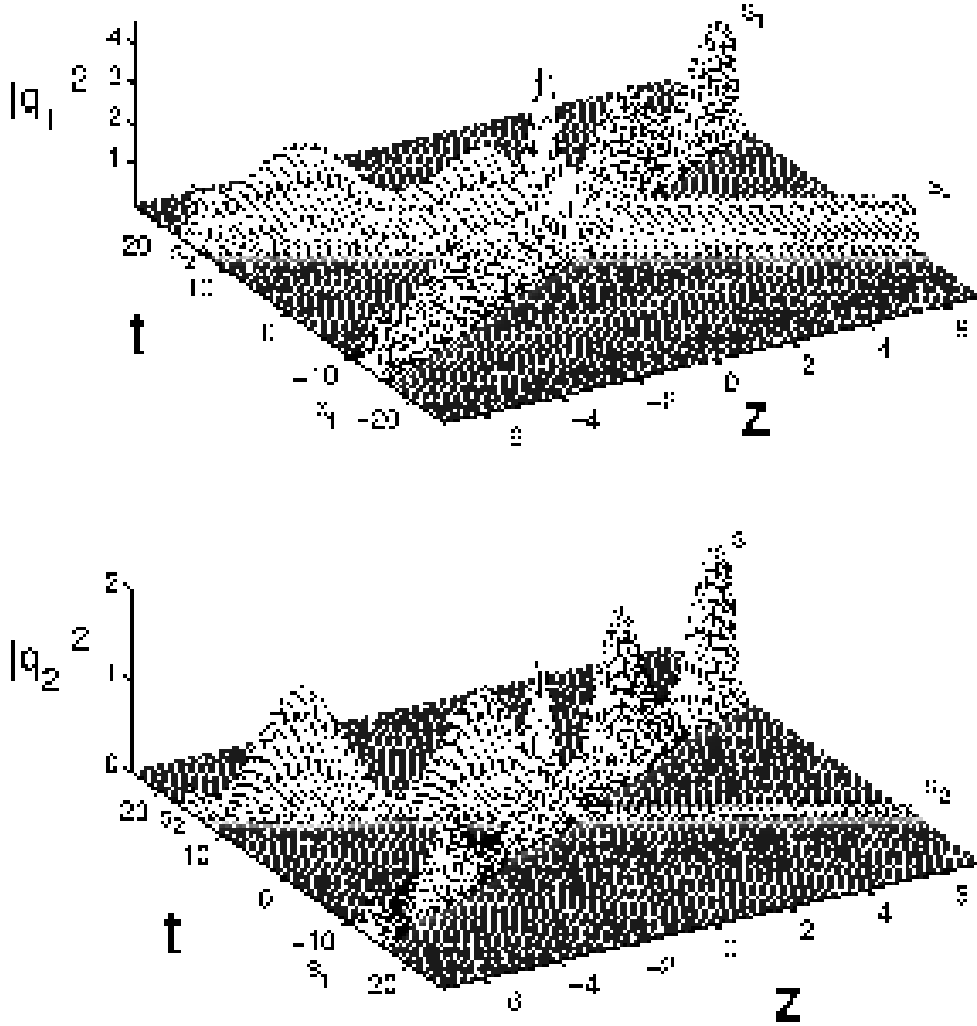


FIG. 4: Suppression of periodic oscillations in S_2 after interaction in a type-II SCC process.

be used to control the switching dynamics. One interesting possibility is complete suppression of oscillation either before or after collision in a particular soliton say “ S_n ” by making any one of $|A_j^{n-}|$ or $|A_j^{n+}|$, $j, n = 1, 2$, to be zero, respectively, with commensurate changes in the other soliton. As the nonsingular condition (7a) of the solution rules out the possibility of making $|A_1^{n\pm}|$ to be zero, the complete suppression of periodic oscillation of intensities in both the components of soliton S_n before (after) collision can be obtained by choosing $|A_2^{n-}|$ ($|A_2^{n+}|$) = 0. This suppression (enhancement) of intensities of a particular soliton in a given component

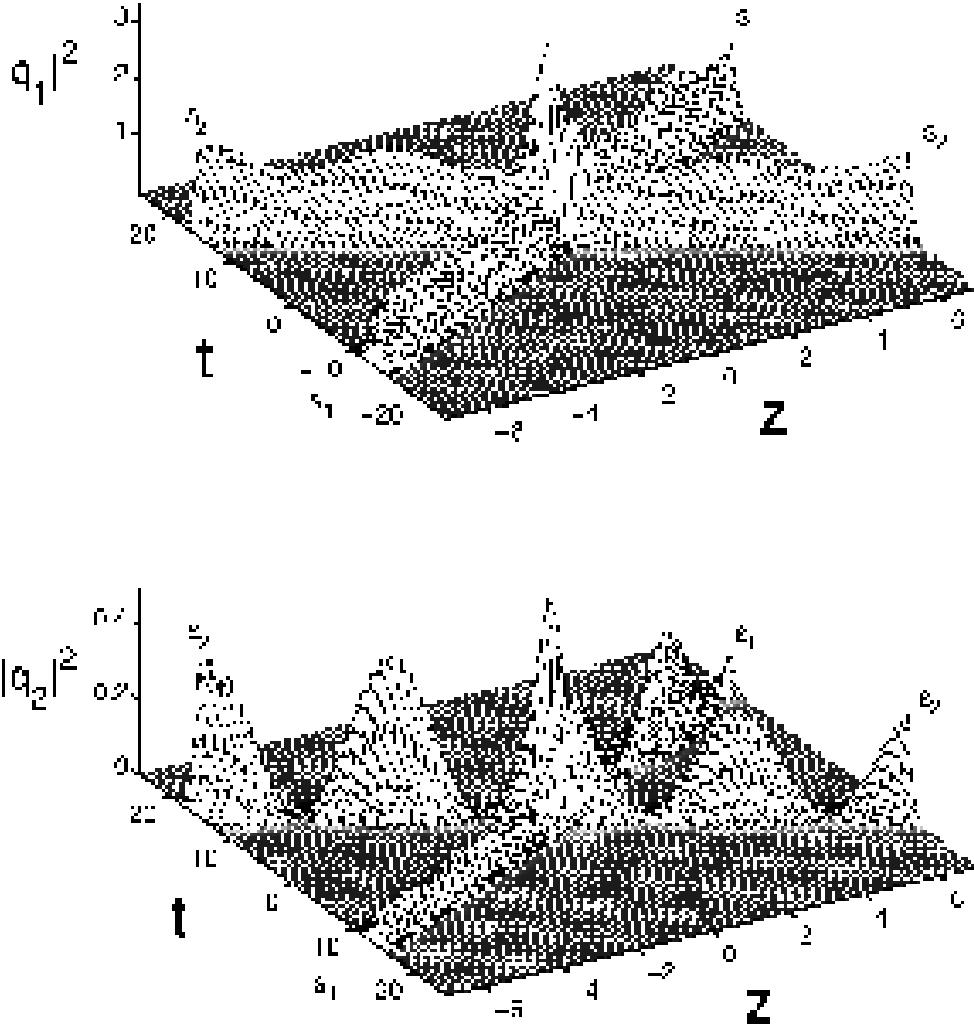


FIG. 5: Suppression of periodic oscillations in S_1 before interaction in a type-II SCC process.

during the type-II SCC results in the enhancement (suppression) of amplitude of periodic oscillations in the other colliding soliton as inferred from Eq. (8). Fig. 4. shows the type-II SCC scenario in which the oscillations in the q_1 and q_2 components of S_2 after interaction are completely suppressed, for the choice $\alpha_1^{(1)}=0.6093+0.9489i$, $\alpha_1^{(2)} = 0.4978 + 0.1540i$, $\alpha_2^{(1)}=0.5403+0.8415i$, $\alpha_2^{(2)} = 0$, $\rho = 1$, $\chi = 0.5$, $k_1 = 1 + i$, $k_2 = 1.1 - i$. The reason for this is that in the absence of coupling terms soliton S_2 undergoes type-II SCC with S_1 and its intensity in q_2 component after interaction is

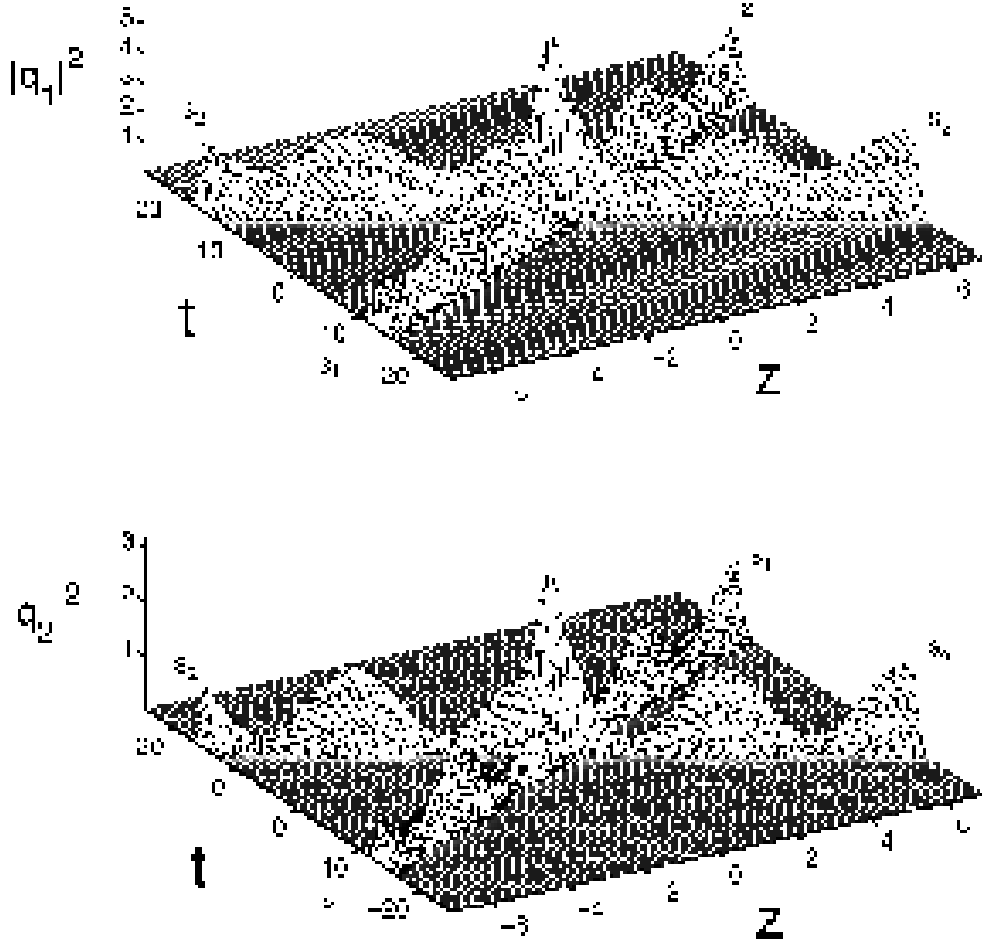


FIG. 6: Elastic collision of bright solitons with periodic oscillations in mixed CNLS system with linear self and cross couplings.

exactly zero for the given parametric choice. Similarly Fig. 5 shows the suppression of oscillations in q_1 and q_2 components of S_1 before interaction for the parametric choice $\alpha_1^{(1)} = 1, \alpha_1^{(2)} = 0, \alpha_2^{(1)} = 1.0201, \alpha_2^{(2)} = 0.2013, \rho = 1, \chi = 0.5, k_1 = 1 + i, k_2 = 1.1 - i$. This kind of switching process arises from the fact that in the absence of coupling the intensity of S_1 in q_2 component (that is, $|A_2^{1-}|^2 k_{1R}^2$) is zero before it collides with S_2 .

3. The standard elastic collision with periodic energy switching only arises for the choice $\frac{\alpha_1^{(1)}}{\alpha_2^{(1)}} = \frac{\alpha_1^{(2)}}{\alpha_2^{(2)}}$. This is shown in Fig. 6 for the parametric choice $\alpha_1^{(1)} = 0.6782 + 1.0562i, \alpha_1^{(2)} = 0.6782 + 1.0562i, \alpha_2^{(1)} = 0.7247 + 0.2242i, \alpha_2^{(2)} = 0.7247 + 0.2242i, \rho = 1,$

$$\chi = 0.5, k_1=1 + i, k_2 = 1.1 - i.$$

In order to appreciate the significance of the present system, we compare the soliton collision behaviour with that of twisted birefringent fibers [27] which involve focusing type nonlinearities. The crucial difference follows from Eq. (9), which says that the energy exchange between the two components (q_1, q_2) before and after collision is constant and as a result a given soliton experiences the same effect (either suppression or enhancement of intensity) in both the components during its collision with other soliton contrary to the twisted birefringent system. Thus the amplitude of oscillation due to coupling can be simultaneously enhanced/suppressed after collision in both the components as a consequence of type II-SCC, a situation which is not possible in twisted birefringent fibers. Another important advantage is the efficiency of switching due to linear couplings. Here the coupling terms influence the energy switching exponentially due to the hyperbolic terms (see Eq. (9)). This suggests large switching of energy with small self coupling strengths, as compared with Manakov system with linear couplings, a desirable property in fiber couplers.

V. CONCLUSION

In this paper, we have shown that the set of mixed 2-CNLS equations with linear self and cross coupling terms can be transformed to the standard integrable mixed 2-CNLS equations by performing the transformation (3a). The bright soliton solutions are obtained by applying this transformation to the recently reported bright soliton solutions of the mixed 2-CNLS equations [24] without linear coupling terms. Our study shows that inclusion of linear self and cross coupling terms lead to periodic energy switching among the components. We have also pointed out that due to the exponential dependence on the coupling terms, the energy switching can be large with small coupling strengths. In a two soliton collision process such periodic energy switching coexists with the type-II shape changing collision behaviour. However the standard elastic collision process can take place with or without periodic energy switching for very specific parametric choices. An important result which follows from the present study is that the shape changing collision of type-II can be used suitably to suppress or enhance the periodic oscillations in the energy switching process completely or partially and also simultaneously in both the components. These results can give further impetus in understanding the Lindner-Fedyanin system in the continuum limit,

and can also find potential applications in fiber couplers and in BECs.

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