

# Motion of Curves and Surfaces and Nonlinear Evolution Equations in (2+1) Dimensions

M. Lakshmanan<sup>a,\*</sup>, R. Myrzakulov<sup>b,c,†</sup>, S. Vijayalakshmi<sup>a</sup> and A.K. Danlybaeva<sup>c</sup>

<sup>a</sup>*Centre for Nonlinear Dynamics, Department of Physics, Bharathidasan University,  
Tiruchirapalli 620 024, India*

<sup>b</sup>*High Energy Physics Institute, National Academy of Sciences, 480082, Alma-Ata-82, Kazakstan*

<sup>c</sup>*Centre for Nonlinear Problems, PO Box 30, 480035, Alma-Ata-35, Kazakstan*

## Abstract

It is shown that a class of important integrable nonlinear evolution equations in (2+1) dimensions can be associated with the motion of space curves endowed with an extra spatial variable or equivalently, moving surfaces. Geometrical invariants then define topological conserved quantities. Underlying evolution equations are shown to be associated with a triad of linear equations. Our examples include Ishimori equation and Myrzakulov equations which are shown to be geometrically equivalent to Davey-Stewartson and Zakharov - Strachan (2+1) dimensional nonlinear Schrödinger equations respectively.

PACS Numbers: 02.30.Jr, 52.35.Sb, 75.10.H, 02.40.Hw

Typeset using REVTeX

The motion of curves in  $E^3$  and the defining equations of surfaces have drawn wide interest in the past, especially since they give nice geometric interpretations of nonlinear evolution equations in (1+1) dimensions [1-6]. There are many physical situations where they play a natural role. Particularly they have important connections with soliton equations solvable by Ablowitz, Kaup, Newell and Segur (AKNS) formalism [1-6]. For a recent renaissance of interest, see for example [7] and References therein. Other examples include dynamics of interfaces, surfaces and fronts, vortex filaments, supercoiled DNAs, magnetic fluxes, deformation of membranes, dynamics of proteins, propagation of flame fronts, and so on [8,9]. Of special interest among these systems is the dynamics of isotropic Heisenberg ferromagnetic spin chains where an interesting equivalence with nonlinear Schrödinger family of equations arise in a natural and physical way [10].

The question then arises as to whether nonlinear evolution equations in (2+1) dimensions of importance, especially spin equations, can be given a similar geometrical setting associated with motion of curves endowed with an additional coordinate or equivalently motion of surfaces. In this letter, we develop a general theory of obtaining nonlinear evolution equations in (2+1) dimensions as the compatibility requirements of the geometric equations defining the motion of curves and surfaces, incidently equivalent to a triad of linear equations. Our analysis also brings out certain natural topological integral invariants associated with the resultant systems. We then consider a class of (2+1) dimensional spin systems including (2+1) dimensional generalized Heisenberg ferromagnetic spin systems and deduce their equivalent nonlinear Schrödinger family including Davey-Stewartson, Zakharov-Strachan and other equations. In this context, we also wish to add that in recent times a general approach (mainly due to Konopelchenko) has appeared in the literature [11, 12] concerning the interpretation of hierarchies of integrable (2+1) dimensional systems as special motions of surfaces in  $E^3$  by inducing the surfaces in it via the solutions to two dimensional linear problems (2D LPs). But in the present paper, we are interested in extending the theory of moving space curve formalism in (1+1) dimensions to (2+1) dimensions.

We consider the space curve in  $E^3$  as in Fig. 1, defined by the Serret-Frenet equations

[13]

$$\vec{e}_{jx} = \vec{D} \wedge \vec{e}_j, \quad \vec{D} = \tau \vec{e}_1 + \kappa \vec{e}_3, \quad j = 1, 2, 3 \quad (1)$$

where  $\vec{e}_1, \vec{e}_2$  and  $\vec{e}_3$  are the unit tangent, normal and binormal vectors respectively to the curve. Here  $x$  is the arclength parametrising the curve. The curvature and torsion of the curve are defined respectively as

$$\begin{aligned} \kappa &= (\vec{e}_{1x} \cdot \vec{e}_{1x})^{\frac{1}{2}}, \\ \tau &= \kappa^{-2} \vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1xx}). \end{aligned} \quad (2)$$

The unit tangent vector  $\vec{e}_1$  is given by  $\vec{e}_1 = \frac{\partial \vec{r}}{\partial x} = \frac{1}{\sqrt{g}} \frac{\partial \vec{r}}{\partial \theta}$  where  $g$  is the metric  $g = \frac{\partial \vec{r}}{\partial \theta} \cdot \frac{\partial \vec{r}}{\partial \theta}$  on the curve such that  $x(\theta, t) = \int_0^\theta \sqrt{g(\theta', t)} d\theta'$ . Here  $\theta$  defines a smooth curve and  $\vec{r}(\theta, t)$  is the position vector of a point on the curve at time  $t$ . Considering the motion of such a twisted curve, the time evolution of the orthogonal trihedral can be easily seen [3] to be

$$\begin{aligned} \vec{e}_{jt} &= \vec{\Omega} \wedge \vec{e}_j, \\ \vec{\Omega} &= \omega_1 \vec{e}_1 + \omega_2 \vec{e}_2 + \omega_3 \vec{e}_3, \quad j = 1, 2, 3 \end{aligned} \quad (3)$$

where  $\omega_i, i = 1, 2, 3$  are some functions of  $\kappa$  and  $\tau$  and their derivatives. Then the compatibility of Eqs. (1) and (3) gives the evolution equations for  $\kappa$  and  $\tau$  [3] as

$$\begin{aligned} \kappa_t &= \omega_{3x} + \tau \omega_2, \\ \tau_t &= \omega_{1x} - \kappa \omega_2, \\ \omega_{2x} &= \tau \omega_3 - \kappa \omega_1. \end{aligned} \quad (4)$$

Now let us pass on to the subject of this letter namely to the theory of curves in (2+1) dimensions by endowing them with an additional spatial variable  $y$ . Alternately this could represent a surface specified by the trihedral  $\vec{e}_j(x, y, t)$  which is set in motion. A starting point of our approach is the observation that the  $y$ -evolution of the trihedral  $\vec{e}_j$  is determined by the following system of linear equations [14] (see also Refs. [15,16]);

$$\begin{aligned}\vec{e}_{jy} &= \vec{\Gamma} \wedge \vec{e}_j, \\ \vec{\Gamma} &= \gamma_1 \vec{e}_1 + \gamma_2 \vec{e}_2 + \gamma_3 \vec{e}_3.\end{aligned}\tag{5}$$

Here  $\gamma_j$ 's are functions to be determined. Using the fact that  $\vec{e}_i$ 's,  $i = 1, 2, 3$  form an orthogonal trihedral and using (1) and (5) in the compatibility condition  $\vec{e}_{jxy} = \vec{e}_{jyx}$ , we obtain

$$\gamma_1 = \partial_x^{-1}(\tau_y + \kappa\gamma_2), \quad \gamma_3 = \partial_x^{-1}(\kappa_y - \tau\gamma_2),\tag{6a}$$

and  $\gamma_2(x, y, t)$  is a solution of the following equation

$$-\kappa\gamma_2 = \tau_y - \left[ \frac{\tau\partial_x^{-1}(\kappa_y - \tau\gamma_2) - \gamma_{2x}}{\kappa} \right]_x,\tag{6b}$$

where  $\partial_x^{-1} \equiv \int_{-\infty}^x dx$ . We also note that Eqs. (1) and (5) may be identified with the Gauss-Weingarten equations of surface theory written in terms of orthogonal coordinates and so Eqs. (6) with the Codazzi-Mainardi equations. On the other hand, the condition  $\vec{e}_{jty} = \vec{e}_{jyt}$  gives rise to the set of equations

$$\begin{aligned}\gamma_{1t} &= \omega_{1y} + \omega_3\gamma_2 - \omega_2\gamma_3, \\ \gamma_{2t} &= \omega_{2y} + \omega_1\gamma_3 - \omega_3\gamma_1, \\ \gamma_{3t} &= \omega_{3y} + \omega_2\gamma_1 - \omega_1\gamma_2.\end{aligned}\tag{7}$$

Choosing  $\gamma_1, \gamma_2, \gamma_3$  consistently so that Eqs. (4) and (7) are compatible, one can obtain (2+1) dimensional evolution equations for  $\kappa$  and  $\tau$ . Further one can easily show that the defining equations for the motion of the trihedral, namely (1), (3) and (5) are equivalent to a set of three Riccati equations

$$z_{lx} = -i\tau z_l + \frac{\kappa}{2} [1 + z_l^2],\tag{8a}$$

$$z_{ly} = -i\gamma_1 z_l + \frac{1}{2} [\gamma_3 + i\gamma_2] z_l^2 + \frac{1}{2} [\gamma_3 - i\gamma_2],\tag{8b}$$

and

$$z_{lt} = -i\omega_1 z_l + \frac{1}{2} [\omega_3 + i\omega_2] z_l^2 + \frac{1}{2} [\omega_3 - i\omega_2],\tag{8c}$$

where  $z_l$ ,  $l = 1, 2, 3$  is a scalar variable obtained through an orthogonal rotation of the trihedral,  $z_l = (e_{2l} + ie_{3l})/(1 - e_{1l})$ ,  $e_{1l}^2 + e_{2l}^2 + e_{3l}^2 = 1$ . Introducing the transformation  $z_l = v_2/v_1$ , we obtain the triad of equivalent linear equations:

$$\begin{aligned} v_{1x} &= \frac{i\tau}{2}v_1 - \frac{\kappa}{2}v_2, \\ v_{2x} &= \frac{\kappa}{2}v_1 - \frac{i\tau}{2}v_2, \end{aligned} \tag{9a}$$

$$\begin{aligned} v_{1y} &= \frac{i\gamma_1}{2}v_1 - \frac{1}{2}(\gamma_3 + i\gamma_2)v_2, \\ v_{2y} &= \frac{1}{2}(\gamma_3 - i\gamma_2)v_1 - \frac{i\gamma_1}{2}v_2, \end{aligned} \tag{9b}$$

$$\begin{aligned} v_{1t} &= \frac{i\omega_1}{2}v_1 - \frac{1}{2}(\omega_3 + i\omega_2)v_2, \\ v_{2t} &= \frac{1}{2}(\omega_3 - i\omega_2)v_1 - \frac{i\omega_1}{2}v_2. \end{aligned} \tag{9c}$$

The compatibility of these linear equations again gives rise to Eqs. (4), (6) and (7). Thus any nonlinear evolution equation obtained through the space curve formulation in (2+1) dimensions is equivalent to a triad of linear equations. Specific examples are given below.

The above formulation then leads us in a natural way to certain topological invariants as given in the following theorem:

**Theorem.** The above constructed evolution equations in (2+1) dimensions possess the following integrals of motion [14]

$$\begin{aligned} K_1 &= \frac{1}{4\pi} \int \int \kappa \gamma_2 dx dy, \\ K_2 &= \frac{1}{4\pi} \int \int \tau \gamma_2 dx dy. \end{aligned} \tag{10a}$$

Proof: The proof is straightforward and follows from the various relations discussed above. For example, from (6a) we get  $(-\kappa \gamma_2)_t = (\tau_t)_y - (\gamma_{1t})_x$ , or  $(-\kappa \gamma_2)_t = (\omega_{1x} - \kappa\omega_2)_y - (\omega_{1y} + \omega_3\gamma_2 - \omega_2\gamma_3)_x$ . Hence follows the first statement of the theorem. Similarly the second one may also be proved. In terms of the unit vector  $\vec{e}_1$  these integrals of motion take the form

$$K_1 = \frac{1}{4\pi} \int \int \vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y}) dx dy,$$

and

$$K_2 = \frac{1}{4\pi} \int \int \frac{[\vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y})][\vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1xx})]}{(\vec{e}_{1x}^2)^{\frac{3}{2}}} dx dy. \quad (10b)$$

So the above quantities  $K_j$  are the conserved integrals - invariants of the (2+1) dimensional evolution equations and they play an important role in the theory. In particular, curves can be classified by the value of the topological invariants  $K_j$  but we will not pursue this aspect further. We now present a few applications of the above presented formalism in finding the geometrically equivalent counterparts of some known (2+1) dimensional spin systems. To this end, firstly, following the Ref. [10] we identify the tangent vector  $\vec{e}_1$  with the unit spin vector  $\vec{e}_1 \equiv \vec{S}(x, y, t)$ . Secondly, we introduce the following complex transformation which is a generalization of the one given in Ref. [10],

$$q(x, y, t) = a(x, y, t) \exp ib(x, y, t), \quad (11)$$

where  $a(x, y, t)$  and  $b(x, y, t)$  are functions of  $\kappa$  and  $\tau$ , to be determined, and obtain the equation for the complex function  $q(x, y, t)$  which turns out to be an interesting integrable (2+1) dimensional nonlinear evolution equation. We want to demonstrate our approach in the following examples of the (2+1) dimensional spin systems.

### A) The Myrzakulov I (M-I) equation

This equation reads as [14,15]

$$\vec{e}_{1t} = (\vec{e}_1 \wedge \vec{e}_{1y} + u\vec{e}_1)_x, \quad \vec{e}_1 = \vec{S}(x, y, t), \quad (12a)$$

$$u_x = -\vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y}). \quad (12b)$$

In this case, we obtain

$$\gamma_2 = \frac{u_x}{\kappa}, \quad (13a)$$

$$\begin{aligned} \vec{\Omega} &= (\omega_1, \omega_2, \omega_3), \\ &= \left( \frac{\kappa_{xy}}{\kappa} - \tau \partial_x^{-1} \tau_y, -\kappa_y, -\kappa \partial_x^{-1} \tau_y \right). \end{aligned} \quad (13b)$$

For the functions  $a(x, y, t)$  and  $b(x, y, t)$  in the transformation (11), we take the form

$$a = \frac{\kappa(x, y, t)}{2}, \quad b = -\partial_x^{-1}\tau(x, y, t). \quad (14)$$

Then the function  $q$  satisfies the following evolution equation,

$$iq_t(x, y, t) = q_{xy} + Vq, \quad V_x = 2(|q|^2)_y, \quad (15)$$

an equation which belongs to the class of equations originally discovered by Calogero [17] and then discussed by Zakharov [18] and recently rederived by Strachan [19]. Many of its properties have received considerable attention recently [20].

### B) The Myrzakulov III (M-III) equation

This equation which is a generalization of Eq. (12) has the form [14]

$$\vec{e}_{1t} = (\vec{e}_1 \wedge \vec{e}_{1y} + u\vec{e}_1)_x + 2f(cf + d)\vec{e}_{1y} - 4cv\vec{e}_{1x}, \quad (16a)$$

$$u_x = -\vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y}), \quad v_x = \frac{1}{4(2fc + d)^2}(\vec{e}_{1x}^2)_y, \quad (16b)$$

where  $f$ ,  $c$  and  $d$  are constants. In this case, using the same form for  $\gamma_i$ 's and  $\omega_i$ 's as in Eq. (13) and with the following choice for the functions  $a(x, y, t)$  and  $b(x, y, t)$ ,

$$a(x, y, t) = \frac{\kappa(x, y, t)}{2(2cf + d)}, \quad b(x, y, t) = 2f(cf + d)x - \partial_x^{-1}\tau, \quad (17)$$

we obtain the evolution equation

$$iq_t(x, y, t) = q_{xy} - 4ic(Vq)_x + 2d^2Vq, \quad V_x = (|q|^2)_y \quad (18)$$

This equation as well as the M-III equation (16) are integrable, that is they have the Lax representations [14] and some properties of both equations were investigated in [14-16,21,22]. Note that Eqs. (16) and (18) admit integrable reductions: when  $c = 0$  they pass on to the M-I and Zakharov equations Eqs. (12) and (15) respectively. If we consider the case  $d = 0$ , then they reduce to the M-II and Strachan equations respectively [23].

### C) The Ishimori equation

This equation has the form [24]

$$\vec{e}_{1t} = \vec{e}_1 \wedge (\vec{e}_{1xx} + \sigma^2\vec{e}_{yy}) + u_x\vec{e}_{1y} + u_y\vec{e}_{1x}, \quad (19a)$$

$$u_{xx} - \sigma^2u_{yy} = -2\sigma^2\vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y}), \quad (19b)$$

where  $\sigma^2 = \pm 1$ . For the case  $\sigma^2 = -1$ , we obtain

$$\begin{aligned}\gamma_2 &= \frac{u_{xx} + u_{yy}}{-2\kappa}, \\ \omega_1 &= \frac{\tau\omega_3 - \omega_{2x}}{\kappa},\end{aligned}\tag{20a}$$

$$\omega_2 = u_x\gamma_2 - \kappa_x + \gamma_{3y} + \gamma_1\gamma_2,\tag{20b}$$

$$\omega_3 = -\kappa\tau + \kappa u_y + u_x\gamma_3 - \gamma_{2y} + \gamma_1\gamma_3.\tag{20c}$$

If we choose  $a(x, y, t)$  and  $b(x, y, t)$  in the form

$$a = \frac{1}{2}[\kappa^2 + \gamma_2^2 + \gamma_3^2 - 2\kappa\gamma_2]^{\frac{1}{2}},\tag{21a}$$

$$b = \partial_x^{-1} \left( \tau - \frac{u_y}{2} + \frac{\gamma_2\gamma_{3x} - \gamma_3\gamma_{2x} - \kappa\gamma_{3x}}{\kappa^2 + \gamma_2^2 + \gamma_3^2 - 2\kappa\gamma_2} \right),\tag{21b}$$

then the function  $q$  satisfies the following equation

$$iq_t(x, y, t) + q_{xx} - q_{yy} - 2q\phi = 0,\tag{22a}$$

$$\phi_{xx} + \phi_{yy} + (|q|^2)_{xx} - (|q|^2)_{yy} = 0,\tag{22b}$$

where  $\phi$  is a function of  $x$ ,  $y$  and  $t$ , which is nothing but the Davey-Stewartson equation II [6].

Note that Eqs. (19) and (22) are known to be gauge equivalent to each other [25]. Here their geometrical equivalence has been established. Similarly equivalence can be established for  $\sigma^2 = 1$ .

#### D) The (2+1) dimensional isotropic Heisenberg ferromagnet model

This equation has the form [26]

$$\vec{e}_{1t} = \vec{e}_1 \wedge (\vec{e}_{1xx} + \vec{e}_{1yy}).\tag{23}$$

In this case, we get

$$\begin{aligned}\omega_1 &= \frac{\tau\omega_3 - \omega_{2x}}{\kappa}, \quad \omega_2 = -\kappa_x - \gamma_{3y} - \gamma_1\gamma_2, \\ \omega_3 &= \gamma_{2y} - \kappa\tau - \gamma_1\gamma_3.\end{aligned}\tag{24}$$

However one finds that  $\gamma_i$ 's can not be uniquely found due to lack of an equation of the form (12b) or (16b) or (19b). So Eq. (6b) does not appear to be solvable, indicating that



no straightforward linearization is available for the (2+1) dimensional isotropic Heisenberg spin equation without introducing constraints.

In conclusion, we have shown that a geometrical formulation can be developed for a class of (2+1) dimensional nonlinear evolution equations through moving space curve formalism. The formulation shows how a class of evolution equations can be associated with a triad of linear equations. Whether such a connection alone is sufficient to prove the integrability of the underlying (2+1) dimensional systems is a further intricate question. Even in (1+1) dimensions, there are some equations which though linearizable are not integrable; for example, the spherically symmetric Heisenberg ferromagnetic spin chain equation is equivalent to a nonlocal nonlinear Schrödinger equation which is of non-Painlevé type and so nonintegrable [27]. So also is the (1+1) dimensional isotropic Landau-Lifshitz equation with Gilbert damping [28] and so on. However the three examples considered in this paper are integrable namely the M-I, M-III and Ishimori equations and their geometrical equivalents namely the Zakharov, Strachan and Davey-Stewartson equations respectively are also integrable. Their linearization can be performed through the set of equations (9) and the associated linear eigenvalue problems can be constructed. On the other hand the pure isotropic Heisenberg spin chain equation in (2+1) dimensions (example D above) seems not to be even linearizable by the space curve formalism without constraints, indicating the necessity of additional scalar fields or nonlocal terms for its linearization. It is hoped that other (2+1) dimensional equations of interest like the (2+1) dimensional Korteweg-de Vries, Nizhnik-Novikov-Veselov, breaking soliton etc. equations and so on may have similar geometrical interpretations in terms of moving space curves or surfaces, for example with restrictions on the value of the curvature  $\kappa$  or torsion  $\tau$ . These possibilities are being explored at present.

Finally it is of interest to consider the connection between our approach and to that of Konopelchenko who uses the (2+1) dimensional nonlinear PDEs to induce integrable dynamics (deformations) of the induced surfaces[11]. In our approach, we consider the compatibility of three linear problems namely equations (1), (3) and (5). Two of them (eqs. (1) and (5)) can be considered as equivalent to general surfaces in orthogonal coordinates. In

the approach of Konopelchenko, for a given 2D LP, the variable coordinates  $X^i$  ( $i = 1, 2, 3$ ) of a surface in  $E^3$  are defined as some integrals over certain bilinear combinations of solutions  $\psi$  of the 2D LP and solutions  $\psi^*$  of the adjoint 2D LP and thus surfaces are induced via the solutions of 2D LPs. Then the compatibility of 2D LPs with time evolution leads to hierarchies of (2+1) dimensional nonlinear evolution equations both for the coefficients of the 2D LP and for the wave function  $\psi$ . This nonlinear evolution equation induces the corresponding evolution of the induced surface. In this sense, we believe that the two approaches differ from each other and give complementary approaches to integrable (2+1) dimensional evolution equations.

This work of M.L. forms part of a Department of Science and Technology, Government of India sponsored research project. R.M. wishes to thank Bharathidasan University for hospitality during his visits to Tiruchirapalli. S.V. acknowledges the receipt of the award Junior Research Fellowship from the Council of Scientific and Industrial Research, India.

## REFERENCES

\* electronic mail: lakshman@bdu.ernet.in

† electronic mail: cnlpmyra@satsun.sci.kz

- [1] G.L. Lamb Jr., Phys. Rev. Lett. **37**, 235 (1976); J. Math. Phys. **18**, 1654 (1977)
- [2] F. Lund and T. Regge, Phys. Rev. D **14**, 1524 (1976)
- [3] M. Lakshmanan, Phys. Lett. A **64**, 353 (1978); J. Math. Phys. **20**, 1667 (1979)
- [4] R. Sasaki, Nucl. Phys. B **154**, 343 (1979)
- [5] A. Sym, Springer Lecture Notes in Physics, edited by R. Martini (Springer, Berlin 1985)  
Vol 239, p 154
- [6] M.J. Ablowitz and P.A. Clarkson, *Solitons, Nonlinear evolution equations and Inverse Scattering* (Cambridge University Press, Cambridge, 1991)
- [7] A. Doliwa and P.M. Santini, Phys. Lett. A **185**, 373 (1994)
- [8] A.R. Bishop, L.J. Cambell and P.J. Channels, in *Fronts, Interfaces and Patterns* (North - Holland, New York, 1984)
- [9] P. Pelcé in *Dynamics of Curved Fronts*, (Academic press, New York, 1986)
- [10] M. Lakshmanan, Phys. Lett. A **61**, 53 (1977);  
M. Lakshmanan, Th. W. Ruijgrok and C. J. Thompson, Physica A **84**, 577 (1976)
- [11] B.G. Konopelchenko, Stud. Appl. Math. **96**, 9 (1996)
- [12] B.G. Konopelchenko, Phys. Lett. A **183**, 153 (1993); J. Math. Phys. **38**, 434 (1997)
- [13] J.J. Stoker, *Differential geometry*, (Wiley-Interscience, New York, 1969)
- [14] R. Myrzakulov, On some integrable and nonintegrable soliton equations of magnets I-IV (HEPI, Alma-Ata) (1987)

- [15] R. Myrzakulov, S. Vijayalakshmi, G.N. Nugmanova and M. Lakshmanan, Phys. Lett. A **233**, 391 (1997)
- [16] R. Myrzakulov and A.K.Danlybaeva; R. Myrzakulov and R.N.Syzdykova, Preprints CNLP, Alma-Ata, 1994
- [17] F. Calogero, Lett. Nuovo Cimento **14**, 43 (1975)
- [18] V. E. Zakharov, *Solitons*, edited by R.K. Bullough and P.J. Caudrey (Springer, Berlin, 1980)
- [19] I. A. B. Strachan, Inv. Prob. **8**, L21 (1992)
- [20] R. Radha and M. Lakshmanan, J. Phys. A: Math. Gen. **30**, 3229 (1997)
- [21] R. Myrzakulov and G. N. Nugmanova, Izvestiya NAS RK. Ser. fiz.-mat. **6**, 32 (1995)
- [22] G. N. Nugmanova, Ph. D. Thesis, Kazak State University, Alma-Ata (1992)
- [23] I. A. B. Strachan, J. Math. Phys. **34**, 243 (1993)
- [24] Y. Ishimori, Prog. Theor. Phys. **72**, 33 (1984)
- [25] B.G. Konopelchenko, *Solitons in multidimensions*, (World Scientific, Singapore, 1993)
- [26] M. Lakshmanan and M. Daniel, Physica A **107**, 533 (1981)
- [27] K. Porsezian and M. Lakshmanan, J. Math. Phys. **32**, 2923 (1991)
- [28] M. Daniel and M. Lakshmanan, Physica A **120**, 125 (1983)

### FIGURE CAPTIONS

Fig. 1 : Motion of a space curve in  $E^3$  with the orthogonal trihedral  $\vec{e}_1$ ,  $\vec{e}_2$  and  $\vec{e}_3$

**Running Title:** Motion of curves/surfaces and  $(2+1)$  NLEEs