On the simplest (2+1) dimensional integrable spin systems and their equivalent nonlinear Schrödinger equations

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Abstract

Using a moving space curve formalism, geometrical as well as gauge equivalence between a (2+1) dimensional spin equation (M-I equation) and the (2+1) dimensional nonlinear Schrödinger equation (NLSE) originally discovered by Calogero, discussed then by Zakharov and recently rederived by Strachan, have been estabilished. A compatible set of three linear equations are obtained and integrals of motion are discussed. Through stereographic projection, the M-I equation has been bilinearized and different types of solutions such as line and curved solitons, breaking solitons, induced dromions, and domain wall type solutions are presented. Breaking soliton solutions of (2+1) dimensional NLSE have also been reported. Generalizations of the above spin equation are discussed.

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I. INTRODUCTION

The two dimensional sigma-models with nontrivial topological structures are known to play a useful role in modern field theory. This set of models constitute a laboratory for studying two dimensional analogues of elementary particles within the framework of classical field theory[1]. In this context, important case studies concern with the existence of localized coherent structures (dromions, lumps, etc.) and other types of soliton-like solutions of some classical nonlinear field models such as, for example, the Ishimori and the other (2+1) dimensional spin systems[1-3].

Generally speaking, in (2+1) dimensions we have a number of remarkable properties, which may not exist in (1+1) dimensionsal counterparts. For example,

1°. These equations possess the so called localized coherent structures (such as dromions, lumps and so on), which may undergo both elastic and inelastic scattering depending upon the initial conditions.

2°.The corresponding spectral parameter (eigenvalue) of the associated linear problem (the Lax representation) can be dependent on the t (time) and y (space) variables and satisfy even nonlinear equations (see, for instance, [3-4]). As a consequence, the original soliton equation can have breaking solutions[4-6]. Besides, in this case, for finding solutions one even needs to use non-isospectral Inverse Scattering Transform(IST)[5].

3°. Each integrable (1+1) dimensional equation may admit several integrable (and nonintegrable) extensions[2,3] in (2+1) dimensions. For example, the KdV equation has integrable extensions such as KP, NNV and breaking soliton equations[6].

4°. Regarding the subject of the present paper, many of the (2+1) dimensional spin equations possess the topological invariant-with the so called topological charge

$$Q = \frac{1}{4\pi}\int dxdy\vec{S}\cdot\vec{S}_x\wedge\vec{S}_y, \tag{1}$$

and their solutions are classified by the integer values of Q_N, $N = 0, \pm1, \pm2,...$ Here, $\vec{S} = (S_1, S_2, S_3)$ and $\mid \vec{S} \mid^2 = S_1^2 + S_2^2 + S_3^2 = 1$ and $\vec{S}_x = \left(\frac{\partial\vec{S}}{\partial x}\right)$, $\vec{S}_y = \left(\frac{\partial\vec{S}}{\partial y}\right)$. This property can be realised, for example, in the (2+1) dimensional analogues of the well known (1+1) dimensional Heisenberg ferromagnet model (or the (1+1) dimensional isotropic Landau - Lifshitz equation (LLE))[7-9]

$$\vec{S}_t = \vec{S} \wedge \vec{S}_{xx}. \tag{2}$$

The Heisenberg ferromagnetic spin equation (2) possesses many useful (2+1) dimensional extensions: Some of them are as follows.

a) The Ishimori equation[10]

$$\vec{S}_t = \vec{S} \wedge (\vec{S}_{xx} + \epsilon^2 \vec{S}_{yy}) + u_x \vec{S}_y + u_y \vec{S}_x, \tag{3a}$$

$$u_{xx} - \epsilon^2 u_{yy} = -2\epsilon^2 \vec{S} \cdot (\vec{S}_x \wedge \vec{S}_y). \tag{3b}$$

b) The M - I equation[5]

$$\vec{S}_t = (\vec{S} \wedge \vec{S}_y + u\vec{S})_x, \tag{4a}$$

$$u_x = -\vec{S} \cdot (\vec{S}_x \wedge \vec{S}_y). \tag{4b}$$

c) The (2+1) dimensional isotropic LLE[11]

$$\vec{S}_t = \vec{S} \wedge (\vec{S}_{xx} + \vec{S}_{yy}). \tag{5}$$

Here u is a scalar function and $\epsilon^2 = \pm 1$. It turns out that eqs. (3) and (4) are integrable, while Eq. (5) is apparently nonintegrable. All the three Eqs. (3) - (5) describe the nonlinear dynamics of the classical spin systems in the plane and in the (1+1) dimensional case reduce to one and the same Eq. (2). Properties of Eqs. (3) are relatively well studied (see, for example, [3,10]). In this paper we wish to concentrate on studying the M-I equation(4). In particular, we wish to identify the geometrically equivalent (2+1) dimensional NLSE for (4) and study the nature of the nonlinear excitations admitted by the spin system.

The paper is organized as follows. In Sec.II we briefly review some necessary informations about (4) relevant to this paper. The geometrical and gauge equivalent counterpart of Eq. (4) is constructed in Sec.III and Sec.IV, respectively. Integrals of motion are discussed in Sec.V. In Sec.VI the Hirota bilinear form of Eq. (4) is derived. The solitons (line and curved), domain walls and dromion-like solutions as well as their breaking analogues are obtained in Sec.VII and breaking solitons and dromions for its equivalent counterpart are obtained in Sec.VIII. In Sec.IX we comment on the possible further extensions and we conclude in Sec.X.

II. THE M-I EQUATION

The Lax representation of Eq. (4) can be shown to take the form[5]

$$\phi_{1x} = U_1\phi_1, \tag{6a}$$

$$\phi_{1t} = V_1\phi_1 + \lambda\phi_{1y}, \tag{6b}$$

with

$$U_1 = \frac{i\lambda}{2}S, \tag{7a}$$

$$V_1 = \frac{\lambda}{4}([S, S_y] + 2iuS). \tag{7b}$$

Here

$$S = \begin{pmatrix} S_3 & rS^- \\ rS^+ & -S_3 \end{pmatrix}, \tag{8}$$

$S^{\pm} = S_1 \pm iS_2$ and λ is the eigenvalue parameter, which satisfies the following nonlinear equation

$$\lambda_t = \lambda\lambda_y. \tag{9}$$

Hence, using the compatibility of Eqs. (6a) and (6b), we get

$$iS_t = ([S, S_y] + 2iuS)_x\,, \tag{10a}$$

$$u_x = -\frac{1}{2i}tr(SS_xS_y), \tag{10b}$$

where tr denotes the trace of the matrix. This system is obviously the matrix form of Eq. (4).

Thus, for solving equation(4), we must use the non-isospectral extension of IST [5]. As a consequence, Eq. (4) admits besides the usual solutions, corresponding to constant solution of Eq. (9), breaking analogues related to the t and y dependence of λ. Note that Eq. (9) itself is the compatibility condition of the following system of the linear equations,

$$f_x = \frac{i\lambda}{2}f, \tag{11a}$$

$$f_t = \lambda f_y. \tag{11b}$$

Another useful form of Eq. (4), can be obtained by using the complex (stereographic) variable $\omega(x, y, t)$ defined through the relations

$$S^+ = S_1 + iS_2 = \frac{2\omega}{1+\mid\omega\mid^2}, \quad S_3 = \frac{1-\mid\omega\mid^2}{1+\mid\omega\mid^2}. \tag{12}$$

In this case, Eq. (4) takes the form

$$i(\omega_t - u\omega_x) + \omega_{xy} - \frac{2\omega^*\omega_x\omega_y}{1+\mid\omega\mid^2} = 0, \tag{13a}$$

$$u_x + \frac{2i(\omega_x \omega_y^* - \omega_x^* \omega_y)}{(1+ \mid \omega \mid^2)^2} = 0. \tag{13b}$$

We will use this form also frequently in our following analysis.

III. L - EQUIVALENT COUNTERPART

It is well known that in (1+1) dimensions there exists geometrical equivalence between spin systems and nonlinear Schrödinger type equations[7,12], which in ref.[5] was called the Lakshmanan eqivalence or shortly the L-equivalence (see also [12-14]). In refs.[5,13-15] a (2+1) dimensional generalization of the L-equivalence was presented. In this section we find the L-equivalent counterpart of Eq. (4). For this purpose, we will extend the geometrical method applicable to (1+1) dimensional systems suitably to the (2+1) dimensional case. We now associate a moving space curve parametrised by the arclength x, and endowed with an additional coordinate y, with the spin system[12,16]. Then the Serret-Frenet equation associated with the curve has the form

$$\vec{e}_{ix} = \vec{D} \wedge \vec{e}_i, \tag{14}$$

where

$$\vec{D} = \tau \vec{e}_1 + \kappa \vec{e}_3 \tag{15}$$

and $\vec{e}_i$'s, $i = 1, 2, 3$, form the orthogonal trihedral.

Mapping the spin variable on the unit tangent vector

$$\vec{S}(x, y, t) = \vec{e}_1, \tag{16}$$

the curvature and the torsion are given by

$$\kappa(x, y, t) = (\vec{S}_x^2)^{\frac{1}{2}}, \tag{17a}$$

$$\tau(x,y,t) = \kappa^{-2}\vec{S}\cdot(\vec{S}_x \wedge \vec{S}_{xx}). \tag{17b}$$

Due to the orthonormality nature of the trihedral, $\vec{e}_{it}.\vec{e}_i = 0$, $\vec{e}_{iy}.\vec{e}_i = 0$, $i,j = 1,2,3$ and using the compatibility condition $\vec{e}_{ixy} = \vec{e}_{iyx}$, we find the equation for the y-part

$$\vec{e}_{iy} = \vec{\gamma} \wedge \vec{e}_i, \tag{18}$$

where $\vec{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$ and

$$\gamma_1 = u + \partial_x^{-1}\tau_y, \tag{19a}$$

$$\gamma_2 = \frac{u_x}{\kappa}, \tag{19b}$$

$$\gamma_3 = \partial_x^{-1}\left(\kappa_y - \frac{\tau u_x}{\kappa}\right). \tag{19c}$$

Alternatively the trihedral $\vec{e}_i(x,y,t)$, $i = 1,2,3$ could be thought of as defining a suitable surface in E^3, (so that Eqs. (14) and (18) represent the Gauss - Weingarten equations in orthogonal coordinates and that the compatibility condition $\vec{e}_{ixy} = \vec{e}_{iyx}$ gives rise to the Codazzi -Mainardi equations), which is then set in motion. Now, from Eq. (4) and using Eqs. (14) and (18), we can easily find the time evolution of the trihedral as

$$\vec{e}_{it} = \vec{\Omega} \wedge \vec{e}_i, \tag{20}$$

with

$$\vec{\Omega} = (\omega_1, \omega_2, \omega_3) = \left(\frac{\kappa_{xy}}{\kappa} - \tau\partial_x^{-1}\tau_y, -\kappa_y, -\kappa\partial_x^{-1}\tau_y\right). \tag{21}$$

Ultimately, the compatibility condition $\vec{e}_{ixt} = \vec{e}_{itx}$, which is also consistent with the relation $\vec{e}_{iyt} = \vec{e}_{ity}$, $i = 1,2,3$ yields the following evolution equations for the curvature and torsion,

$$\kappa_t = -(\kappa\tau)_y - \kappa_x \partial_x^{-1} \tau_y, \tag{22a}$$

$$\tau_t = \left[\frac{\kappa_{xy}}{\kappa} - \tau \partial_x^{-1} \tau_y\right]_x + \kappa\kappa_y. \tag{22b}$$

On making the complex transformation[7]

$$\psi(x, y, t) = \frac{\kappa(x, y, t)}{2} \exp\left[-i \int_{-\infty}^{x} \tau(x', y, t) dx'\right], \tag{23}$$

the set of equations (22) reduces to the following (2+1) dimensional NLSE

$$i\psi_t = \psi_{xy} + r^2 V \psi, \tag{24a}$$

$$V_x = 2\partial_y \mid \psi \mid^2. \tag{24b}$$

Here, $r^2 = +1$, that is, we have the attractive type NLSE (The case $r^2 = -1$ corresponds to the repulsive case). Eq. (24) belongs to the class of equations discovered by Calogero[17] and then discussed by Zakharov[18] and recently rederived by Strachan (for $r^2 = +1$)[19]. Its Painlevé property and some exact solutions were also obtained [20]. N-soliton solutions of Eq. (24) for both the cases ($r^2 = \pm 1$) can be found in ref.[21]. Thus, we have proved that Eq. (24) is equivalent to Eq. (4) in the geometrical sense.

A. LINEARIZATION

Introducing now the complex variable corresponding to an orthogonal rotation

$$z_l = \frac{e_{2l} + i e_{3l}}{1 - e_{1l}}, \quad e_{1l}^2 + e_{2l}^2 + e_{3l}^2 = 1, \quad l = 1, 2, 3$$

the spatial and temporal evolution of the trihedral (eqs. 14, 18 and 20) can be rewritten as a set of the following three Riccati equations:

$$z_{lx} = -i\tau z_l + \frac{\kappa}{2}\left[1 + z_l^2\right], \tag{25a}$$

$$z_{ly} = -i\gamma_1 z_l + \frac{1}{2}\left[\gamma_3 + i\gamma_2\right] z_l^2 + \frac{1}{2}\left[\gamma_3 - i\gamma_2\right], \tag{25b}$$

$$z_{lt} = -i\omega_1 z_l + \frac{1}{2}\left[\omega_3 + i\omega_2\right] z_l^2 + \frac{1}{2}\left[\omega_3 - i\omega_2\right]. \tag{25c}$$

It is easy to check that Eq. (25a) is equivalent to the Serret-Frenet equations (14), (25b) is equivalent to the y-variation of the trihedral Eq. (18), while the temporal evolution (25c) is equivalent to (20).

Further introducing the transformation

$$z_l = \frac{v_2}{v_1}, \tag{26}$$

Eq. (25) can be written as a system of three coupled two component first order equations,

$$\begin{pmatrix} v_{1x} \\ v_{2x} \end{pmatrix} = \begin{pmatrix} \frac{i\tau}{2} & \frac{-\kappa}{2} \\ \frac{\kappa}{2} & \frac{-i\tau}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \tag{27a}$$

$$\begin{pmatrix} v_{1y} \\ v_{2y} \end{pmatrix} = \begin{pmatrix} \frac{i\gamma_1}{2} & \frac{-1}{2}(\gamma_3 + i\gamma_2) \\ \frac{1}{2}(\gamma_3 - i\gamma_2) & \frac{-i\gamma_1}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}, \tag{27b}$$

$$\begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix} = \begin{pmatrix} \frac{i\omega_1}{2} & \frac{-1}{2}(\omega_3 + i\omega_2) \\ \frac{1}{2}(\omega_3 - i\omega_2) & \frac{-i\omega_1}{2} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}. \tag{27c}$$

Once again one can check that the compatibility of the three sets of equations (27) gives rise to the evolution equation (22).

IV. GAUGE EQUIVALENT COUNTERPART

Next, it is also of interest to note that Eqs. (4) and (12) are gauge equivalent to each other in the sense of Zakharov and Takhtajan[22]. To obtain the gauge equivalent counterpart of Eq. (4), in the usual way we consider the following gauge transformation

$$\phi_1 = g^{-1}\phi_2, \tag{28}$$

where $g(x, y, t)$ and $\phi_2(x, y, t, \lambda)$ are arbitrary (2×2) matrix functions of the type defined on a compact manifold $S^2 = SU(2)/U(1)$. Substituting Eq. (28) into Eq. (6), after some algebra we get the following system of linear equations for ϕ_2,

$$\phi_{2x} = U_2\phi_2, \tag{29a}$$

$$\phi_{2t} = V_2\phi_2 + \lambda\phi_{2y} \tag{29b}$$

with

$$U_2 = \frac{i\lambda}{2}\sigma_3 + G, \quad G = \begin{pmatrix} 0 & \phi \\ -r^2\phi^* & 0 \end{pmatrix}, \quad r^2 = +1, \tag{30a}$$

$$V_2 = -i\sigma_3\left(\frac{VI}{2} + G_y\right), \quad I = diag(1,1), \tag{30b}$$

$$V = 2\partial_x^{-1}\partial_y\left(\mid \psi \mid^2\right). \tag{30c}$$

The compatibility condition of Eq. (29) along with (9) becomes (24), that is, Eq. (4) and Eq. (24) are gauge equivalent to each other. The above transformation is in fact reversible and we can similarly prove that Eq. (24) is gauge equivalent to Eq. (4). It is also of interest to note that the set of linear Eqs. (27) can be recast in the form (29) after suitable transformations.

Next, we present some important formulae which are just consequences of the geometrical/gauge equivalence of Eqs. (4) and (24). We have

$$tr(S_x^2) = 8 \mid \psi \mid^2 = 2\vec{S}_x^2. \tag{31a}$$

In a similar manner we find that

$$-2i\vec{S}\cdot(\vec{S}_x\wedge\vec{S}_{xx}) = tr(SS_xS_{xx}) = 4(\psi^*\psi_x - \psi\psi^*_x). \tag{31b}$$

These relations are obviously equivalent to Eq. (23). One notes that these are of the same form as in the case of (1+1) dimensional Heisenberg spin chain [7,8].

V. INTEGRALS OF MOTION

The spin Eq. (4) allows an infinite number of integrals of motion as a consequence of integrability. These integrals can be consructed using for example the Lax representation(6). However some of the integrals can be constructed using that L-equivalence property. Now from (22a), it follows that

$$(\kappa^2)_t = [-\kappa^2\partial_x^{-1}\tau_y]_x + [-k^2\tau]_y. \tag{32a}$$

Hence we get the first integral

$$K_1 = \int \kappa^2 dxdy. \tag{32b}$$

Similarly (22b) leads to

$$K_2 = \int \kappa^2\tau dxdy. \tag{33}$$

In terms of the spin vector the corresponding two conservation laws are

$$\left(\vec{S}_x^2\right)_t + \partial_x\left[\vec{S}_x^2\partial_x^{-1}\left(\frac{\vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}}{\vec{S}_x^2}\right)_y\right] + \partial_y\left[\vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}\right] = 0, \tag{34}$$

$$\left[\vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}\right]_t + \partial_x\left[\frac{(\vec{S}_x^2)_x(\vec{S}_x^2)_y}{4\vec{S}_x^2} + \vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}\partial_x^{-1}\left(\frac{\vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}}{\vec{S}_x^2}\right)_y\right]$$
$$+\partial_y\left\{\frac{(\vec{S}_x^2)_x^2}{4\vec{S}_x^2} + \frac{\left(\vec{S}\cdot\vec{S}_x\wedge\vec{S}_{xx}\right)^2}{\vec{S}_x^2} - \frac{\left(\vec{S}_x^2\right)_{xx}}{2} - \frac{\vec{S}_x^4}{4}\right\} = 0. \tag{35}$$

Note that these integrals have the same forms as in the (1+1) dimensional case. More interesting integrals of purely (2+1) dimensional nature can be obtained from the condition

$$\vec{e}_{jxy} = \vec{e}_{jyx}. \tag{36}$$

Making use of the various relations in Sec.III and after some algebra we have

$$(-\kappa\gamma_2)_t = (-\gamma_{1t})_x + (\tau_t)_y, \tag{37}$$

$$(-\tau\gamma_2)_t = (\gamma_{3t})_x + (-\kappa_t)_y. \tag{38}$$

These equations give two integrals

$$K_3 = \int (-\kappa\gamma_2) dxdy, \tag{39}$$

and

$$K_4 = \int (-\tau\gamma_2) dxdy. \tag{40}$$

In terms of the spin vector $\vec{S}$ these integrals of motion take the forms

$$K_1 = \int (\vec{S}_x^2) dxdy, \tag{41}$$

$$K_2 = \int \vec{S} \cdot \vec{S}_x \wedge \vec{S}_{xx} dxdy, \tag{42}$$

$$K_3 = \int \vec{S} \cdot (\vec{S}_x \wedge \vec{S}_y) dxdy \tag{43}$$

and

$$K_4 = \int \frac{[\vec{S} \cdot (\vec{S}_x \wedge \vec{S}_y)][\vec{S} \cdot (\vec{S}_x \wedge \vec{S}_{xx})]}{(\vec{S}_x^2)^{\frac{3}{2}}} dxdy. \tag{44}$$

Note that K_3 is the topological charge given by Eq. (1) to within a constant. One can proceed to find the other integrals of motion using the eigenvalue problem(29).

VI. HIROTA BILINEAR FORM

In order to solve the Cauchy initial value problem of Eq. (4), it will be of interest to investigate the system in the framework of the inverse spectral transform method, for example, by the d-bar dressing method[2,3]. However, for our present purpose we concentrate on exact analytic solutions of Eq. (4). In doing so, it is convenient to rewrite Eq. (4) or its equivalent stereographic form (13) in the Hirota bilinear form. On writing

$$\omega = \frac{g}{f}, \tag{45}$$

Eq. (4) or Eq. (13) becomes

$$(iD_t - D_xD_y)(f^* \circ g) = 0, \tag{46a}$$

$$(iD_t - D_xD_y)(f^* \circ f - g^* \circ g) = 0, \tag{46b}$$

$$D_x(f^* \circ f + g^* \circ g) = 0, \tag{46c}$$

while the potential u takes the form

$$u(x,y,t) = -\frac{iD_y(f^* \circ f + g^* \circ g)}{f^* \circ f + g^* \circ g}, \tag{46d}$$

where g and f are complex valued functions. Here D_x is the Hirota bilinear operator, defined by

$$D_x^k D_y^m D_t^n (f \circ g) = (\partial_x - \partial_{x\prime})^k (\partial_y - \partial_{y\prime})^m (\partial_t - \partial_{t\prime})^n f(x,y,t) g(x,y,t) \|_{x=x\prime, y=y\prime, t=t\prime} \tag{47}$$

Using the above definition of the D-operator, we get from (31d) that

$$u_x = -2i\left[D_y(f \circ g)D_x(f^* \circ g^*) - c.c\right]. \tag{48a}$$

In terms of g and f, the spin field takes the form

$$S^+ = \frac{2f^*g}{\mid f\mid^2 + \mid g\mid^2}, \quad S_3 = \frac{\mid f\mid^2 - \mid g\mid^2}{\mid f\mid^2 + \mid g\mid^2}. \tag{48b}$$

Eq. (46) represents the starting point to obtain interesting classes of solutions for the spin system (4).The construction of the solutions is standard. One expands the functions g and f as a series

$$g = \epsilon g_1 + \epsilon^3 g_3 + \epsilon^5 g_5 + \cdots\cdot, \tag{49a}$$

$$f = 1 + \epsilon^2 f_2 + \epsilon^4 f_4 + \epsilon^6 f_6 + \ldots\ldots \tag{49b}$$

Substituting these expansions into (46 a,b,c) and equating the coefficients of ϵ, one obtains the following system of equations from (46a):

$$\epsilon^1 : ig_{1t} + g_{1xy} = 0, \tag{50a}$$

$$\epsilon^3 : \left[i\partial_t + \partial_x\partial_y\right] g_3 = \left[iD_t - D_xD_y\right](f_2^*.g_1), \tag{50b}$$

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$$\epsilon^{2n+1} : \left[i\partial_t + \partial_x\partial_y\right] g_{2n+1} = \sum_{k+m=n}\left[iD_t - D_xD_y\right](f_{2k}^*.g_{2m+1}), \tag{50c}$$

and from (46b):

$$\epsilon^2 : i\partial_t(f_2^* - f_2) - \partial_x\partial_y(f_2^* + f_2) = [iD_t - D_xD_y]\,(g_1^*.g_1), \tag{51a}$$

$$\epsilon^4 : i\partial_t(f_4^* - f_4) - \partial_x\partial_y(f_4^* + f_4) = [iD_t - D_xD_y]\,(g_1^*.g_3 + g_3^*.g_1 - f_2^*.f_2), \tag{51b}$$

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$$\epsilon^{2n} : i\partial_t(f_{2n}^* - f_{2n}) - \partial_x\partial_y(f_{2n}^* + f_{2n}) = (iD_t - D_xD_y)\left(\sum_{n_1+n_2=n-1} g_{2n_1+1}^*.g_{2n_2+1}\right)$$

$$-(iD_t - D_xD_y)\left(\sum_{m_1+m_2=n} f_{2m_1}^*.f_{2m_2}\right). \tag{51c}$$

Further from (46c), we have the following:

$$\epsilon^2 : \partial_x(f_2^* - f_2) = -D_x(g_1^*.g_1), \tag{52a}$$

$$\epsilon^4 : \partial_x(f_4^* - f_4) = -D_x(g_1^*.g_3 + g_3^*.g_1 + f_2^*.f_2), \tag{52b}$$

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$$\epsilon^{2n} : \partial_x(f_{2n}^* - f_{2n}) = -D_x\left[\sum_{n_1+n_2=n-1} (g_{2n_1+1}^*.g_{2n_(2+1)} + \sum_{n_1+n_2=n} f_{2n_1}^*.f_{2n_2})\right]. \tag{52c}$$

Solving recursively the above equations, we obtain many interesting classes of solutions to Eq. (4).

VII. SOLUTIONS OF THE SPIN SYSTEM

Using the results of the previous section, we are in a position to construct many exact solutions such as solitons, domain walls, breaking solitons and induced dromions of Eq. (4). To obtain such solutions, we can use Eqs. (46) as the starting point.

A. The 1-line soliton and curved soliton solutions

On solving Eq. (50a), we obtain

$$g_1 = \sum_{j=1}^{N} \exp \chi_j, \quad \chi_j = a_j x + b_j(y,t) + c_j, \tag{53a}$$

where $b_j(y,t)$ is an arbitrary function of (y,t) satisfying the relation

$$b_j(y,t) = b_j(\rho) = b_j(y + i a_j t), \tag{53b}$$

and a_j and c_j are complex constants. In order to construct the one soliton solution, we take the case of $N = 1$ in (53a) and substitute it in Eq. (51a). We obtain

$$f_2 = \exp(\chi_1 + \chi_1^* + \psi). \tag{54a}$$

Using Eqs. (51a) and (52a), it is found that

$$\exp \psi = -\frac{a_1^2}{(a_1 + a_1^*)^2}. \tag{54b}$$

If we use the above forms of g_1 and f_2 in Eqs. (50b), (51b) and (52b), we can see that $g_j = 0$ for $j \geq 3$ and $f_j = 0$ for $j \geq 4$. By substituting the values of g_1 and f_2 in Eqs. (48a) and (48b), we obtain the expressions for the 1-soliton solution for the spin components and for the potential for example with the choice $\exp c_1 = \frac{2a_{1R}}{a_1^*}$ as

$$S^+(x,y,t) = \frac{2a_{1R}}{a_{1R}^2 + a_{1I}^2} \exp i\chi_{1I} \left[i a_{1I} - a_{1R} tanh \chi_{1R} \right] sech \chi_{1R}, \tag{55a}$$

$$S_3(x,y,t) = 1 - \frac{2a_{1R}^2}{a_{1R}^2 + a_{1I}^2} sech^2 \chi_{1R}, \tag{55b}$$

and the potential as

$$u(x,y,t) = \frac{2a_{1R}}{a_{1R}^2 + a_{1I}^2} \left(a_{1I} b_{1R}' - a_{1R} b_{1I}' \right) sech^2 \chi_{1R}. \tag{55c}$$

It follows from Eqs. (55) that one can identify two types of solitons:

1. Line solitons:

If we choose the function $b_1(y,t)$ in Eq. (53b) as

$$b_1(y,t) = b_1 y + i b_1 a_1 t, \tag{56a}$$

then

$$\chi_1 = a_1 x + b_1 y + i a_1 b_1 t + c_1 \tag{56b}$$

where b_1 is now a complex constant, and Eqs. (55) correspond to the line solitons. In this case the spin vector $\vec{S} \rightarrow (0,0,1)$ for fixed t as x, $y \rightarrow \pm\infty$, except along the line

$$\chi_{1R} = a_{1R} x + b_{1R} y - (a_{1R} b_{1I} + a_{1I} b_{1R}) t + c_{1R} = 0, \tag{57}$$

where it is still bounded.

2. Curved solitons:

However for arbitrary form of $b_1(\rho) = b_1(y + i a_1 t)$ in Eq. (53b) and for fixed (y,t), it follows from (55) that $\vec{S} \rightarrow (0,0,1)$ as $x, y \rightarrow \pm\infty$ and the wavefront itself is defined by the equation

$$\chi_{1R} = a_{1R} x + b_{1R}(\rho) + c_{1R} = 0. \tag{58}$$

We may call such solitons (which do not decay along the curve(58)) as curved solitons[23].

B. The 2-soliton and N-soliton solutions

To generate a 2 line or curved soliton solution (2-SS), we take $N = 2$ in (53a) and hence g_1 takes the form

$$g_1 = \exp\chi_1 + \exp\chi_2. \tag{59}$$

Substituting (59) in (50)-(52), after some calculation we obtain

$$f_2 = N_{11}\exp(\chi_1 + \chi_1^*) + N_{12}\exp(\chi_1 + \chi_2^*) + N_{21}\exp(\chi_1^* + \chi_2) + \\ N_{22}\exp(\chi_2 + \chi_2^*), \tag{60a}$$

$$g_3 = L_{12}N_{11}N_{12}\exp(\chi_1 + \chi_1^* + \chi_2) + L_{12}N_{22}N_{21}\exp(\chi_1 + \chi_2 + \chi_2^*), \tag{60b}$$

$$f_4 = L_{12}L_{12}^*N_{11}N_{12}N_{21}N_{22}\exp(\chi_1 + \chi_1^* + \chi_2 + \chi_2^*), \tag{60c}$$

where

$$N_{rs} = -\frac{a_r^2}{(a_r + a_s^*)^2}, \quad L_{rs} = -\frac{(a_r - a_s)^2}{a_s^2}, \tag{60d}$$

and $g_j = 0$ for $j \geq 5$ and $f_j = 0$ for $j \geq 6$. Inserting (60) into (48b) we get

$$S^+(x, y, t) = 2\frac{(1 + f_2^* + f_4^*)(g_1 + g_3)}{|\, 1 + f_2 + f_4\,|^2 + |\, g_1 + g_3\,|^2}, \tag{61a}$$

$$S_3(x, y, t) = \frac{|\, 1 + f_2 + f_4\,|^2 - |\, g_1 + g_3\,|^2}{|\, 1 + f_2 + f_4\,|^2 + |\, g_1 + g_3\,|^2}, \tag{61b}$$

and similarly the expression for the potential can also be obtained from (48a) or (46d).

Finally by taking g_1 as

$$g_1 = \sum_{j=1}^{N} \exp\chi_j$$

and extending the above procedure, one can obtain the N-SS also.

C. The domain wall type solution

The soliton solutions of (55) and (61) correspond to the boundary condition

$$\vec{S}(x,y,t) = (0.0,1), \;\; as \;\; x, \;\; y \to \pm\infty. \tag{62}$$

Another class of physically interesting solutions are the domain wall type solutions, which have the asymptotic form

$$\vec{S}(x,y,t) = (0,0,\pm 1), as \;\; x, \;\; y \to \pm\infty \tag{63}$$

In order to obtain domain wall solutions in the present model, we make the choice

$$\omega(x,y,t) = g(x,y,t), \quad f(x,y,t) = 1. \tag{64}$$

Then, Eq. (46) reduces to

$$ig_t + g_{xy} = 0, \tag{65a}$$

$$g^*_x g_y + g^*_y g_x = 0, \tag{65b}$$

$$g^*_x g - g^* g_x = 0, \tag{65c}$$

which is consistent with Eq. (13). Alternately, we can use another substitution

$$\omega(x,y,t) = \frac{1}{f(x,y,t)}, \quad g(x,y,t) = 1 \tag{66}$$

Here also it follows from (46) that

$$if^*_t - f^*_{xy} = 0 \tag{67a}$$

$$f_x^* f_y + f_x f_y^* = 0 \tag{67b}$$

$$f_x^* f - f^* f_x = 0. \tag{67c}$$

Comparing Eqs. (65) and (67), we see that if $\omega(x, y, t)$ is a solution of Eq. (13), so also

$$\omega'(x, y, t) = \pm \frac{1}{\omega(x, y, t)} \tag{68}$$

are solutions of Eq. (13). This is an obvious consequence of the fact that Eq. (13) is invariant under inversion.

Now, we find the simplest non-trivial solutions for example of Eq. (65). Let us take the ansatz

$$g = \exp\left(ax + iby - abt\right) \tag{69}$$

where a, b are real constants. The components of the spin vector $\vec{S}$ are given by

$$S^+(x, y, t) = \frac{\exp iby}{cosh[a(x - bt - x_0)]}, \tag{70a}$$

$$S_3(x, y, t) = -tanh[a(x - bt - x_0)]. \tag{70b}$$

We can also have a more general solution of the form

$$g = \exp\left[ax + im(y, t)\right], \tag{71}$$

where a is a real constant and $m(y, t)$ is an arbitrary function of y and t. From Eq. (65a), it follows that

$$m = m(\rho) = m(y + iat), \rho = y + iat. \tag{72}$$

Expressions for the spin components are then given by

$$S^{+}(x,y,t) = \frac{\exp[iRe(m(\rho))]}{cosh[ax - Im(m(\rho))]}, \tag{73a}$$

$$S_3(x,y,t) = -tanh[ax + Re(m(\rho))], \tag{73b}$$

and the potential is

$$u(x,y,t) = -2m'_R\{1 + \exp\left[-2(ax - m_I)\right]\}^{-1}, \tag{74}$$

where the $'$ denotes the differentiation with respect to the real part of the argument. Naturally even more general solutions can be obtained by taking more general forms for $g(x,y,t)$ than (69) or (71).

D. The breaking soliton solution

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We have already noted in Sec.II that for the present system (4), we have a non-isospectral problem, as the spectral parameter λ satisfies Eq. (9). The above presented solutions all correspond to the constant solution of Eq. (9), namely $\lambda = \lambda_1 =$ constant. One may consider other interesting solutions of Eq. (9). For example, one can have a special solution

$$\lambda = \lambda_1 = \delta(y,t) + i\xi(y,t) = \frac{y + k + i\eta}{q - t}, \tag{75}$$

where q, k and η are real constants. Corresponding to this case, we may call the resulting solutions of Eqs. (4) and (24) as breaking solitons[4]. Using the Hirota method, one can also construct the breaking 1-SS of Eq. (4) associated with (75). For this purpose, we take g_1 in the form

$$g = g_1 = \exp\chi, \quad \chi = ax + m + c = \chi_R + i\chi_I, \tag{76}$$

where $a = a(y,t)$, $m = m(y,t)$ and $c = c(t)$ are functions to be determined. Substituting (76) into the first of Eq. (53), we get

$$ia_t + aa_y = 0, \quad im_t + am_y = 0, \quad iA_t + Aa_y = 0, \tag{77}$$

where $A = \exp(c)$. Particular solutions of Eqs. (77) have the forms

$$a = -i\lambda = \frac{\eta - i(y+k)}{q-t}, \quad m = m\left(\frac{y+k+i\eta}{q-t}\right), \quad A = \frac{A_0}{q-t}, \tag{78}$$

where η, k, q and A_0 are some constants. From Eqs. (50)-(52), we obtain

$$f_2 = B\exp 2\chi_R, \quad B = \frac{(y+k+i\eta)^2}{4\eta^2}. \tag{79}$$

Now, we can write the breaking 1-SS of Eq. (4) (using equations (48b), (76)-(79)),

$$S^+(x,y,t) = \frac{\exp\left[i\chi_{1I} + \ln\frac{2\eta}{y+k+i\eta}\right] sech\left[\chi_{1R} + \ln\frac{y+k+i\eta}{2\eta}\right]}{1 + \frac{\eta^2}{(y+k)^2+\eta^2} sech\left[\chi_{1R} + \ln\frac{y+k+i\eta}{2\eta}\right] sech\left[\chi_{1R} + \ln\frac{y+k-i\eta}{2\eta}\right]}, \tag{80a}$$

$$S_3(x,y,t) = \frac{1 - \frac{\eta^2}{(y+k)^2+\eta^2} sech\left[\chi_{1R} + \ln\frac{y+k+i\eta}{2\eta}\right] sech\left[\chi_{1R} + \ln\frac{y+k-i\eta}{2\eta}\right]}{1 + \frac{\eta^2}{(y+k)^2+\eta^2} sech\left[\chi_{1R} + \ln\frac{y+k+i\eta}{2\eta}\right] sech\left[\chi_{1R} + \ln\frac{y+k-i\eta}{2\eta}\right]}, \tag{80b}$$

where $\chi_1 = \chi$ as defined in Eq. (76). We see that the solution (80) corresponds to an algebraically decaying solution for large x, y.

E. Localized coherent structures (dromions)

Next, we present the dromion type localized solutions of Eq. (4), the so-called induced localized structures/or induced dromions[23] for the potential $u(x,y,t)$. This is possible by utilising the freedom in the choice of the arbitrary functions b_{1R} and b_{1I} of b_1 in Eq. (55c) and (53b). For example, if we make the ansatz

$$b_{1I}(\rho_R) = kb_{1R}(\rho_R) = tanh(\rho_R), \tag{81}$$

$$u = 2\eta(\xi - \eta k) sech^2 \rho_R sech[\eta x + tanh \rho_R - \eta x_0], \tag{82}$$

where $\rho_R = y - a_{1I} t$ and k is a constant. Similarly, the expressions for the spin can be obtained from Eqs. (55 a,b). The solution (82) for $u(x, y, t)$ decays exponentially in all the directions, eventhough the spin $\vec{S}$ itself is not fully localized. Analogously we can construct another type of "induced dromion" solution with the choice

$$b_{1I} = k b_{1R} = \int \frac{d\rho_R}{(\rho_R + \rho_0)^2 + 1} + b_0, \tag{83}$$

where ρ_0 and b_0 are constants, so that

$$u(x, y, t) = \frac{2\eta(\xi - ky)}{(\rho_R + \rho_0)^2 + 1} sech^2 \left[\eta x + \int \frac{d\rho_R}{(\rho_R + \rho_0)^2 + 1} - \eta x_0 \right]. \tag{84}$$

Proceeding in this way we can construct even more general solutions and multidromions for the potential.

VIII. SOLUTIONS OF (2+1) DIMENSIONAL NLSE

In this section, we wish to consider briefly the corresponding solutions of the equivalent generalized NLSE Eq. (24). Already this equation has received some attention in the literature. The following types of solutions are available[23,24]:

a) Line solitons,

b) Induced dromions.

Now, we can construct the N-breaking soliton solutions of Eq. (24) as well. As an example, let us obtain the 1-breaking soliton solution of Eq. (24). The Hirota bilinear form of Eq. (24) can be obtained by using the transformation.

$$\psi = \frac{h}{\phi} \tag{85}$$

as [20,24]

$$[iD_t + D_x D_y](h \circ \phi) = 0, \tag{86a}$$

$$D_x^2(\phi \circ \phi) = 2hh^*. \tag{86b}$$

We look for the 1-breaking soliton solution in the following form:

$$h = \exp \chi, \tag{87a}$$

$$\phi = 1 + \phi_2, \tag{87b}$$

where$\chi = b(y,t)x + n(y,t) + c(t)$. Substituting (87) into (86), we get

$$ib_t + bb_y = 0, \tag{88a}$$

$$in_t + bn_y = 0, \tag{88b}$$

$$iB_t + Bb_y = 0, \tag{88c}$$

and

$$\phi_2 = \frac{1}{(b+b^*)^2} \exp \chi + \chi^* = \exp 2(b_R x + n_R + \chi_0), \tag{89}$$

where $\exp 2\chi_0 = \frac{1}{4b_R^2}$, $B = \exp c(t)$ and $b_R = b_R(t) = Re(b)$. Now, the formula (85) provides us the 1-breaking soliton solution of Eq. (24),

$$\psi(x,y,t) = \frac{b_R(t) \exp i\,[b_I(y,t)x + n_I(y,t) + c_0]}{cosh\,[b_R x + n_R(y,t) + \chi_0]}, \tag{90}$$

where $b(y,t) = b_R + ib_I$, $n(y,t) = n_R + in_I$ and $B(t)$ are the solutions of Eqs. (88). Just as in the case of Eq. (77), if we take the following particular solutions of the system of Eqs. (88);

$$b = -i\lambda = \frac{\eta - i(y+k)}{q-t}, n = \frac{y+k+i\eta}{q-t}, B = \frac{B_0}{(q-t)}, \tag{91}$$

then the 1-breaking soliton solution of Eq. (24) takes the form

$$\psi(x,y,t) = -\frac{\eta}{q-t} \exp i \left[-\frac{y+k}{q-t}x + n_I(y,t) + c_0 \right] sechZ, \tag{92}$$

where $Z = \frac{\eta}{q-t}x + n_R(y,t) + \chi_0$ and c_0, χ_0 are constants.

Similarly, we obtain the breaking N-SS of (24). In this case we can take the ansatz

$$g_1 = \sum_{j=1}^{N} \exp \chi_j \tag{93}$$

with $\chi_j = b_j(y,t)x + n_j(y,t) + c_j(t)$. Inserting (93) into (92), one is lead to

$$ib_{jt} + b_j b_{jy} = 0, \tag{94a}$$

$$in_{jt} + b_j n_{jy} = 0, \tag{94b}$$

$$iB_{jt} + B_j b_{jy} = 0, \tag{94c}$$

Proceeding as before, one can obtain breaking N-soliton solution.

IX. SIMPLEST INTEGRABLE EXTENSIONS

As mentioned above in ref.[5] (see also [25,26]) a new class of (2+1) dimensional integrable spin equations was proposed. In particular, Eq. (4) is a particular reduction of the following so called M-III equation (according to the notations of ref.[5])

$$\vec{S}_t = (\vec{S} \wedge \vec{S}_y + u\vec{S})_x + 2l(cl+d)\vec{S}_y - 4cV\vec{S}_x + \vec{S} \wedge A\vec{S}, \tag{95a}$$

$$u_x = -\vec{S} \cdot (\vec{S}_x \wedge \vec{S}_y), \tag{95b}$$

$$V_x = \frac{1}{4(2lc+d)^2}(\vec{S}_x^2)_y, \tag{95c}$$

where l, c and d are constants and A is the anisotropy tensor. This equation possesses some integrable reductions. For example, one obtains

a) the isotropic M - I equation (4), when c = 0, A = 0;

b) the isotropic M - II equation, when d = 0, A = 0;

c) the isotropic M - III equation, when $c \neq 0 \neq d$, $A = 0$;

d) the anisotropic M - I equation, when c = 0

and so on. All these equations are integrable in the sense that each one of such reduction has a Lax representation[5]. As an example, let us present the associated linear problem for the isotropic M - III equation[5],

$$\psi_{1x} = U_1\psi_1, \tag{96a}$$

$$\psi_{1t} = V_1\psi_1 + (2c\lambda^2 + 2d\lambda)\psi_{1y}, \tag{96b}$$

with

$$U_1 = [ic(\lambda^2 - l^2) + id(\lambda - l)]S + \frac{c(\lambda - l)}{2cl + d}SS_x, \tag{97a}$$

$$V_1 = [2c(\lambda^2 - l^2) + 2d(\lambda - l)]B + \lambda^2 F_2 + \lambda F_1 + F_0, \tag{97b}$$

where

$$F_2 = -4ic^2 VS,$$

$$F_1 = -4icdVS - \frac{4c^2V}{2cl+d}VSS_x - \frac{ic}{2cl+d}S\{(SS_x)_y - [SS_x, B]\}, \tag{98}$$

$$F_0 = -lF_1 - l^2 F_2.$$

Here

$$B = \frac{1}{4}([S, S_y] + 2iuS), S = \vec{S} \cdot \vec{\sigma}$$

From these equations one can deduce the corresponding Lax representations of the isotropic M -I and M - II equations by choosing c = 0 and d = 0, respectively. We note that Eq. (95) for the isotropic M-III equation reduction case is gauge[27] and L-equivalent[14] to the following equation

$$i\phi_t = \phi_{xy} - 4ic(V\phi)_x + 2d^2 V\phi, \tag{99a}$$

$$V_x = (\mid \phi \mid^2)_y. \tag{99b}$$

This equation has two integrable cases: a) if c=0, Eq. (99) reduces to Eq. (24); b) if d=0, we get the Strachan equation[19]. The Lax representations corresponding to (99) is obtained as follows.

$$\psi_{2x} = U_2 \psi_2, \tag{100a}$$

$$\psi_{2t} = V_2 \psi_2 + (2c\lambda^2 + 2d\lambda)\psi_{2y}, \tag{100b}$$

with

$$U_2 = i[(c\lambda^2 + d\lambda)\sigma_3 + (2c\lambda + d)Q], \tag{101a}$$

$$V_2 = \lambda^2 B_2 + \lambda B_1 + B_0. \tag{101b}$$

Here

$$B_2 = -4ic^2 V\sigma_3,$$

$$B_1 = -4idcV\sigma_3 + 2c\sigma_3 Q_y - 8ic^2 VQ, \tag{102}$$

$$B_0 = \frac{d}{2c}B_1 - \frac{d^2}{4c^2}B_2,$$

$$Q = \begin{pmatrix} 0 & \phi \\ -\phi^* & 0 \end{pmatrix}.$$

X. CONCLUSIONS

In this paper we have obtained many interesting classes of exact solutions of Eq. (4) and its equivalent counterpart Eq. (24), after estabilishing their L- equivalence and gauge equivalence. The equivalence concept was also extended to more general cases in Sec.IX. Another interesting class of solutions are the periodic solutions which for Eq. (4) are given by

$$S_1(x,y,t) = \frac{CD - AB}{AD - BC}, S_2(x,y,t) = -i\frac{CD + AB}{AD - BC},$$

$$S_3(x,y,t) = \frac{AD + BC}{AD - BC}, \tag{103}$$

where

$$A = \theta(z - \frac{r}{2} - \frac{\delta}{2}, \tau), \quad B = \theta(z + \frac{r}{2} - \frac{\delta}{2}, \tau),$$

$$C = \theta(z - \frac{r}{2} + \frac{\delta}{2}, \tau), \quad D = \theta(z + \frac{r}{2} + \frac{\delta}{2}, \tau). \tag{104}$$

Here, $\theta(z,\tau)$ is the Riemann θ functions and r, δ are some constants. More detailed investigation of the periodic solution(103) will be considered elsewhere.

Also we would like to note that the extended version of the L-equivalence method which we used in section III works out for many other (2+1) dimensional spin equations as well, for instance, to the Ishimori equation[14]. In the later case, the unit vectors $\vec{e}_k$, $k = 1, 2, 3$, satisfy the Eqs. (14) and (18), while the vector $\vec{e}_1 \equiv \vec{S}$ satisfies the Ishimori equation

$$\vec{e}_{1t} = \vec{e}_1 \wedge (\vec{e}_{1xx} + \epsilon^2 \vec{e}_{1yy}) + u_x \vec{e}_{1y} + u_y \vec{e}_{1x}, \tag{105a}$$

$$u_{xx} - \epsilon^2 u_{yy} = -2\epsilon^2 \vec{e}_1 \cdot (\vec{e}_{1x} \wedge \vec{e}_{1y}). \tag{105b}$$

Proceeding as in sec.III, we can obtain an evolution equation for the curvature κ and torsion τ in the form

$$\kappa_t = \omega_{3x} + \tau\omega_2, \tag{106a}$$

$$\tau_t = \omega_{1x} - \kappa\omega_2, \tag{106b}$$

where

$$\omega_1 = \frac{\tau\omega_3 - \omega_{2x}}{\kappa}, \quad \omega_2 = u_x\gamma_2 - \kappa_x - \epsilon^2\gamma_{3y} + \gamma_1\gamma_2,$$

$$\omega_3 = -\kappa\tau + \kappa u_y + u_x\gamma_3 + \epsilon^2\gamma_{2y} - \gamma_1\gamma_3, \quad \gamma_2 = \frac{(u_{xx} - \epsilon^2 u_{yy})}{2\epsilon^2\kappa}$$

and the values of γ_1 and γ_3 can be obtained from the relations (19a) and (19c) respectively. Then the complex transformation

$$\phi = a \exp ib, \tag{107}$$

where

$$a = \frac{1}{2}[\kappa^2 + \gamma_2^2 + \gamma_3^2 - 2\kappa\gamma_2]^{\frac{1}{2}}, \tag{108a}$$

$$b=\partial_x^{-1}(\tau-\frac{u_y}{2}+\frac{\gamma_2\gamma_{3x}-\gamma_3\gamma_{2x}-\kappa\gamma_{3x}}{\kappa^2+\gamma_2^2+\gamma_3^2-2\kappa\gamma_2}) \tag{108b}$$

satisfies the Davey-Stewartson equation for $\epsilon^2=-1$,

$$i\phi_t=\phi_{yy}-\phi_{xx}+2\phi u, \tag{109a}$$

$$u_{yy}+u_{xx}=(\mid\phi\mid)^2_{xx}-(\mid\phi\mid)^2_{yy}, \tag{109b}$$

where u is a function of x, y and t. Here κ and τ have the forms(17). Thus the geometrical equivalence between (105) and (109) can be estabilished using the generalized transformation (107).

To summarize, in this paper, we have shown the equivalence of the (2+1) dimensional spin equation (4) and the generalized (2+1) dimensional NLSE (24). Besides we have found interesting classes of exact solutions for Eq. (4) and NLSE(24). However, many questions remain open and deserve further investigation. These include the existence of, for example of localized coherent structures (other than the induced dromion-like solution presented here), the determination of the Hamiltonian structure and the finding of possible physical applications of the solutions obtained above. Another interesting problem is the classification of solitons by the values of the topological charge. These questions are being pursued further.

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