Proc. Indian Acad. Sci. (Engg. Sci.), Vol. 3, Part 1, March 1980, pp. 31–36. © Printed in India.

Continuous and dispersed phase coefficients for heat and mass transfer involving single-file drops

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MS received 25 September 1978; revised 1 May 1979

Abstract. An attempt has been made in this paper at modelling analogous situations of heat or mass transfer involving single-file drops in liquid-liquid systems. Experimental data on continuous and dispersed phase coefficients have been correlated.

Keywords. Single file drops; dispersed phase coefficients; heat transfer; mass transfer.

1. Introduction

The interrelationships between heat and mass transfer processes have received considerable attention in recent years. Various models have been proposed in literature for predicting both continuous phase and the dispersed phase coefficients for heat and mass transfer. Studies pertaining to heat or mass transfer to and from liquid drops are of interest in the design of equipment involving dispersion of drops in continuous media by distributors or nozzles. However, no attempt has been made so far to obtain suitable and analogous relationships for estimation of heat and mass transfer rates in direct contact liquid-liquid systems.

2. Background of the problem

In heat or mass transfer, the transfer during free rise or the travel stage of the mobile drops is important for the design of column contactors. Theoretical and semi-empirical correlations have been proposed by many investigators for heat transfer (Calderbank & Korchinski 1956; Handlos & Baron 1957; Elzinga & Banchero 1961; Sideman & Shabtai 1964; Raghavan *et al* 1978) as well as mass transfer (Geddes 1946; Kronig & Brink 1950; Garner *et al* 1959; Garner & Hale 1955; Ruby and Elgin 1955; Handlos & Baron 1957; Steinberger & Treybal 1960; Griffith 1960; Venkateshwar Rao & Kaparthi 1966; Hughmark 1967; Maheshwari *et al* 1976) involving liquid drops. Outside or inside transfer coefficients are usually expressed in terms of Nusselt (for heat transfer) or Sherwood (for mass transfer) numbers, which are generally adequately described by dimensionless equations of the following form

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A list of symbols appears at the end of the paper.

$$\mathrm{Nu} = C\,(\mathrm{Re})^{a}\,(\mathrm{Pr})^{b}, \tag{1}$$

$$\mathrm{Sh} = C\,(\mathrm{Re})^{a}\,(\mathrm{Sc})^{b}, \tag{2}$$

where the constant C and the exponents a and b assume different values depending on the data covered by the different authors. The literature survey reveals that no successful attempt has been made to correlate the combined data on both heat and mass transfer owing to the differences observed in the interpretation of respective transfer rates. An attempt at modelling the transfer process applicable to analogous situations of heat and mass transfer will therefore be of interest.

In two recent reports (Raghavan *et al* 1978 andMaheshwari *et al* 1976) the following correlations for the continuous and dispersed phase coefficients for heat and mass transfer have been proposed involving analogous groups and exponents on the basis of Higbie's penetration theory, showing 0·5 power on diffusivity

$$(\mathrm{Nu})_c = \beta\,(\mathrm{Re})_c^{1\cdot 25}\,(\mathrm{Pr})_c^{0\cdot 5}, \tag{3}$$

$$(\mathrm{Nu})_d = \beta\,(\mathrm{Re})_d^{1\cdot 25}\,(\mathrm{Pr})_d^{0\cdot 5}, \tag{4}$$

$$(\mathrm{Sh})_c = \alpha\,(\mathrm{Re})_c^{1\cdot 25}\,(\mathrm{Sc})_c^{0\cdot 5}, \tag{5}$$

$$(\mathrm{Sh})_d = \alpha\,(\mathrm{Re})_c^{1\cdot 25}\,(\mathrm{Sc})_d^{0\cdot 5}. \tag{6}$$

Equations (3) to (6) indicate that though the groups and their exponents are analogous for heat and mass transfer, the constants β and α were different, *viz*, $\beta=0{\cdot}001$ and $\alpha=0{\cdot}0026$. Hence any attempt at using equations (3) and (4) for analogous situations of mass transfer (or using equations (5) and (6) for analogous situations of heat transfer) would show discrepancies with experimental values since the constant α for mass transfer was 2·6 times larger than the constant β for heat transfer. The aim of the present work was to obtain analogous equations with identical constants and exponents such that fair prediction of either heat or mass transfer data is possible assuming that the constants C, a and b in equations (1) and (2) may not be the same for continuous and dispersed phase coefficients for heat and mass transport.

3. Continuous phase coefficients

In this analysis the heat transfer data of Calderbank & Korchinski (1956), available for the system—mercury drops falling into aqueous glycerol—were used to obtain a relationship for prediction of h_c, the continuous phase individual heat transfer coefficient. The data on mercury drops given by Calderbank and Korchinski were chosen as the basis for obtaining the continuous phase coefficient since these data were stated to be devoid of end effects and the drop phase resistance to heat transfer was shown to be a negligible fraction of the total resistance in these experiments. These data also covered a wide range of Reynolds numbers [$100 < (\mathrm{Re})_c < 3000$] and Prandtl numbers [$5{\cdot}45 < (\mathrm{Pr})_c < 188$] by varying the size of mercury drops and the concentration of aqueous glycerol, respectively.

The type of equation represented by (1) was chosen for application to the h_c data of Calderbank and Korchinski and by statistical analysis of the data the correlation constants and exponents were evaluated by least-square fit. The following correlation resulted which satisfactorily fits the h_c data,

$$(\mathrm{Nu})_c = 0{\cdot}11\,(\mathrm{Re})_c^{0{\cdot}78}\,(\mathrm{Pr})_c^{0{\cdot}44}. \tag{7}$$

Equation (7) fitted the data with an average deviation of ±6·42% and a maximum deviation of 19·29%. The values of the constants and the exponents of (7) were obtained by multiple regression analysis.

An analogous equation for k_c, the individual mass transfer coefficient for the continuous phase corresponding to (7) may be assumed as

$$(\mathrm{Sh})_c = 0{\cdot}11\,(\mathrm{Re})_c^{0{\cdot}78}\,(\mathrm{Sc})_c^{0{\cdot}44}. \tag{8}$$

The exponent of 0·44 on Schmidt group fairly agrees with the values reported in the literature (Ruby & Elgin 1955; Garner *et al* 1959) for mass transfer from circulating drops.

4. Dispersed phase coefficients

If one could estimate h_c and k_c by the use of (7) and (8), the corresponding h_d and k_d data may be evaluated from reported overall coefficient data on U_0 and K_{0d} by the use of the following additivity equations.

$$1/U_0 = (1/h_d) + (1/h_c), \tag{9}$$

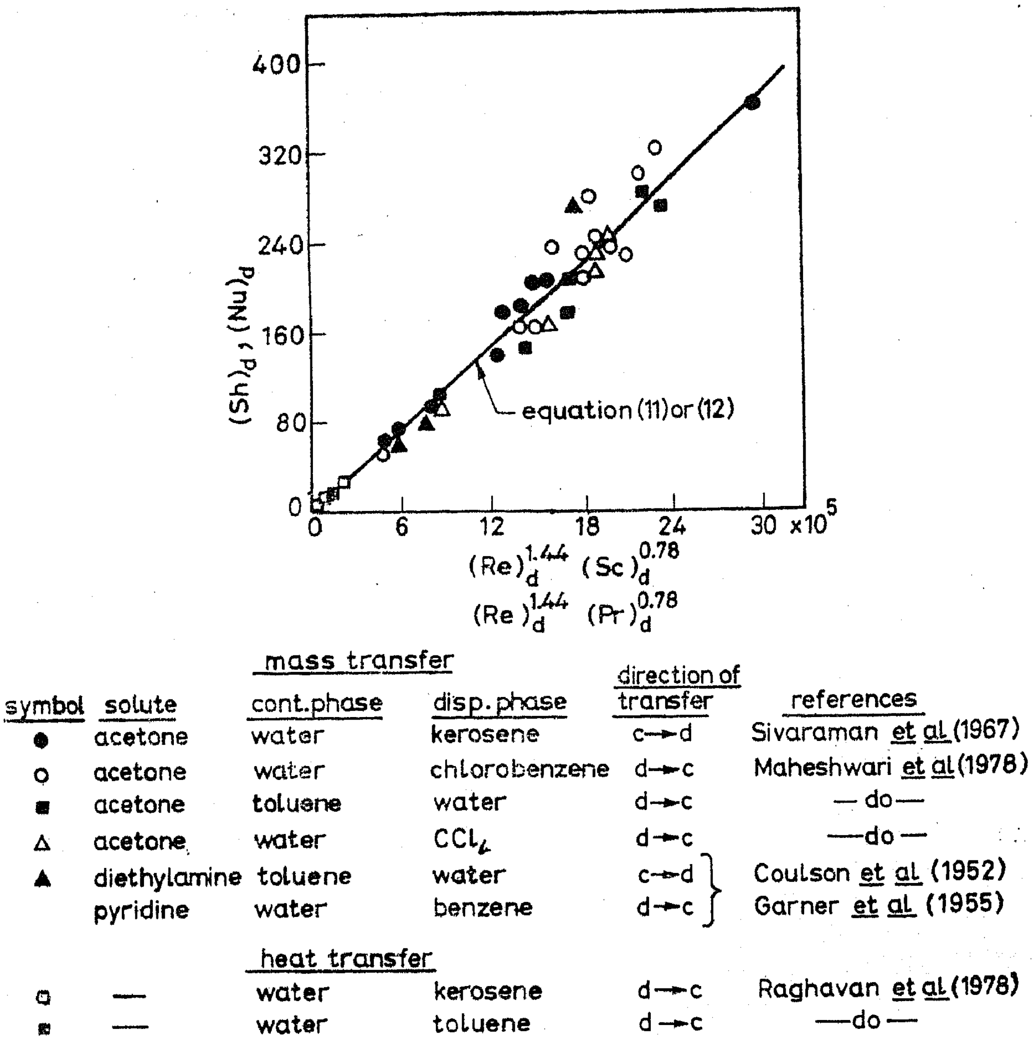
$$1/K_{0d} = (1/k_d) + (m/k_c), \tag{10}$$

where m is the equilibrium distribution coefficient for mass transfer defined as C_d^*/C_t. In the present analysis, the reported data on overall coefficients for heat transfer and mass transfer involving single-file drop systems have been used. The heat transfer data on U_0 covered are that of Raghavan *et al* (1978) for the two systems toluene-water and kerosene-water. The mass transfer data on K_{0d} used in the analysis were those of Sivaraman *et al* (1967) and Maheshwari *et al* (1976) covering six different ternary systems. The analysis of h_d by using (7) and (8) and k_d by using (9) and (10) resulted in the following analogous correlations when both heat and mass transfer data were analysed together

$$(\mathrm{Nu})_d = 0{\cdot}000123\,(\mathrm{Re})_d^{1{\cdot}44}\,(\mathrm{Pr})_d^{0{\cdot}78}, \tag{11}$$

$$(\mathrm{Sh})_d = 0{\cdot}000123\,(\mathrm{Re})_d^{1{\cdot}44}\,(\mathrm{Sc})_d^{0{\cdot}78}. \tag{12}$$

The combined data showed an average deviation of ±12·77% with a maximum deviation of 26·51%. It is also to be noted that the sample mass transfer data of Handlos & Baron (1957), Garner & Hale (1955) and Coulson & Skinner (1952) when



symbol	solute	cont. phase	disp. phase	direction of transfer	references
		mass transfer			
●	acetone	water	kerosene	c→d	Sivaraman *et al.* (1967)
○	acetone	water	chlorobenzene	d→c	Maheshwari *et al.* (1978)
■	acetone	toluene	water	d→c	— do —
△	acetone	water	CCl$_4$	d→c	— do —
▲	diethylamine	toluene	water	c→d	Coulson *et al.* (1952)
	pyridine	water	benzene	d→c	Garner *et al.* (1955)
		heat transfer			
□	—	water	kerosene	d→c	Raghavan *et al.* (1978)
■	—	water	toluene	d→c	— do —

Figure 1. Correlation of dispersed phase coefficients for heat or mass transfer by equations (11) and (12).

Table 1. Range of variables covered in the analysis of mass and heat transfer data involving single file drops.

Variable	Mass transfer	Heat transfer
d_N	0·028–0·625	0·040–0·476
d_e	0·220–0·880	0·33–0·76
u_t	8–17	11–15
K_{od}	0·002–0·010	—
U_o	—	3·59–10·30
μ_c, μ_d	0·715–1·180	0·525–1·287
ρ_c, ρ_d	0·784–1·582	0·795–0·870
D_c, D_d	1·38–2·51	—
C_{pc}, C_{pd}	—	0·41–0·48
k'_c, k'_d	—	$3\cdot5\times10^{-4}$
$(Sh)_d$	4·25–321·0	—
$(Nu)_d$	—	3·3–21·7
$(Re)_d$	230–1160	260–1600
$(Sc)_d$	300–800	—
$(Pr)_d$	—	5·0–17

analysed in this manner gave a satisfactory fit according to equation (12). Figure 1 shows a plot of the experimental coefficients for the heat or mass transfer inside the drops as correlated by equations (11) and (12) respectively. The range of variables covered in the present analysis of heat and mass transfer data is given in table 1.

5. Conclusion

The results of the present analysis of heat and mass transfer data with single-file drops in liquid-liquid systems indicate that continuous and dispersed phase coefficients for transfer of heat or mass could be satisfactorily predicted by analogous equations involving a Reynolds number based on droplet diameter and terminal velocity of the droplet, and the pertinent physical property groups for heat and mass transport.

KV expresses thanks to the University Grants Commission, New Delhi for the award of a fellowship.

List of symbols

C	concentration of the liquid phases (g/cm^3)
C^*	concentration of solute at equilibrium (g/cm^3)
c_p	specific heat (cal/g °C)
D	diffusivity (cm^2/s)
d_e	equivalent drop diameter (cm)
h	area-based heat transfer coefficient for the individual phase (cal/cm^2/s/°C)
k	area-based mass transfer coefficient for the individual phase (cm/s)
k'	thermal conductivity (cal/s cm^2, °C/cm)
K_{0d}	overall area-based mass transfer coefficient (cm/s)
m	distribution coefficient (C_d^*/C_c)
U_0	overall heat transfer coefficient (cal/s/cm^2/°C)
u_t	terminal velocity of drop (cm/s)
α	constant in equations (5) and (6)
β	constant in equations (3) and (4)
ν	kinematic viscosity (μ/ρ cm^2/s)
μ	viscosity (poise)
ρ	density (g/cm^3)

Dimensionless groups

$(\mathrm{Nu})_c$, $(\mathrm{Nu})_d$	Nusselt number, $(h_c d_e/k'_c)$, $(h_d d_e/k'_d)$
$(\mathrm{Pr})_c$, $(\mathrm{Pr})_d$	Prandtl number, $(C_{pc}\mu_c/k'_c)$, $(C_{pd}\mu_d/k'_d)$
$(\mathrm{Re})_c$, $(\mathrm{Re})_d$	Reynolds number, $(d_e u_t \rho_c/\mu_c)$, $(d_e u_t \rho_d/\mu_d)$
$(\mathrm{Sc})_c$, $(\mathrm{Sc})_d$	Schmidt number, $(\mu_c/\rho_c D_c)$, $(\mu_d/\rho_d D_d)$
$(\mathrm{Sh})_c$, $(\mathrm{Sh})_d$	Sherwood number, $(k_c d_e/D_c)$, $(k_d d_e/D_d)$

Subscripts

c continuous phase
d dispersed phase

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