# Existence of anticipatory, complete and lag synchronizations in time-delay systems

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**Abstract.** Existence of different kinds of synchronizations, namely anticipatory, complete and lag type synchronizations (both exact and approximate), are shown to be possible in time-delay coupled piecewise linear systems. We deduce stability condition for synchronization of such unidirectionally coupled systems following Krasovskii-Lyapunov theory. Transition from anticipatory to lag synchronization via complete synchronization as a function of coupling delay is discussed. The existence of exact synchronization is preceded by a region of approximate synchronization from desynchronized state as a function of a system parameter, whose value determines the stability condition for synchronization. The results are corroborated by the nature of similarity functions. A new type of oscillating synchronization that oscillates between anticipatory, complete and lag synchronization, is identified as a consequence of delay time modulation with suitable stability condition.

## 1. Introduction

Synchronization of coupled chaotic systems is a field of growing interest in view of its potential applications in diverse area of research activities such as secure communication, cryptography, controlling, long term prediction of chaotic systems, etc.[1, 2, 3, 4, 5, 6]. Since the identification of chaotic synchronization different kinds of them have been proposed in interacting chaotic systems both theoretically and experimentally: complete synchronization [7, 8], generalized synchronization [9, 10], phase synchronization [11, 12], lag synchronization [13, 14] and anticipatory synchronization[15, 16].

One of the most important applications of chaos synchronization is secure communication. It has been shown that in secure communication based on simple low dimensional chaotic systems with only one positive Lyapunov exponent, the hidden message can be unmasked by dynamical reconstruction of the chaotic signal using nonlinear dynamical forecasting methods or by using some simple return maps [17, 18]. One way to overcome this problem is to consider chaos synchronization in higher dimensional systems having multiple positive Lyapunov exponents, based on the consideration that increased randomness and unpredictability of the hyperhcaotic signals will make it more difficult to extract the masked message. Recently, chaotic time-delay systems have been suggested as good candidates for secure communication [19] as the time-delay systems exhibit the intriguing characteristics of increase in the embedding dimension and the number of positive Lyapunov exponents with the time-delay inspite of its small number of physical variables. Therefore the study of chaos synchronization in time-delay systems is of considerable practical significance.

Recently, we have shown that even a single scalar delay equation with piecewise linear function can exhibit hyperchaotic behavior even for small values of time-delay [20] and hence it is of interest to consider chaos synchronization in such scalar piecewise linear delay differential equations with appropriate coupling between them. For the present study, we will consider chaos synchronization between two such time-delay systems with unidirectional coupling and having two different time-delays: one in the individual systems, namely, feedback delay $\tau_1$ and the other in the coupling term $\tau_2$. We have arrived at the stability condition for synchronization following Krasovskii-Lyapunov theory, which shows that the stability condition is independent of the delay times, and demonstrate that there exist transitions between three different kinds of synchronization, namely, anticipatory, complete and lag synchronizations by simply tuning the second time-delay parameter in the coupling, for a fixed set of other system parameters satisfying appropriate stability condition. The results have been corroborated by the nature of similarity functions. To enhance the security, delay time modulation is introduced at the feedback delay and the stability of the synchronization manifold with delay time modulation is calculated. We have also identified a new type of oscillating synchronization as a consequence of delay time modulation.

## 2. Scalar piecewise linear delay system

We consider the following first order delay differential equation introduced by Lu and He [21] and discussed in detail in references [20, 22, 23],

$$\dot{x}(t) \quad = \quad -ax(t) + bf(x(t-\tau)), \tag{1}$$

where $a$ and $b$ are parameters, $\tau$ is the constant time-delay and $f$ is an odd piecewise linear function defined by the relation

$$f(x) = \begin{cases} 0, & x \leq -4/3 \\ -1.5x - 2, & -4/3 < x \leq -0.8 \\ x, & -0.8 < x \leq 0.8 \\ -1.5x + 2, & 0.8 < x \leq 4/3 \\ 0, & x > 4/3 \end{cases} \tag{2}$$

This system exhibits hyperchaotic behavior for the parameter values $a = 1.0, b = 1.2$ and $\tau = 5$ and the hyperchaotic nature was confirmed by the existence of multiple positive Lyapunov exponents. The first ten maximal Lyapunov exponents as a function of $\tau$ for the chosen parameter values is shown in Fig. 1. In our present study, we have included three linear parts for the function $f(x)$ in Eq. 2. One may also carry out the studies with two linear parts alone. However, the dynamics in the three linear part case is richer and so we confine our study to this case alone here.

## 3. Stability condition for synchronization

Now let us consider the following unidirectionally coupled drive $x_1(t)$ and response $x_2(t)$ systems with two different time-delays $\tau_1$ and $\tau_2$ as feedback and coupling time-delays, respectively,

$$\dot{x_1}(t) = -ax_1(t) + b_1 f(x_1(t-\tau_1)), \tag{3}$$

$$\dot{x_2}(t) = -ax_2(t) + b_2 f(x_2(t-\tau_1)) + b_3 f(x_1(t-\tau_2)), \tag{4}$$

where $b_1, b_2$ and $b_3$ are constants, $a > 0$, and $f(x)$ is of the same form as in Eq. (2).

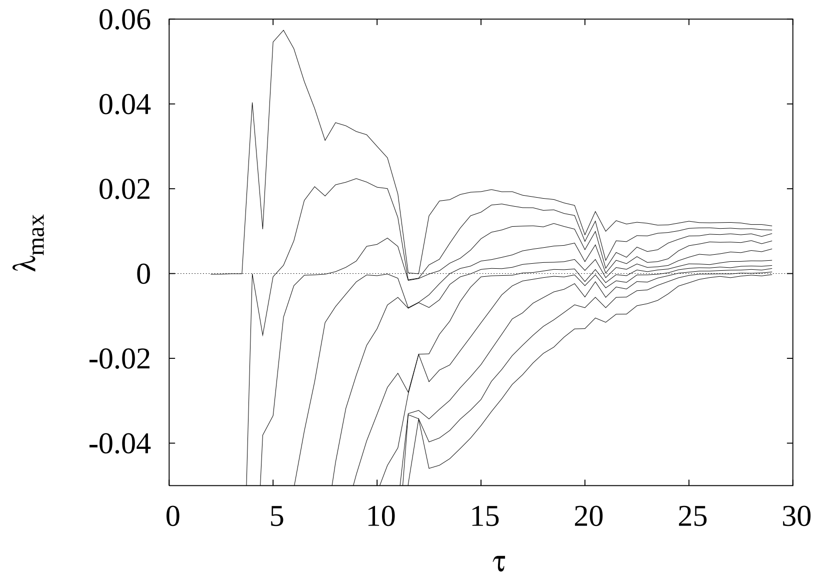


**Figure 1.** The first ten maximal Lyapunov exponents $\lambda_{max}$ of the scalar time-delay equation (1) for the parameter values $a = 1.0, b = 1.2, \tau \in (2, 29)$.

Now we can deduce the stability condition for synchronization of the two time-delay systems Eqs. (3) and (4) in the presence of the delay coupling $b_3 f(x_1(t-\tau_2))$. The time evolution of the difference system with the state variable $\Delta = x_{1\tau} - x_2$ , where $x_{1\tau} = x_1(t-\tau), \tau = \tau_2 - \tau_1$, can be written for small values of $\Delta$ by using the evolution Eqs. (3) as

$$\dot{\Delta} = -a\Delta + (b_2 + b_3 - b_1)f(x_1(t-\tau_2)) + b_2 f'(x_1(t-\tau_2))\Delta_{\tau_1}, \tag{5}$$

In order to study the stability of the synchronization manifold, we choose the parametric condition,

$$b_1 = b_2 + b_3, \tag{6}$$

so that the evolution equation for the difference system $\Delta$ becomes

$$\dot{\Delta} = -a\Delta + b_2 f'(x_1(t-\tau_2))\Delta_{\tau_1}. \tag{7}$$

The synchronization manifold is locally attracting if the origin of this equation is stable. Following Krasovskii-Lyapunov functional approach [24, 25], we define a positive definite Lyapunov functional of the form

$$V(t) = \frac{1}{2}\Delta^2 + \mu \int_{-\tau_1}^{0} \Delta^2(t+\theta)d\theta, \tag{8}$$

where $\mu$ is an arbitrary positive parameter, $\mu > 0$. Note that $V(t)$ approaches zero as $\Delta \to 0$.

To estimate a sufficient condition for the stability of the solution $\Delta = 0$, we require the derivative of the functional $V(t)$ along the trajectory of Eq. (7),

$$\frac{dV}{dt} = -a\Delta^2 + b_2 f'(x_1(t-\tau_2))\Delta\Delta_{\tau_1} + \mu\Delta^2 - \mu\Delta_{\tau_1}^2, \tag{9}$$

to be negative. After simple algebra, one can find the sufficient condition for asymptotic stability as

$$a > |b_2 f'(x_1(t-\tau_2))| \tag{10}$$

along with the condition (6) on the parameters $b_1, b_2$ and $b_3$.

Now from the form of the piecewise linear function $f(x)$ given by Eq. (2), we have,

$$|f'(x_1(t-\tau_2))| = \begin{cases} 1.5, & 0.8 \leq |x_1| \leq \frac{4}{3} \\ 1.0, & |x_1| < 0.8 \end{cases} \tag{11}$$

Consequently the stability condition (10) becomes $a > 1.5|b_2| > |b_2|$ along with the parametric restriction $b_1 = b_2 + b_3$.

Thus one can take $a > |b_2|$ as a less stringent condition for (10) to be valid, while

$$a > 1.5|b_2|, \tag{12}$$

as the most general condition specified by (10) for asymptotic stability of the synchronized state $\Delta = 0$. The condition (12) indeed corresponds to the stability condition for exact anticipatory, identical as well as lag synchronizations for suitable values of the coupling delay $\tau_2$. It may also be noted that the stability condition (12) is independent of the both the delay parameters $\tau_1$ and $\tau_2$.

## 4. Anticipatory synchronization

To start with, we demonstrate that there exists a region of anticipatory synchronization when $\tau_2 < \tau_1$ at the synchronization manifold. For this purpose, we have fixed the value of feedback delay $\tau_1$ at $\tau_1 = 0.25$ while the other parameters are fixed as $a = -0.16, b_1 = 0.2, b_2 = 0.1, b_3 = 0.1$ and the coupling delay $\tau_2$ is considered as a control parameter. For all values of $\tau_2 < \tau_1$, we find from the evolution equation (7), the difference variable $\Delta = x_1(t-\tau) - x_2(t)$ with $\tau = \tau_2 - \tau_1 < 0$ approaches zero asymptotically, so that $x_1(t+\tau) = x_2(t)$, thereby demonstrating the existence of anticipatory synchronization with anticipating time as $\tau$. The existence of anticipatory synchronization is characterized by the nature of similarity functions for different values of $b_2$, the parameter whose value determines the stability of the synchronization manifold as may be seen from Eqn. (12).

Now we use the notion of similarity function to characterize anticipatory synchronization which was originally introduced by Rosenblum *et al.* [11] to characterize lag synchronization. Similarity function is defined as a time-averaged difference between the variables $x_1$ and $x_2$ (with mean values being subtracted) taken with the positive time shift $\tau$ for lag synchronization and negative time shift $-\tau$ for anticipatory synchronization,

$$S_a^2(\tau) = \frac{\langle [x_2(t-\tau) - x_1(t)]^2 \rangle}{[\langle x_1^2(t)\rangle \langle x_2^2(t)\rangle]^{1/2}}, S_l^2(\tau) = \frac{\langle [x_2(t+\tau) - x_1(t)]^2 \rangle}{[\langle x_1^2(t)\rangle \langle x_2^2(t)\rangle]^{1/2}}, \tag{13}$$

where $\langle x \rangle$ means time average over the variable $x$. If the minimum value of $S_a(\tau)$ reaches zero, that is $S_a(\tau) = 0$, then there exists a time shift $-\tau$ between the two signals $x_1(t)$ and $x_2(t)$ such that $x_2(t) = x_1(t+\tau)$, demonstrating the existence of anticipatory synchronization between the drive $x_1$ and the response $x_2$ signals. Fig. 2 shows the similarity function $S_a(\tau)$ as a function of the coupling delay $\tau_2$ for four different values of $b_2$, the parameter whose value determines the stability condition given by Eq. (12), while satisfying the parametric condition $b_1 = b_2 + b_3$. Curves 1 and 2 are plotted for the values of $b_2 = 0.18 (> a = 0.16 > a/1.5)$ and $b_2 = 0.16 (= a > a/1.5)$, respectively, where the minimum values of $S_a(\tau)$ is found to be greater than zero, indicating that there is no exact time shift between the two signals $x_1(t)$ and $x_2(t)$. Note that in the both cases the stringent stability condition (12) and the less stringent condition $a > |b_2|$ are violated. Curve 3 corresponds to the value of $b_2 = 0.15$ ( $< a > a/1.5$), where the minimum value of $S_a(\tau)$ is almost zero, but not exactly zero (as may be seen in the inset of Fig. 2, inwhich case only the less stringent condition $a > |b_2|$ is satisfied while the stringent stability condition is ruled out), indicating an approximate anticipatory

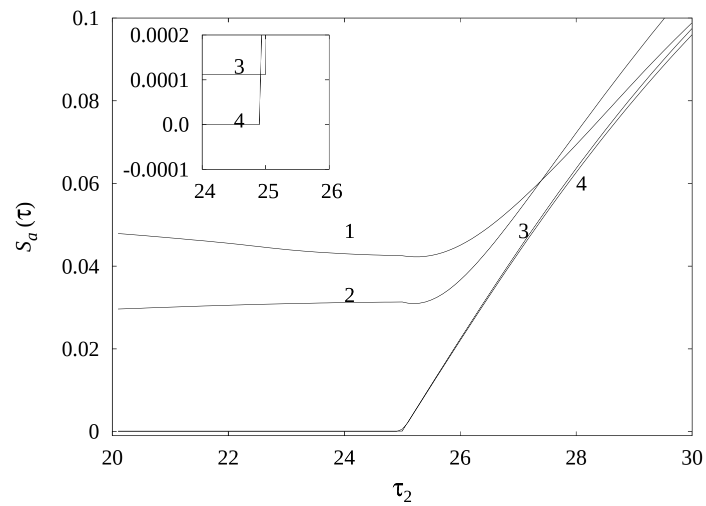


**Figure 2.** Similarity function $S_a(\tau)$ for different values of $b_2$, the other system parameters are $a = 0.16, b_1 = 0.2$ and $\tau_1 = 25.0$. (Curve 1: $b_2 = 0.18, b_3 = 0.02$, Curve 2: $b_2 = 0.16, b_3 = 0.04$, Curve 3: $b_2 = 0.15, b_3 = 0.05$ and Curve 4: $b_2 = 0.1, b_3 = 0.1$).

synchronization $x_1(t) \approx x_2(t-\tau)$. On the other hand the curve 4 is plotted for the value of $b_2 = 0.1(< a/1.5)$, satisfying the general stability criterion, Eq. (12). It shows that the minimum of $S_a(\tau) = 0$, thereby indicating that there exists an exact time shift between the two signals demonstrating anticipatory synchronization. The anticipating time is found to be equal to the difference between the coupling and feedback delay times, that is, $\tau = \tau_2 - \tau_1$. Note that $S_a(\tau) = 0$ for all values of $\tau_2 < \tau_1$, indicating anticipatory synchronization for a range of delay coupling. A further significance is that the anticipating time $\tau = \tau_2 - \tau_1$ is an adjustable quantity as long as $\tau_2 < \tau_1$, which can be tuned suitably to satisfy experimental situations. Thus anticipating of any future chaotic state can be made possible. We also note that the above choice of parameters (Curve 4) in Fig. 2 is only a specific example exhibiting exact anticipatory synchronization. Any other choice of parameters satisfying the condition $b_1 = b_2 + b_3$ subject to $a > 1.5|b_2|$ is good enough to obtain the above phenomenon.

The existence of exact synchronization is preceded by a region of approximate synchronization from desynchronized state as a function of the parameter $b_2$, whose value determines the stability condition for synchronization. We have also found [23]that the existence of approximate synchronization from desynchronized state is characterized by transition from on-off intermittency to periodic structure in probability distribution of laminar phase as shown by Zhan *et al*[14].

## 5. Complete synchronization

Now, we show that when the value of coupling delay $\tau_2$ equals the value of feedback delay $\tau_1$, that is, $\tau_2 = \tau_1$, there exists complete synchronization. We find from the evolution equation (7), the difference variable $\Delta = x_1(t-\tau) - x_2(t)$ with $\tau = \tau_2 - \tau_1 = 0$ approaches zero asymptotically, so that $x_1(t) = x_2(t)$, thereby demonstrating the existence of complete synchronization between the drive $x_1(t)$ and the response $x_2(t)$. Here also, we have identified that the emergence of approximate complete synchronization is associated with the transition form on-off intermittency to periodic structure in the laminar phase distribution as a function of the parameter $b_2$.

## 6. Lag synchronization

Next, we show that for the value of coupling delay $\tau_2$ greater than feedback delay $\tau_1$, the system exhibits lag synchronization. For $\tau_2 > \tau_1$, we find from the evolution equation (7), the difference variable $\Delta = x_1(t-\tau) - x_2(t)$ with $\tau = \tau_2 - \tau_1 > 0$ approaches zero asymptotically, so that $x_1(t-\tau) = x_2(t)$, thereby demonstrating the existence of lag synchronization with lag time equal

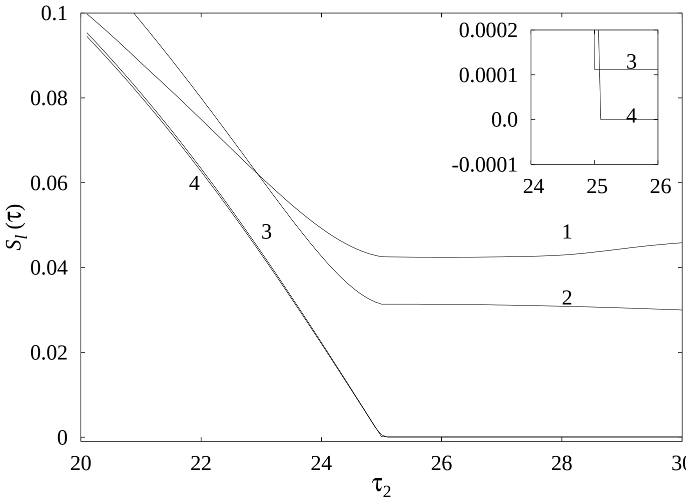


**Figure 3.** Similarity function $S_l(\tau)$ for different values of $b_2$, the other system parameters are $a = 0.16, b_1 = 0.2$ and $\tau_1 = 25.0$. (Curve 1: $b_2 = 0.18, b_3 = 0.02$, Curve 2: $b_2 = 0.16, b_3 = 0.04$ Curve 3: $b_2 = 0.15, b_3 = 0.05$ and Curve 4: $b_2 = 0.1, b_3 = 0.1$).

to $\tau = \tau_2 - \tau_1$. The existence of lag synchronization is characterized by the similarity function $S_l(t)$ defined earlier in Eqn. (13). Fig. 3 shows the similarity function $S_l(\tau)$ Vs coupling delay $\tau_2$ for four different values of $b_2$. Curves 1 and 2 show the similarity function $S_l(\tau)$ for the values of $b_2 = 0.18$ and 0.16, respectively. The minimum of similarity function $S_l(\tau)$ occurs for values of $S_l(\tau) > 0$, where the minimum does not occur at zero and hence there is a lack of exact lag time between the drive and response signals indicating asynchronization. Curve 3 corresponds to the value of $b_2 = 0.15$ (which is less than $a$ but greater than $a/1.5$), where the minimum values of $S_l(\tau)$ is almost zero, but not exactly zero (as may be seen in the inset of Fig. 3), so that $x_1(t) \approx x_2(t + \tau)$. However for the value of $b_3 = 0.1$, for which the general condition (12) is satisfied, the minimum of similarity function becomes exactly zero (Curve 4) indicating that there is an exact time shift (Fig. 3) between drive and response signals $x_1(t)$ and $x_2(t)$, respectively, confirming the occurrence of lag synchronization. Again the above choice of parameters (Curve 4) in Fig. 3 is only a specific example exhibiting exact lag synchronization. Any other choice of parameters satisfying the condition (6) subject to $a > 1.5|b_2|$ is good enough to obtain the above phenomenon.

We have also confirmed that as in the case of anticipatory synchronization, when the parameter $b_2$ varies, the onset of exact lag synchronization is preceded by a region of approximate lag synchronization, which is characterized by a transition from on-off intermittency of the desynchronized state to a periodic structure in the laminar phase distribution.

## 7. Stability condition for synchronization with delay time modulation

To improve the security of the above type of synchronized systems, we have extended our studies by choosing time-delay as a function of time. Recently, the concept of delay time modulation was introduced by Kye *et al* [26, 27] and they have shown that reconstruction of phase space of the time-delay systems is hardly possible as a consequence of delay time modulation.

Now, we will introduce the delay time modulation in the systems (3) and (4) at the feedback delays $\tau_1(t)$ in the form [28]

$$\tau_1(t) \quad = \quad \tau_0 + \tau_a \sin(\omega t), \qquad (14)$$

where $\tau_0$ is the zero frequency component, $\tau_a$ is the amplitude and $\omega/2\pi$ is the frequency of the modulation. Here the coupling delay $\tau_2$ is kept constant.

The stability condition for synchronization of the systems (3) and (4) with delay time modulation can be obtained as (by following the same procedure as for the case of constant

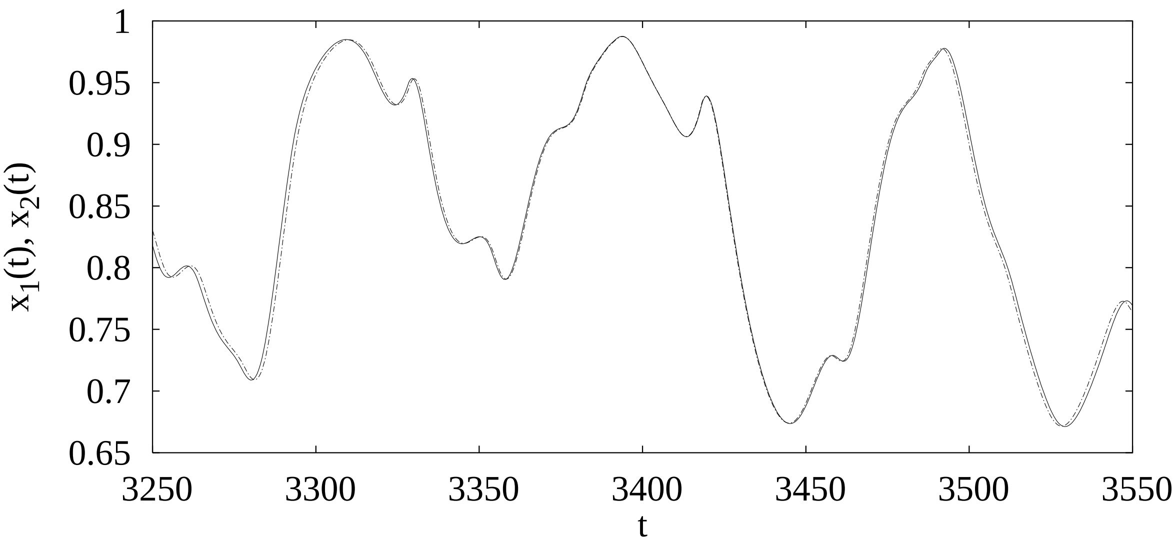


**Figure 4.** Oscillating synchronization from lag to anticipatory synchronization via complete synchronization for $\tau_0 = 100, \tau_a = 90$ and $\omega = 10^{-4}$. Drive $x_1(t)$ is represented by —— and response $x_2(t)$ by — · —.

feedback delay given in section 3),

$$a > \left| \frac{b_2 f'(x_1(t-\tau_2))}{\sqrt{(1-\tau_1')}} \right| \tag{15}$$

along with the condition (6) on the parameters $b_1, b_2$ and $b_3$. Once again as in the previous case, from the form of piecewise function $f(x)$, one can take $a > \left|\frac{b_2}{\sqrt{(1-\tau_1')}}\right|$ as a less stringent condition for (15) to be valid, while

$$a > 1.5 \left| \frac{b_2}{\sqrt{(1-\tau_1')}} \right| \tag{16}$$

as the most general condition specified by (15) for asymptotic stability of the synchronized state.

## 8. Oscillatory synchronization as a consequence of delay time modulation

We have fixed the value of $\tau_0 = 100, \tau_a = 90$ and $\omega = 10^{-4}$ with the same value of $a$ and $b$ as previously studied. For the value of $b_2 = 0.01$ the general stability condition (15) is satisfied. For the chosen values of $\tau_0$ and $\tau_a$, one can find that $\tau_1$ oscillates between $(\tau_1(t) = \tau_0 + \tau_a \sin(\omega t) = 100 \pm 90)$ 10 and 190. Hence for any value of coupling delay $\tau_2$ less than 10, $\tau_2$ is always less than $\tau_1(t)$, one can find from the evolution equation (7) the difference variable $\Delta = x_1(t-\tau(t)) - x_2(t)$ with $\tau(t) = \tau_2 - \tau_1(t)$ approaches zero asymptotically, so that $x_1(t+\tau(t)) = x_2(t)$, implying the existence of anticipatory synchronization with the time dependent anticipating time $\tau(t) = \tau_2 - \tau_1(t)$. Similarly for any value of coupling delay $\tau_2$ greater than 190, that is for $\tau_2$ always greater than $\tau_1(t)$, we have $x_1(t-\tau(t)) = x_2(t)$, demonstrating the existence of lag synchronization. On the other hand, when the coupling delay $\tau_2$ is chosen in

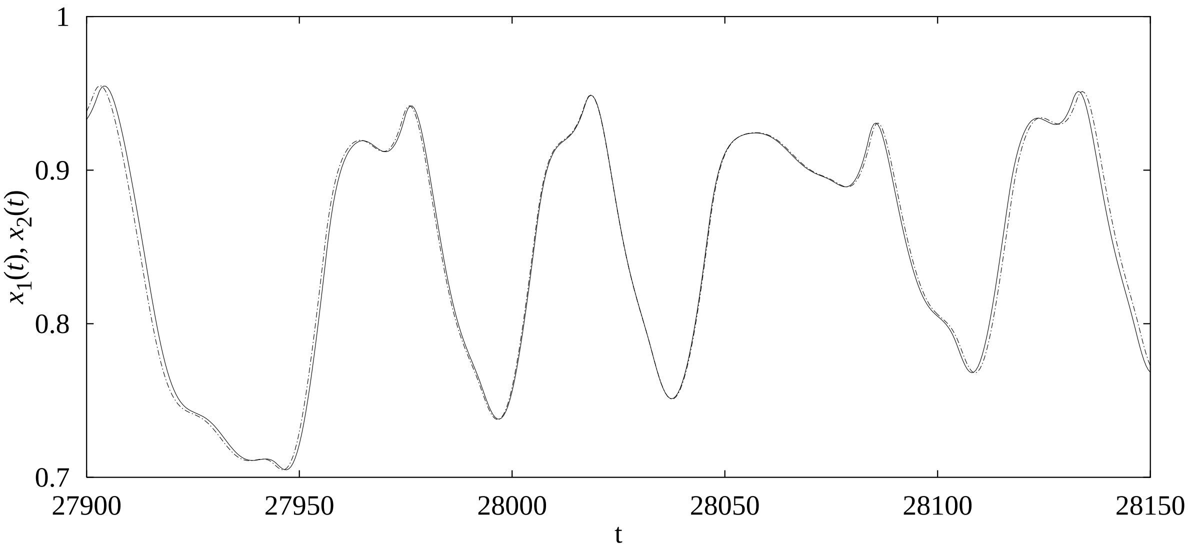


**Figure 5.** Oscillating synchronization from anticipatory to lag synchronization via complete synchronization for $\tau_0 = 100, \tau_a = 90$ and $\omega = 10^{-4}$. Drive $x_1(t)$ is represented by —— and response $x_2(t)$ by — · —.

between 10 and 190, one is able to find that the feedback delay $\tau_1(t)$ is less than $\tau_2$ for some time (inwhich case $\tau(t) = \tau_2 - \tau_1(t) > 0$, so that there exists lag synchronization $x_1(t - \tau(t)) = x_2(t)$) and as $\tau_1(t)$ increases eventually, it approaches $x_2(t)$ at a certain time, where $\tau(t) = \tau_2 - \tau_1(t) = 0$, so that $x_1(t) = x_2(t)$, that is there exists complete synchronization for a specific value of time. As $\tau_1(t)$ exceeds the value of $\tau_2$, now $\tau(t)$ becomes negative, $\tau(t) = \tau_2 - \tau_1(t) < 0$ with $x_1(t + \tau(t)) = x_2(t)$), there exists anticipatory synchronization. Therefore as time evolves there is oscillation between lag, complete and anticipatory synchronization with time dependent anticipating and lag times. Fig. 4 shows the evolution of the drive $x_1(t)$ and the response $x_2(t)$ at the transition between lag to anticipatory synchronization via complete synchronization, whereas Fig. 5 shows the evolution of the drive $x_1(t)$ and the response $x_2(t)$ at the next transition between anticipatory to lag via complete synchronization. Thus as a consequence of delay time modulation there exists a new type of oscillating synchronization that oscillates between lag, complete and anticipatory synchronization with varying anticipating and lag times.

## 9. Summary and conclusion

In summary, we have obtained the stability condition for synchronization in a system of two coupled piecewise linear differential equation following Krasovskii-Lyapunov functional approach for both the case of constant and time dependent delay. We have demonstrated that there exists regimes of anticipatory, complete and lag synchronizations with coupling delay $\tau_2$ as control parameter for constant delay case with anticipating and lag time as $\tau = \tau_2 - \tau_1$. The exact synchronization regimes are preceded by a regime of approximate synchronization from the desynchronized state as a function of parameter $b_2$, which determines the stability condition. The existence of anticipatory and lag synchronizations are characterized by the similarity functions. In the case of time dependent delay, we have demonstrated that there exists regimes of anticipatory and lag synchronization with time dependent anticipating and lag times for suitable values of coupling delay $\tau_2$. We have also shown the existence of new type of oscillating synchronization that oscillates between lag, complete and anticipatory synchronizations as a consequence of delay time modulation, when the value of coupling delay $\tau_2$ is chosen in between $\tau_1 = \tau_0 - \tau_a$ and $\tau_1 = \tau_0 + \tau_a$ with varying anticipating and lag times. We believe that this new type of oscillating synchronization with time dependent anticipating and lag times will enhance the security of secure communication and now we are implementing this experimentally through electronic circuits.


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## References


[1] Pikovsky A S, Rosenblum M G and Kurths J 2001 *Synchronization - A Unified Approach to Nonlinear Science* (Cambridge: Cambridge University Press)
[2] Lakshmanan M and Murali K 1996 *Chaos in Nonlinear Oscillators: Controlling and Synchronization* (Singapore: World Scientific)
[3] Lakshmanan M and Rajasekar S 2003 *Nonlinear Dynamics: Integrability, Chaos and Patterns* (New York: Springer)
[4] Hayes S, Grebogy C and Ott E 1993 *Phys. Rev.* L **70** 3031
[5] Pyragas K 1993 *Phys. Lett.* A **181** 203
[6] Kittel A, Pyragas K and Richter R 1994 *Phys. Rev.* E **50** 262
[7] Fujisaka H and Yamada T 1983 *Prog. Theor. Phys.* **69** 32
[8] Pecora L M and Carroll T L 1990 *Phys. Rev.* L **64** 821
[9] Rulkov N F, Sushchik M M, Tsimring L S and Abarbanal H D I 1995 *Phys. Rev.* E **51** 980
[10] Brown R 1998 *Phys. Rev.* L **81** 4835
[11] Rosenblum M G, Pikovsky A S and Kurths J 1996 *Phys. Rev.* L **76** 1804
[12] Yalcinkaya T and Lai Y C 1997 *Phys. Rev.* L **79** 3885
[13] Rosenblum M G, Pikovsky A S and Kurths J 1997 *Phys. Rev.* L **78** 4193
[14] Zhan M, Wei G W and Lai C H 2002 *Phys. Rev.* E **65** 036202
[15] Voss H U 2001 *Phys. Rev.* L **87** 014102
[16] Masoller C 2001 *Phys. Rev.* L **86** 2782
[17] Short K M and Parker A T 1998 *Phys. Rev.* L **58** 1159
[18] Perez G and Cerdera H A 1995 *Phys. Rev.* L **74** 1970
[19] Mensour B and Longtin A 1998 *Phys. Lett.* A **244** 59
[20] Senthilkumar D V and Lakshmanan M 2005 *Int. J. Bifurcation and Chaos* **15** (to appear)
[21] Lu H and He Z 1996 *IEEE Trans. Circuits Syst.,* I: *Fundam. Theory Appl.* **43** 700
[22] Thangavel P, Murali K and Lakshmanan M 1998 *Int. J. Bifurcation and Chaos* **8** 2481
[23] Senthilkumar D V and Lakshmanan M 2005 *Phys. Rev.* E **71** 016211
[24] Krasovskii N N 1963 *Stability of Motion* (Stanford: Stanford University Press)
[25] Pyragas K 1998 *Phys. Rev.* E **58** 3067
[26] Kye W H, Choi M, Kim M W, Lee S Y, Rim S, Kim C M and Park Y J 2004 *Phys. Lett.* A **322** 338
[27] Kye W H, Choi M, Rim S, Kurdoglyan M S, Kim C M and Park Y J 2004 *Phys. Rev.* E **69** 055202(R)
[28] Shahverdiev E M and Shore K A 2005 *Phys. Rev.* E **71** 016201