ENGINEERING ASPECTS OF FOAM COLUMNS: RECENT TRENDS*

DILIP DESAI and R. KUMAR

Department of Chemical Engineering, Indian Institute of Science, Bangalore 560012, India.

ABSTRACT

The engineering interest in gas-liquid foams, during the past two decades, has revolved around the idea of industrially exploiting the special features of foam. This article overviews the recent attempts at understanding and predicting the behaviour of the simplest and most commonly employed foam device, the foam column. Mainly two aspects which form the backbone of the potential uses of foam are discussed. These are the analysis of gas absorption accompanied by a chemical reaction, which is discussed first; and the question of liquid holdup in a semi-batch foam column, which is discussed next. Finally, future research trends are mentioned.

INTRODUCTION

Foam is an agglomeration of gas bubbles entrapped in a liquid matrix. The bubbles are not spherical and are separated from each other by thin liquid films, thus generating a structure in which the relative movement of bubbles is quite restricted. Though the quantity of liquid present in the foam is quite small as compared to that of the gas, the liquid forms the continuous phase.

Foams, bubbles and thin liquid films have interested physicist and physical chemists for more than a century now; Plateau's pioneering work[1] being the first landmark in man's understanding of foams. Their interest, however, has been mainly from the points of view of equilibrium shapes of bubbles, minimization problems, capillary action, adsorption at the gas-liquid interface, thinning of soap films, nature and range of molecular forces in thin liquid films, etc. Excellent books and reviews covering these interest are available[2-13].

Foam as a nuisance

Many applied scientists, especially chemical engineers, however, consider foam as a nuisance; as it poses operational problems in their process plants. Foam causes process difficulties, for example, in distillation columns[14], evaporators[15], stirred vessels[16], sewage treatment plants[17], fermentative synthesis of antibiotics[18], pulping operations and handling of black liquor in paper industry[19], pumping of petroleum out of a well[20], and in the manufacture of sugar[21] and phosphoric acid[22]. Recently, it has also been found that foam causes problems in the treatment of dyspepsia; and antifoaming agents (like polydimethyl siloxane) have to be added to antacid tablets[23]. Earlier applied research was, therefore, mainly conducted with a final view to obviate foams in process equipments[17-23].

Uses of foams

In the last two decades, however, the emphasis has shifted. Engineers have started wondering whether the nuisance can be converted into an asset. And, indeed, foam does have certain remarkable features which make it a suitable candidate for many uses. These characteristic features of foam are: (i) low liquid content (less than 3%), (ii) large amount of gas entrapped in a liquid matrix, (iii) adsorption of surface active solutes at the gas-liquid interface, (iv) very high gas-liquid interfacial area per unit volume of the liquid, (v) reasonably rigid structure, and (vi) very high viscosity. These characteristics have suggested a number of possible applications. They are:

(i) *As a dust collecting device:* Investigations[24-28] have revealed that foam is a useful device to separate dust out of dust-laden gases. The device is also effective for submicron-size particles.

(ii) *Plant protection:* Foams of gelatin, citrus pectin and starch phosphates have found applications in protecting plants from frost[29,30].

(iii) *Foam fractionation or foam separation:* This is a very promising, relatively new separation technique; and has been extensively investigated[8a,31-42]. It is particularly useful in cases where classical separation techniques cannot be employed because of the very low concentration of the solute in the solution.

(iv) *As a gas-liquid contacting device:* Its high interfacial area makes foam a good gas-liquid contactor; especially when a large amount of gas is to be contacted with a small amount of liquid; *e.g.* in the effluent gas cleaning[43-47].

(v) *Paper making:* A new technique, called the Radfoam process, for paper making and manufacture of non-woven fabrics which utilizes foam as a fibre suspending medium has also been reported[48]. Foams are also increasingly employed in textile printing and processing because of their rigidity and low liquid content[49].

Figure 1. Modes of operation of a foam column.

Foam columns

Many of these applications of foam require that foam be continuously generated in a column. A simple way to do this is to sparge a gas through a distributor, like sieve plate, into a shallow pool of surface active solution kept at the bottom of the column. The gas emerges into the shallow pool in the form of bubbles which rise through the liquid and generate a foam layer on top of the pool. The stability of the foam is controlled by the addition of the requisite amount of appropriate surfactant. The foam column can be operated in different modes shown in figure 1.

Two commonly studied modes are the semi-batch and co-current modes. In the semi-batch mode, the foam continuously forms at the foam-liquid pool interface; and continuously breaks at the foam-gas interface. The liquid thus released flows back through the foam into the pool. The gas has a constant input into and output from the
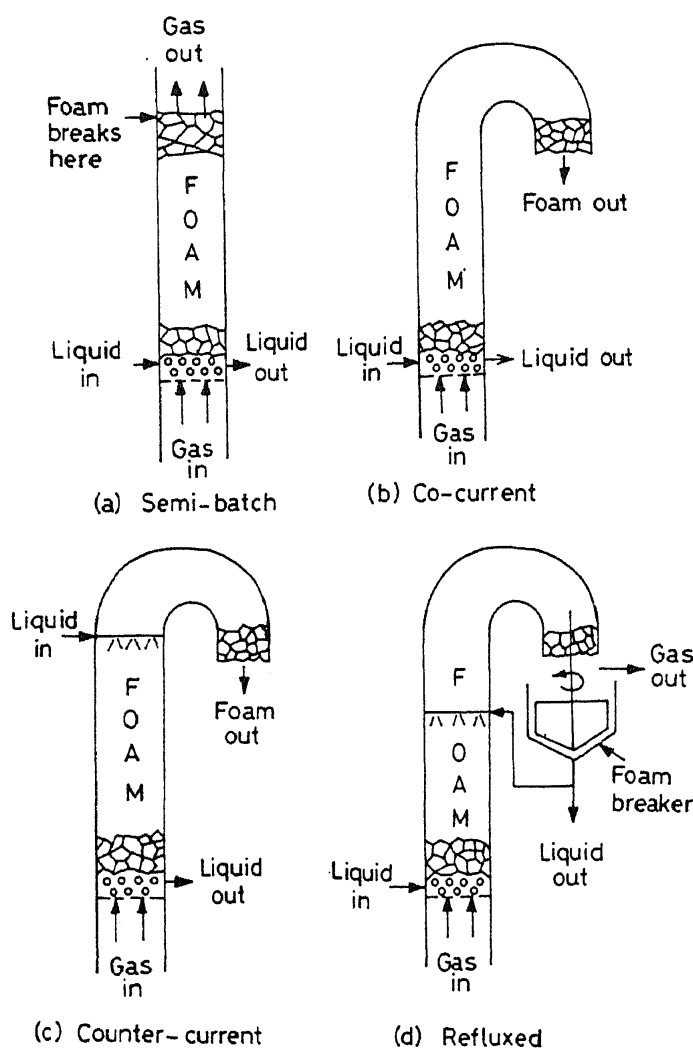
foam column. In the co-current mode, the foam is continuously formed and removed from the foam column.

To employ these foam columns effectively for the uses mentioned earlier, some basic understanding of their behaviour with respect to system, operating and equipment parameters is necessary. And to gain such an understanding, it is necessary to keep in mind certain fundamental aspects of foam. These are described next; and after that the questions of mass transfer and liquid holdup in the foam column are discussed.

Some fundamental aspects of foam

The regular features of the structure of foam are well-described by the famous laws of geometry of bubble agglomerates postulated by Plateau[1a,13,50]. He suggested these laws from the observations on the structural stability and mini-

mization of surface energy. The laws are: (i) Three and only three liquid films meet at an edge. And the angle at which they meet each other is 120°. (ii) Four and only four of these edges meet at a point. And the angle at which they meet each other is 109° 28′ 16″ (the tetrahedral angle).

It has been suggested that an idealized foam consisting of equal-sized regular pentagonal dodecahedron shaped bubbles closely meets the requirement of the above laws; and hence actual foam would consist of such bubbles[5,8b,9]. This has been borne out fairly well by observations on actual foams[51-53]. Consequently, many engineering models of foam behaviour[34,53-59] have employed this dodecahedral idealization. Figure 2 shows the idealized foam bubble.
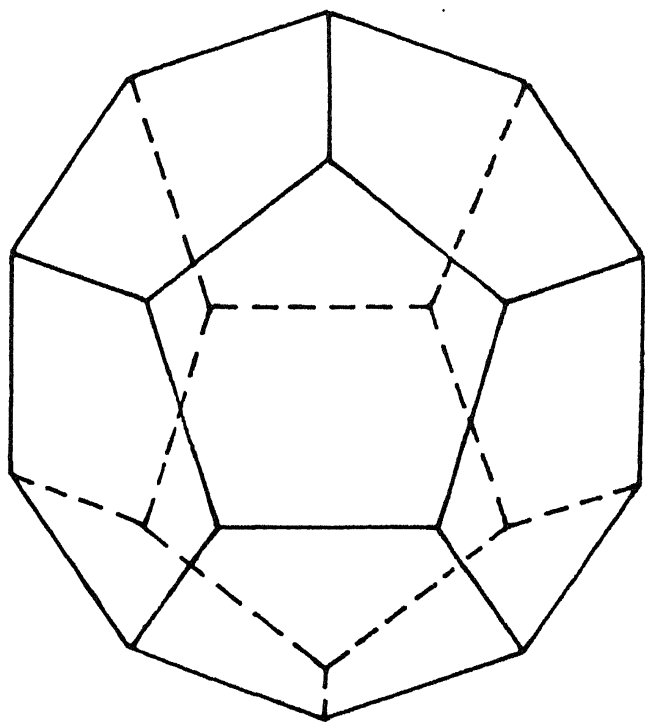

Figure 2. Regular pentagonal dodecahedral foam bubble.

Figure 3 shows a cross-section at a place in the foam where three films meet. The edge at which the three films meet is called a 'Plateau border'. These Plateau borders constitute a net-work through which the liquid at the top of the foam drains down to the bottom of the foam[34,53-62]. In fact, at every point within it, the foam continuously drains its liquid contents downward because of gravity. This drainage of foam liquid through its Plateau borders plays an important

part in the considerations of both the mass transfer and the liquid holdup in the foam.

It is also evident from figure 3 that the gas-liquid interface of the Plateau border is concave outward; whereas, that of the liquid film is flat. Consequently, according to Laplace's law of capillary pressure[3,5,8c], the pressure within the Plateau border is less than the pressure at the centre of the film. This causes the liquid film to continuously thin out until the pressure gradient is counter balanced by some other effects. This thinning of foam films is also important in the considerations of the mass transfer and liquid holdup in the foam.
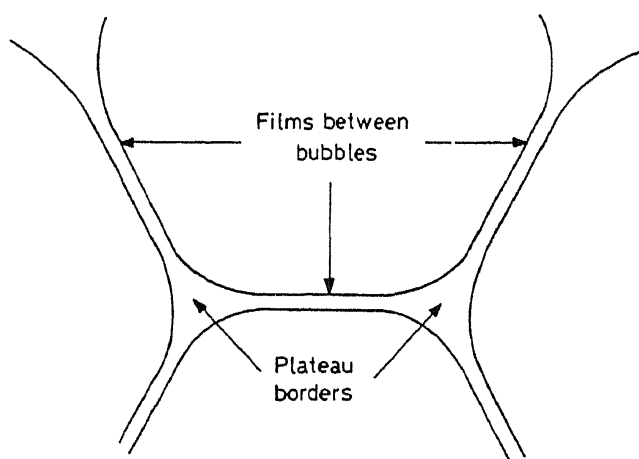

Figure 3. Films and plateau borders of foam.

GAS ABSORPTION ACCOMPANIED BY A CHEMICAL REACTION IN A SEMI-BATCH FOAM COLUMN

As mentioned earlier, investigations[43-47] have appeared in the literature indicating that, under certain conditions, a semi-batch foam column as a medium for gas absorption can be more effective than a conventional packed tower or a bubble cap column. Much of this work is empirical in nature and tries to develop useful correlations for physical characteristics of foam beds and the mass transfer coefficients etc. A theoretical analysis, based on the idealised structure of foam described earlier, has recently been suggested by Biswas and Kumar[63]. These authors consider a semibatch foam bed reactor as made up of two parts *viz* the storage part and the foam

part. There is continuous inflow and outflow of liquid from the storage section. The flow of liquid is maintained in such a way that there is always a constant volume (V) in the storage section. A part of the liquid goes from the storage section into the foam, reacts with the gas and returns in partially converted form to the storage. According to the dodecahedral idealization of foam, the liquid matrix is made up of thin flat films and the Plateau borders (PBs); and the films continuously drain into the Plateau borders because of capillary pressure, as mentioned earlier. As the surface area of the PBs is negligible when compared to that of the films, it is assumed that no mass transfer takes place from gas to the PBs. Thus, the gas rising in the foam section diffuses and reacts with the liquid in the film. As the film is continuously draining, its thickness goes on reducing as a function of height. To account for this, the foam section was subdivided into a number of subsections.

The idealisation of the foam bed reactor is represented in figure 4.

The material balance equations for B i.e. the reactant in the liquid phase are:

Storage section:

$$-V \frac{dC_B}{dt} = (Q_1 + Q)C_B - QC_{Bo} - Q_1 C'_B \quad (1)$$

Foam section:

$$Q_1 C'_B = \sum_{i=1}^{m} q_i C_{Bi} \quad (2)$$

where

$$Q_1 = \sum_{i=1}^{m} q_i \quad (3)$$

To obtain the C_B in the storage as a function of time, it is necessary to express C'_B as a function of C_B. This is done by considering the reaction in a film surrounded on both sides by a limited amount of gaseous reactant. The pertinent equation for this is

$$\frac{\partial C_A}{\partial t_c} = D_A \frac{\partial^2 C_A}{\partial x^2} - \lambda C_A \quad (4)$$

Initial condition:

At　　　$t_c = 0, \quad -a \leqslant x \leqslant a, \quad C_A = 0$　　　(4a)

Boundary conditions:

at $t_c > 0, \quad x = \pm a, \quad \frac{1}{12} V_b \frac{\partial C^*_A}{\partial t_c} = \mp D_A S \frac{\partial C_A}{\partial x}$

(4b)

where　　$C_A = k C^*_A$

The solution of (4) is

$$\frac{M}{M_\infty} = 1 - \sum_{n=1}^{\infty} \frac{\exp(p_n t_c)}{\left[1 + \frac{a}{21} + \frac{p_n}{2D_A K_n^2} + \frac{p_n^2 1a}{2D_A^2 K_n^2} \right]}$$

(5)

where p_n's are the non-zero roots of

$$\frac{1 p_n}{D_A} = K_n \tan K_n a \text{ and } K_n^2 = -\frac{p_n + \lambda}{D_A} \quad (6)$$

The concentrations of the unreacted liquid reactant, $C_{B_1}, C_{B_2} \ldots$ and C_{B_m} at the ends of different foam sections with cumulative contact times $t_{c_1}, (t_{c_1} + t_{c_2})$ etc can be expressed as:

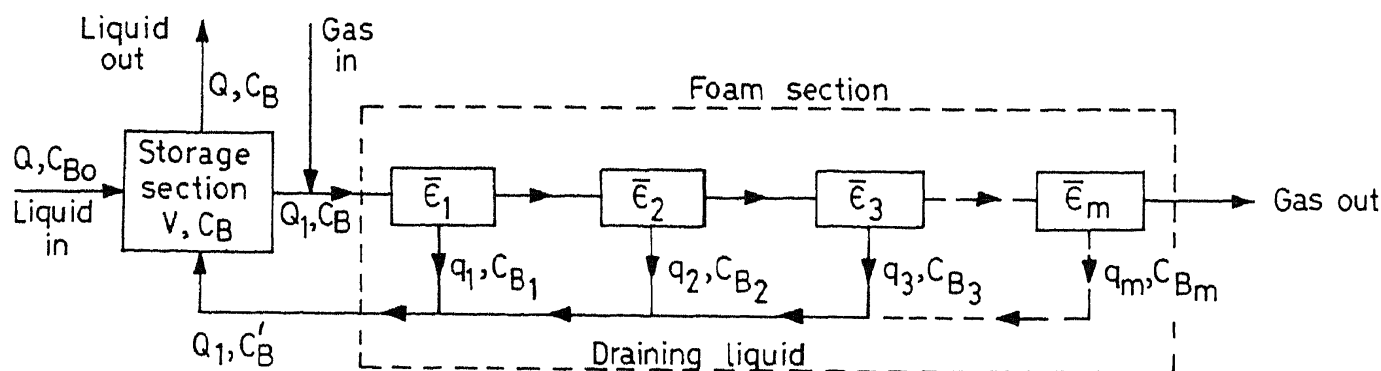$$C_{B_1} = C_B - \frac{2 y_A M_1}{V_1} \quad (7)$$

Figure 4. Model representation of a foam column reactor.

Similar expressions are available for other subsections also. From C_{Bi} values, the C'_B is obtained which on substitution in (4) yields an equation:

$$\frac{dC_B}{dt} + P_1 C_B = P_2 \tag{8}$$

where $P_1 = Q/V$ and

$$P_2 = \frac{1}{V}\left[QC_{Bo} - \frac{2y_A Q_1 M_1}{V_1} \cdots \right.$$

$$\left. - \frac{2y_A Q_1 - (q_1 + q_2 \cdots q_{m-1})(M_m - M_{m-1})}{V_m} \right]$$

Equation (8), when solved with the initial condition

At $t = 0$, $C_B = C_{Bo}$, yields

$$C_B = C_{Bo}e^{-P_1 t} + \frac{P_2}{P_1} - \frac{P_2}{P_1}e^{-P_1 t} \tag{9}$$

The outlet gas concentration can be obtained straightway and works out to be:

$$C_G = C_0 - \frac{12 M_m}{V_b} \tag{10}$$

The expressions for q_m, v_m, a_m, 1 and t_{c_m} are:

$$q_m = \frac{\bar{\varepsilon}_m Q_G}{(1 - \bar{\varepsilon}_m)} - \frac{\bar{\varepsilon}_{m+1} Q_G}{(1 - \bar{\varepsilon}_{m+1})} \tag{11}$$

$$v_m = \frac{v_b \bar{\varepsilon}_m}{6(1 - \bar{\varepsilon}_m)} \tag{12}$$

$$a_m = \frac{\pi \bar{\varepsilon}_m r_b}{7.537(1 - \bar{\varepsilon}_m)\tan 54°} \tag{13}$$

$$1 = \frac{v_b}{10.05\, kr_b^2 \tan 54°} \tag{14}$$

$$t_{c_m} = \frac{H\pi r_c^2(1 - \bar{\varepsilon}_m)}{Q_G} \tag{15}$$

This model has been verified by Biswas and Kumar[63] by studying the reaction between CO_2 and aqueous solution of NaOH using a semi-batch foam bed reactor. A typical comparison between the experimental variation of C_B with time with that predicted by theory is shown in figure 5. The model is found to be quite satisfactory.
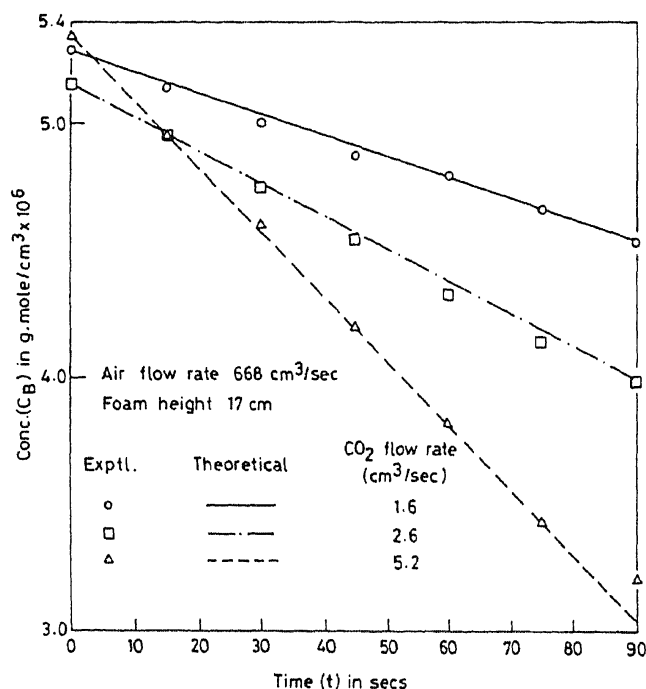

Figure 5. Plot of C_B vs t.

Recently, Bhaskarwar and Kumar[64] have simplified the model, based on the findings of Desai and Kumar[65], who found that for liquids of low viscosity, the liquid hold up varies sharply in the first few centimeters from the liquid pool and thereafter remains virtually constant. Thus, there is no need to have too many subsections in the foam. Instead the foam can be considered as a single section only. They have tested the simplified model by studying the oxidation of sodium sulfide with air at 75°C. The value of the second order reaction velocity constant is already available in literature[66]. Two sets of their data are presented in figures 6 and 7, along with the theoretical predictions. Figure 6 gives conversion versus time plot when sodium dodecyl sulphate is used as the surfactant; whereas figure 7 presents similar results for octyl phenoxy polyethoxy ethanol. It is interesting to see that for a fixed time, as the height of the foam bed is increased, figure 6 indicates that the conversion is decreased, whereas figure 7 indicates just the opposite trends. The reason for the opposing trends is the behaviour of ε with height for the two surfactants. The model also predicts the same trends as obtained experimentally.

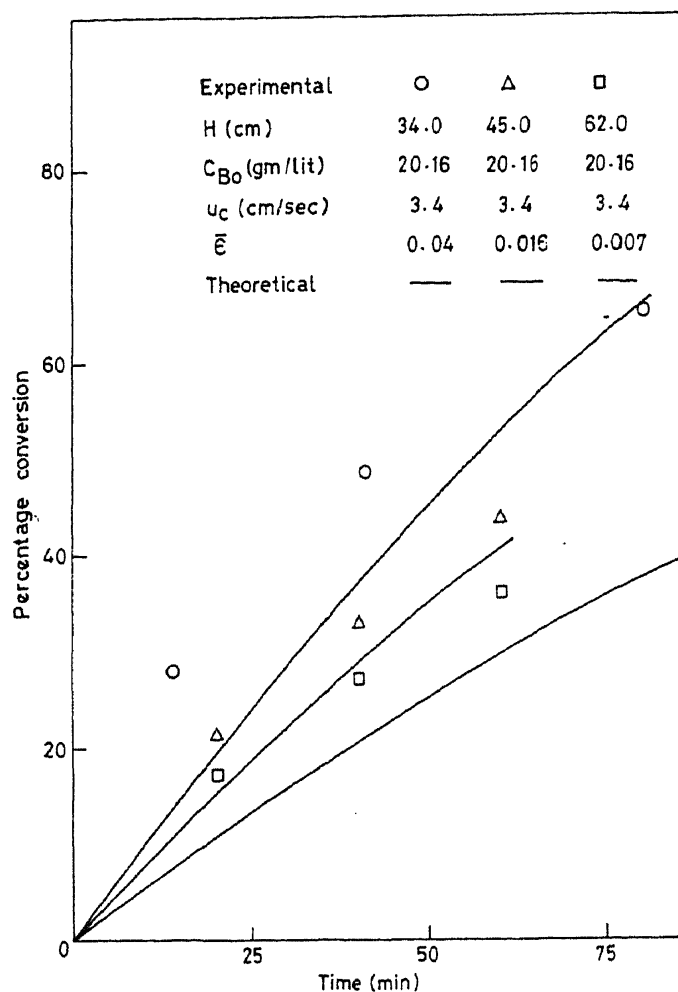Both Biswas[63] and Bhaskarwar[64] have studied

Experimental	O	△	□
H (cm)	34.0	45.0	62.0
C_{Bo} (gm/lit)	20.16	20.16	20.16
u_c (cm/sec)	3.4	3.4	3.4
$\bar{\epsilon}$	0.04	0.016	0.007
Theoretical	—	—	—

Figure 6. Effect of height of foam bed on conversion (surfactant-sodium dodecyl sulphate).

Experimental	△	□	O
H (cm)	34.0	45.0	62.0
C_{Bo}(gm/lit)	11.74	11.74	11.74
u_c (cm/sec)	3.4	3.4	3.4
$\bar{\epsilon}$	0.025	0.023	0.019
Theoretical	—	—	—

Figure 7. Effect of height of foam bed on conversion (surfactant-octyl phenoxy polyethoxyethanol).

second order reactions by treating them as pseudo first order ones. The conditions for treating them as pseudo first order ones[67] were met and pseudo first order rate constant was found as the product of the true second order rate constant and C_B. As C_B varied as a function of time, the integration of (4) was done over short successive intervals of time.

LIQUID HOLDUP IN SEMI-BATCH FOAM COLUMNS

Analysis of a column used for any of the purposes mentioned earlier requires a knowledge about two basic variables. These are the bubble size and the liquid holdup. Prediction of bubble sizes has been investigated extensively and considerable predictive capabilities to estimate bubble sizes from the first principles already exist[68,69]. However, the prediction of the liquid
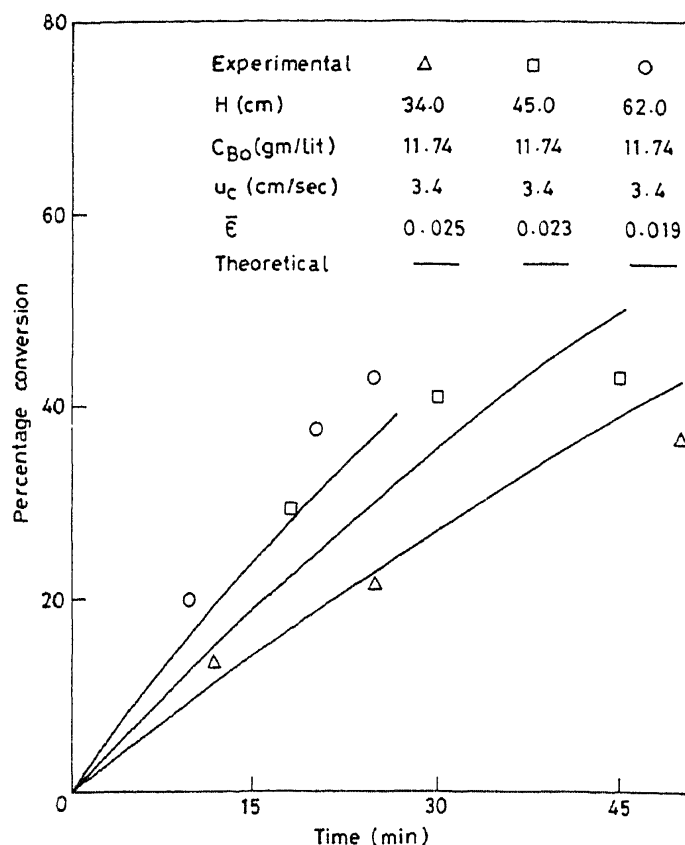
holdup in foam columns, without recourse to any adjustable parameters, has only recently been attempted[65]. This attempt is described below:

Hydrodynamics of a Plateau border

A very interesting and fundamental feature of foam is that it has draining pipes (Plateau borders) which do not have solid (or rigid) walls. All earlier models[53-59] of liquid holdup and foam drainage had to have an empirical constant to account for this non-rigidity of Plateau border walls. Only Leonard and Lemlich[34] took into account the non-rigidity of Plateau border walls. However, they solved the problem numerically and did not present the average velocity through the Plateau border as a function of the surface viscosity (which is the measure of non-rigidity of the walls), thus making their work unamenable for use in later models on foam. This problem of flow through a Plateau border has recently been solved[70] as follows:

The cross-section of the Plateau border is idealized as an equilateral triangle with vertices of
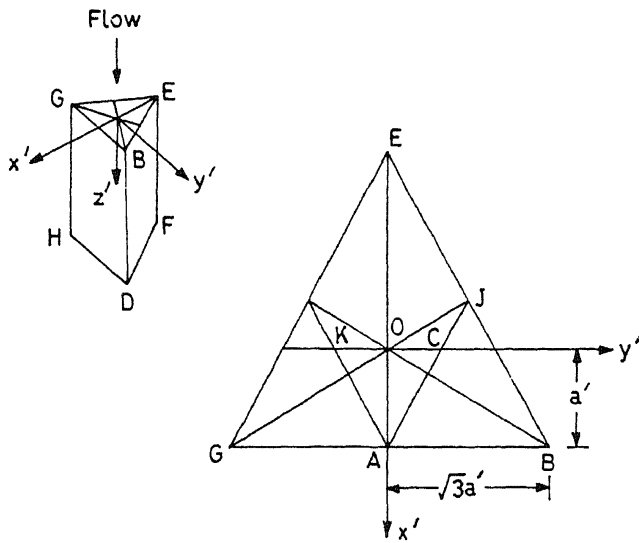
Figure 8. Details of the idealised Plateau border.

the triangle having zero velocity. Figure 8 shows the details of the idealized Plateau border. From the comparison of figures 3 and 8, it is evident that the idealized shape simplifies the situation considerably without introducing any note-worthy deviation from the real shape. Also, of the whole triangle GEB, only triangle AOB need be considered, because from the flow through this, the whole flow field can be generated through considerations of symmetry.

Equation: For steady, incompressible, viscous flow through the Plateau border, the pertinent form of Navier-Stoke's equations, in dimension-less form is:

$$\frac{\partial^2 v}{\partial x_1^2} + \frac{\partial^2 v}{\partial y_1^2} = 3G \qquad (16)$$

where

$$v = v_{z'}/v_{max}; \quad x_1 = x'/\sqrt{3}a'; \quad y_1 = y'/\sqrt{3}a';$$

$$v_{max} = v_{z'} \text{ at } x' = 0; \quad y' = 0 \qquad (17)$$

and

$$G = \frac{1}{\mu}\frac{\partial p}{\partial z'}\frac{a'^2}{v_{max}} \qquad (18a)$$

Boundary Conditions: The pertinent boundary conditions, in dimensionless form, are

(i) at $x_1 = 1/\sqrt{3}$; $y_1 = 1$; $v = 0$ (19a)
(zero velocity at the corner B)

(ii) at $y_1 = 0$; $0 \leqslant x_1 \leqslant 1/\sqrt{3}$;
$$(\partial v/\partial y_1) = 0$$
(symmetry across the line AO) (19b)

(iii) at $x_1 = 1/\sqrt{3}$; $0 \leqslant y_1 \leqslant 1.0$
$$(\partial^2 v/\partial y_1^2) = \sqrt{3} \cdot \alpha \cdot (\partial v/\partial x_1) \qquad (19c)$$

where $\alpha = \mu \cdot a'/\mu_s$ (18b)

(from the momentum balance at the gas-liquid interface; assuming no surface tension gradient on the gas-liquid interface)

(iv) on the line $y_1 = \sqrt{3}x_1$; $0 \leqslant y_1 \leqslant 1.0$;

$$\sqrt{3} \cdot \frac{\partial v}{\partial x_1} - \frac{\partial v}{\partial y_1} = 0 \qquad (19d)$$

(symmetry across the line OB)

(v) at $x_1 = 0$, $y_1 = 0$, $v = 1.0$ (19e)
(from the definition of v)

Solution

The solution is obtained by a method of successive approximation (or a boundary collo-cation method) and is given by

$$v = \left(-\frac{G}{4}\right)\left(\frac{4}{3} - 3x_1^2 - 3y_1^2 - \sqrt{3}x_1^3\right.$$
$$\left. + 3\sqrt{3}x_1 y_1^2\right)$$

$$+ \sum_{n=1,3,5}^{\infty} A_n \cdot \cosh\left(\frac{n\pi x_1}{2}\right) \cdot \cos\left(\frac{n\pi y_1}{2}\right)$$

$$+ \sum_{i=1,2,3}^{K} C_i \phi_i(x_1, y_1) \qquad (20)$$

where A_n's and C_i's are constants, and K is the order of approximation desired. $\phi_i(x_1, y_1)$ rep-resents a polynomial of order i in x_1, y_1; and could be given by the following general expression:

$$\phi_i(x_1, y_1) = \sum_{j=0,2,4}^{i} a_{ij}[x_1^{(i-j)} \cdot y_1^j - (1/\sqrt{3})^{(i-j)}]$$

$$i = 1, 2, 3 \qquad K \qquad (21)$$

where

$$a_{i,j+2} = -a_{i,j}\frac{(i-j)(i-j-1)}{(j+2)(j+1)} \qquad (22a)$$

and $a_{i,0} = 1$ (22b)

The constants A_n's and C_i's are to be evaluated from boundary conditions[70]. This solution reduces to the known solution of slow steady viscous flow through a triangular pipe, when surface viscosity is infinite (*i.e.* when the pipe walls are rigid).

The main findings of this analysis are succintly represented by figure 9. Figure 9 relates a velocity co-efficient (which is the ratio of the average velocity through the Plateau border when the surface viscosity is finite to the average velocity when the surface viscosity is infinite, *i.e.* the walls are rigid) with the inverse of dimensionless surface viscosity, α, defined by (18b). It is evident that for a very low value of surface viscosity the average velocity through the Plateau border can be as high as about 40 times the average velocity obtained with a rigid pipe. Figure 9 also shows the limited experimental
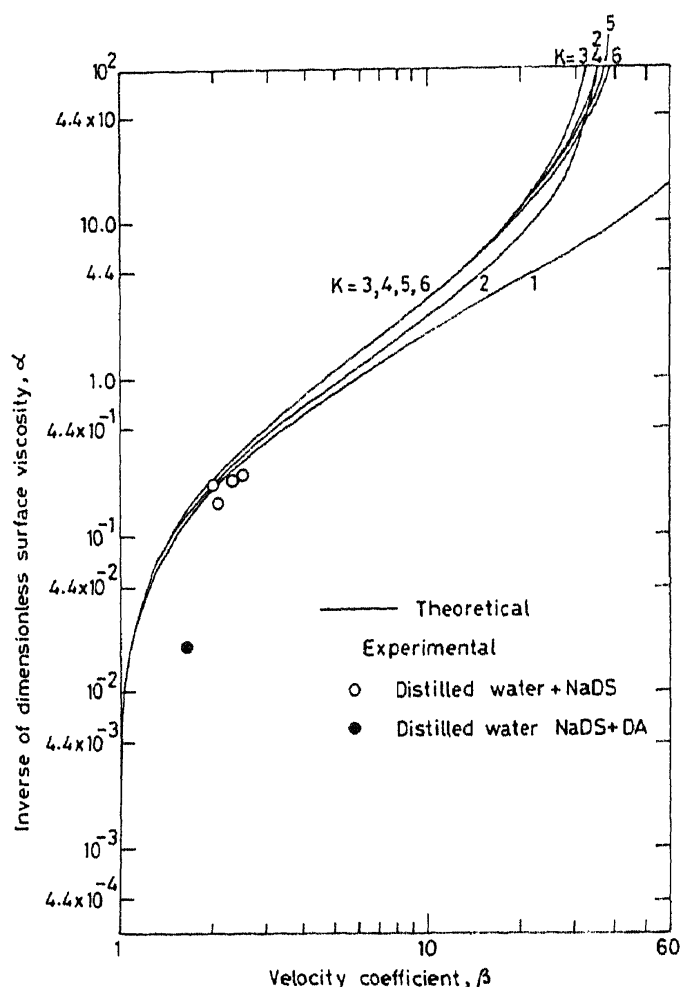
verification of the theory. The curve of α versus β with $K = 6$ is a general one; and for easy use in theories on foam, it is available[65] as a polynomial in α.

Foam Drainage Mechanism

Another very interesting feature of the foam is that it has a drainage network consisting of collapsible pipes in the sense that the cross-sectional area of a pipe can be increased by increasing the flow through it and *vice versa*. This fact, however, raises a question as to whether all the Plateau border surrounding a gas bubble are incorporated in the drainage network of foam. All the earlier models on foam drainage mechanism[34,53–58] assume that all the Plateau borders around a bubble participate equally in the drainage. This view, however, leads to much higher prediction of liquid holdups in foams than observed experimentally. And experimental results are explained only by introducing arbitrary adjustable parameters[53,54,58] or by assuming much lower values of surface viscosity[35,36] than observed by independant experiments[71,72]. A recent model[65] on liquid holdup in semi-batch foams adopts a different view. These investigators suggest that all the Plateau borders around a gas bubble are not included in the drainage net-work. Their idealization of the drainage mechanism is given in figure 10. They
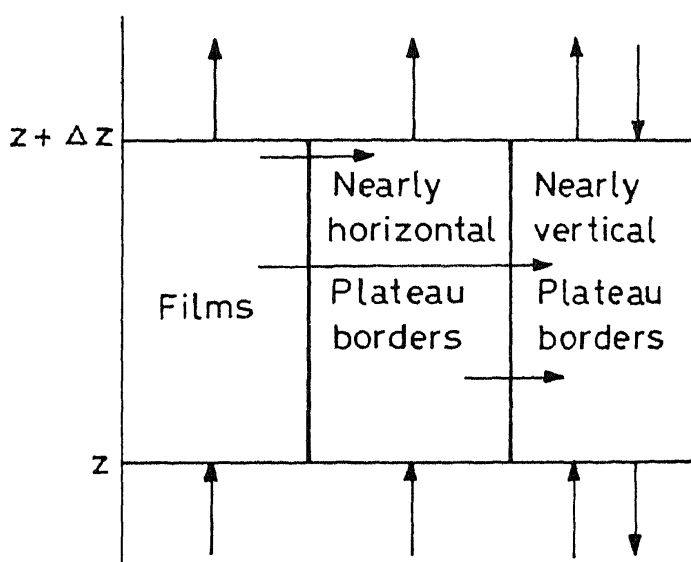
Figure 9. Variation of velocity coefficient with the dimensionless surface viscosity.

Figure 10. A schematic representation of the proposed mechanism of drainage.

distinguish between two kinds of Plateau borders: Nearly vertical Plateau borders and nearly horizontal Plateau borders. The drainage network of foam consists of only nearly vertical Plateau borders. The nearly horizontal Plateau borders merely transfer the liquid from the films and their own liquid into the nearly vertical Plateau borders. Based on this idealization and taking the structural parameters of foam into account, the following equations have been written which can predict the liquid holdup profiles in a semi-batch foam column.

$$-\frac{dw_1}{dz} - K_1^* w_1^3 = 0 \tag{23}$$

$$-\frac{dw_2}{dz} + \frac{1}{2} K_1^* w_1^3 - K_2^* w_2^2 = 0 \tag{24}$$

$$-\frac{dw_3}{dz} + K_3^* \cdot \frac{d}{dz}[w_3^2] + \frac{1}{2} K_1^* w_1^3 + K_2^* w_2^2 = 0 \tag{25}$$

where

$$w_1 = \frac{\varepsilon_F}{1 - \varepsilon_T} \tag{26a}$$

$$w_2 = \frac{\varepsilon_{HPB}}{1 - \varepsilon_T} \tag{26b}$$

$$w_3 = \frac{\varepsilon_{VPB}}{1 - \varepsilon_T} \tag{26c}$$

and

$$\varepsilon_T = \varepsilon_F + \varepsilon_{HPB} + \varepsilon_{VPB} \tag{27}$$

and constants K_1^*, K_2^* and K_3^* can be evaluated from system, operational and equipment parameters[65].

This model has been verified experimentally[65]. A typical comparison between the experimentally obtained holdup profiles with those obtained by theory is shown in figure 11. It is seen that the agreement between the two is fairly encouraging. It is also seen that model adequately predicts the effect of liquid viscosity on the liquid holdup profiles. The performance of the theory when other parameters (like superficial gas velocity, surface viscosity etc) are varied is equally encouraging.

Future research trends
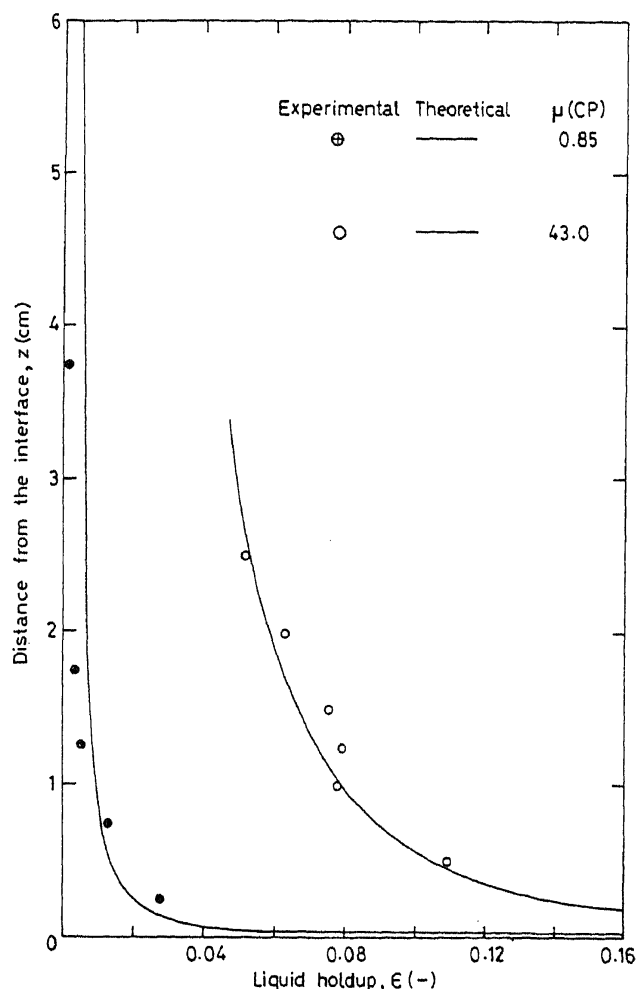
Systematic analysis of foam bed column has

Figure 11. Effect of viscosity on the liquid holdup profiles.

just begun and is still in a stage of infancy. Significant amount of effort is required before adequate understanding, resulting in more robust models, emerges. Some of the unresolved problems requiring immediate attention and offering intellectual challenge are: (i) general model for the reactor, (ii) reactor involving multiple phases, (iii) co-current foam reactor analysis, (iv) recirculation phenomenon and its inclusion in the analysis, (v) foam stability analysis, (vi) effect of addition of solid particles, (vii) prediction of dust collection efficiency, (viii) how to use foam for gas separation, (ix) study of desorption in foam beds etc.

It is premature to conclude whether foam columns will find large scale applications in industry. The present trend, however, indicates that such chances are bright.

NOMENCLATURE

$2a$	: thickness of a liquid film, cm
$a_1, a_2, a_3, \ldots$ and a_m	: halves of "$2a$" in subsections $1, 2, 3,$ $\ldots$ and m respectively, cm
a'	: one third of the median of the equilateral triangle GEB, cm.
$a_{i,j}$'s	: constants, Eqn. (21).
A_n's	: constants, Eqn. (20).
C_0	: initial concentration of reactant A in gas phase, g mole/cm^3
C_A	: concentration of reactant A in liquid phase, g mole/cm^3
C_A^*	: concentration of reactant A in gaseous phase, g mole/cm^3
C_B	: concentration of liquid phase reactant B, g mole/cm^3
C_{B_0}	: initial concentration of liquid-phase reactant B, g mole/cm^3
$C_{B_1}, C_{B_2}, C_{B_3} \ldots$ and C_{B_m}	: concentrations of liquid-phase reactant B at the ends of the subsections $1, 2, 3 \ldots$ and m respectively, g mole/cm^3
C_B'	: concentration of reactant B in feed back liquid, g mole/cm^3
C_G	: exit concentration of unreacted gas, g mole/cm^3
C_i's	: constants, Eqn. (20)
D_A	: diffusion coefficient of reactant A in liquid phase, cm^2/sec
G	: dimensionless pressure drop defined by Eqn. (18a)
H	: height of any foam subsection, cm
k	: equilibrium distribution factor
K	: order of approximation, a constant, Eqn. (20)
K_n	: defined in Eqn. (6)
K_1^*, K_2^*, K_3^*	: constants eqns. (23)–(25)
l	: equals $V_b/12\,ks$, cm
m	: number of a subsection of the foam layer
M	: total amount of solute, both free to diffuse and immobilized in half the liquid film of surface area s at contact time t_c, g mole
$M_1, M_2, M_3 \ldots$	: amounts of reactant A absorbed in

and M_m	half the film for contact times t_{c_1}, $(t_{c_1}+t_{c_2})$, $(t_{c_1}+t_{c_2}+t_{c_3}+ \ldots \ t_{c_m})$ respectively, g mole
M_∞	: total amount of reactant A, both free to diffuse and immobilized in half the liquid film of surface area s after infinite time, g mole
n	: integer number $0, 1, 2, 3, \ldots$
p	: pressure, dynes.cm^{-2}
P_n	: defined in eqn. (6)
P_1	: equals Q/V
P_2	: defined in text
$q_1, q_2, q_3 \ldots$ and q_m	: flow rates of liquid drained from subsections $1, 2, 3, \ldots$ and m respectively, cm^3/sec
Q	: flow rate of liquid entering the foam bed reactor, cm^3/sec
Q_G	: flow rate of gas through the reactor column, cm^3/sec
Q_1	: total flow rate of draining liquid, cm^3/sec
r_b	: average radius of a bubble, cm
r_c	: average radius of the reactor column, cm
s	: surface area of a liquid film, cm^2
t	: reaction time, sec
t_c	: contact time, sec
t_{c_m}	: contact time for any subsection m, sec
v	: dimensionless velocity in direction z'
$v_{z'}$	: velocity in the direction z', cms.sec^{-1}
v_{max}	: velocity in direction z' at the origin, cms.sec^{-1}
V	: storage volume, cm^3
$V_1, V_2, \ldots V_m$	: volumes of liquid films in subsections $1, 2, \ldots m$ respectively, cm^3
V_b	: volume of a bubble, cm^3
w_1, w_2, w_3	: dimensionless variables defined by eqns. (26)
x	: distance from the centre of the liquid film, cm
x'	: coordinate axis x', cms

x_1	: dimensionless co-ordinate axis x_1, eqn. (17)
y'	: coordinate axis y', cms
y_1	: dimensionless co-ordinate axis y_1, eqn. (17)
y_A	: stoichiometric factor, i.e. moles of liquid phase reactant B consumed per mole of gaseous reactant A
z	: distance from the foam-liquid pool interface, cm
z'	: co-ordinate axis z', cm

Greek symbols :

α	: inverse of dimensionless surface viscosity defined by eqn. (18b)
β	: dimensionless velocity coefficient defined in the text
ε	: volume fraction of liquid at a plane in the foam column
$\bar{\varepsilon}_m$	: average value of ε for subsection m
ε_F	: volume fraction of liquid in films
ε_{HPB}	: volume fraction of liquid in nearly horizontal Plateau borders
ε_{VPB}	: volume fraction of liquid in nearly vertical Plateau borders
ε_T	: volume fraction of liquid defined by eqn. (27)
λ	: pseudo-first order kinetic rate constant, $\sec^{-1}$
μ	: viscosity of the liquid, $gm.cm^{-1}.\sec^{-1}$
μ_s	: surface viscosity of the gas-liquid system, $gm.\sec^{-1}$
π	: equals 3.142
ϕ_i	: a polynomial function defined by eqn. (21)

11 October 1983

1. Plateau, J. A. F., *Statique des Liquids*, Paris, 1873. a) Vol. I, Cha. 5.
2. Boys, C. V., *Soap bubbles, their colours and forces which mold them*, Dover Publ. Inc. New York, 1959.
3. Isenberg, C. *The science of soap films and soap bubbles*, Tieto Ltd., Avon, England, 1978.
4. Mysels, K. J., Shinoda, K. and Frankel, S., *Soap films, Studies of their thinning and a Bibliography*, Pergamon Press, New York, 1959.
5. Adamson, A. W., *Physical chemistry of surfaces*, John Wiley and Sons, New York, 1976.
6. Osipow, L. I., *Surface chemistry – theory and industrial applications*, Reinhold Publ. Corp., New York, 1962.
7. Kitchner, J. A., *Foams and free liquid films*, in J. F. Danielli *et al.* (eds), Recent Progress in Surface Science', Vol. I, Academic Press, New York, 1964.
8. Bikerman, J. J., *Foams*, Springer-Verlag, Berlin, 1973. (a) p. 263, (b) p. 62, (c) p. 3.
9. Manegold, E., *Schaum*, (Heidelberg: Strassenbau, Chimie und Technik Verlag), 1953.
10. Schelukdo, A., *Advn. Colloid Interface Sci.*, 1967, **1**, 391.
11. Clunie, J. S., Goodman, J. F. and Ingram, B. T., *Thin liquid films*, in Matijevic (ed.), 'Surface and Colloid Science', Vol. 3, Wiley-Interscience, New York, 1971.
12. Vrij, A., Joosten, J. G. H. and Fijnaut, H. M., *Light scattering from thin liquid films* in 'Advances in Chemical Physics', (eds), I. Prigogine, and S. A. Rice, John Wiley and Sons, New York, 1981, Vol. 48, p. 329.
13. Ross, S., *Ind. Engg. Chem.* 1969, **61**, 48.
14. Perry, J. H. (ed.), 'Chemical Engineer's Handbook', 4th Edition, McGraw-Hill Book Co., New York, 1963, p. 18.
15. Perry, J. H. (ed.), 'Chemical Engineer's Handbook', 4th Edition, McGraw-Hill Book Co., New York, 1963, p. 11.
16. Mehta, V. D. and Sharma, M. M., *Chem. Engg. Sci.*, 1971, **26**, 461.
17. Scott, J. and Westland, R. J., *Chem. Abstr.* 1959, **53**: 624.
18. Rumyantsev, V. I. and Ezhov, V. E., *Chem. Abstr.* 1965, **63**: 16127.
19. Carpenter, W. L. and Geelman, S., *Tappi*, 1967, **50**, 83A.
20. Chittum, J. F., *U.S. Patent*: 1963, 3 108634; 3 108635.
21. Zhukov, K. S., *Chem. Abstr.* 1954, **48**, 2400.
22. Riggs, O. L., *U.S. Patent*: 1971, 3 578402.
23. Burton, J. S., *Foams and their clinical implications* in R. J. Akers, (ed.), 'Foams', Academic Press, New York, 1976, p. 127.
24. Pozin, M. E. *et al.*, *J. Appl. Chem. USSR*, 1955, **28**, 805, 877, 1071, 1187.
25. Jaheri, M. and Calvert, S., *J. Air Pollut. Control Assoc.*, 1968, **18**, 2110.

26. Bransky, D. W. and Diwoky, F. F., *Ref. Nat Gas Manuf.* 1960, **19**, 191.

27. Quon, J. E. and Phillips, R. A., paper presented at A.C.S. Meeting, 1964.

28. Zemskov, I. F., Stepano, A. F. and Meklikh, A. V., *J. Appl. Chem. USSR*, 1962, **35**, 11.

29. Braud, H. J. and Chesness, J. L., *Trans. ASAE*, 1970,, **13**, 1.

30. Butler, S. W., *Can. Patent* 1970, 850, 049 (Cl. 134–44).

31. Lemlich, R. (ed.), 'Adsorptive bubble separation techniques', Academic Press, 1972.

32. Rubin, E. and Gaden, E. L., in H. M. Schoen, (ed.), 'New Chemical Engineering Separation Techniques', Interscience, New York, 1962.

33. Lemlich, R. and Brumer, C. A., *Ind. Engg. Chem. Funda.* 1963, **2**, 297.

34. Leonard, R. A. and Lemlich, R., *A. I. Ch. E. J.* 1965, **11**, 18.

35. Shih, F. and Lemlich, R., *A. I. Ch. E. J.* 1967, **13**, 751.

36. Shih, F., and Lemlich, R., *Ind. Engg. Chem. Funda.*, 1971,, **10**, 254.

37. Haas, P. A. and Johnson, H. F., *A. I. Ch. E. J.*, 1965, **11**, 319.

38. Rubin, E., LaMantia, C. R., and Gaden, E. H. Jr., *Chem. Engg. Sci.* 1967, **22**, 1117.

39. Grieves, R. B., and Bhattacharya, D., *Can. J. Chem. Engg.* 1965, **41**, 286.

40. Grieves, R. B., *The Chem. Engg. J.* 1975, **9**, 93.

41. Goldberg, M. and Rubin, E., *Separation Sci.*, 1972, **7**, 51.

42. Lemlich, R., *Ind. Engg. Chem.*, October 1968, **60**, 16.

43. Metzner, A. B. and Brown, L. F., *Ind. Engg. Chem.* 1956, **48**, 2040.

44. Helsby, F. W. and Birt, D. C. P., *J. Appl. Chem.* 1955, **5**, 347.

45. Jackson, J., *Brit. Chem. Engg.* 1963, **8**, 319.

46. Zurakowski, S. and Glaser, R., *Chem. Abstr:* 1966, **64**: 3214.

47. Sieu, H. and Wasilewski, P., *Chem. Abstr:* 1972, **76**: 156568 w.

48. Punton, V. W., 'The use of an aqueous foam as a fibre-suspending medium in quality papermaking', in R. J. Akers, (ed.), 'Foams', Academic Press, New York, 1976, p. 179.

49. Bandyopadhyay, N., *Colourage* May 6, 1982, **29**, 3.

50. Ahmad, S. I., *Separation Sci.*, 1975, **10**, 649.

51. Desch, C. M., *Rec. Trav. Chim.*, 1923, **42**, 882.

52. Matzke, E. B., *Am. J. Bot.* 1946, **33**, 58.

53. Ho, G. E., Muller, R. L. and Prince, R. G. H., in 'Distillation – 1969', (London: The Instn. Chem. Engrs., 1969) p. 2.10.

54. Haas, P. A. and Johnson, H. F., *Ind. Engg. Chem. Fundam.*, 1967, **6**, 225.

55. Hartland, S. and Barber, A. D., *Trans. Instn. Chem. Engg.* 1974, **52**, 43.

56. Ho, G. E. and Prince, R. G. H., in *Proc. of Symp. on Bubbles and Foams – 1971*. (Nuremberg: Verein Deutscher Ingenieure).

57. Chanda, M. and Kumar, R., Paper presented at 23rd Annual Meeting, Ind. Inst. of Chem. Engrs. 1970.

58. Steiner, L., Hunkeler, R., and Hartland, S., *Trans. Insti. Chem. Engrs.* 1977, **55**, 153.

59. Miles, G. D., Shedlovsky, L. and Ross, J., *J. Phys. Chem.* 1945, **49**, 93.

60. Exerowa, D. Khristov, Khr. and Penev, I., in R. J. Akers, (ed.), 'Foams', p. 109, Academic Press, New York, 1976.

61. Khristov, Khr. I., Exerowa, D. R. and Krugljakov, P. M., *J. Colloid and Interf. Sci.*, 1981, **79**, 584.

62. Barber, A. D. and Hartland, S., *Trans. Instn. Chem. Engg.* 1975, **53**, 106.

63. Biswas, J. and Kumar, R., *Chem. Engg. Sci.* 1981, **36**, 1547.

64. Bhaskarwar, A. N. and Kumar, R. *Chem. Engg. Sci. (in Press)*

65. Desai, D. and Kumar, R. *Chem. Engg. Sci.* 1983, **38**, 1525.

66. Chandrasekaran, K. and Sharma, M. M., *Chem. Engg. Sci.* 1976, **31**, 861.

67. Danckwerts, P. V., *Gas-liquid Reactions,* McGraw-Hill Book Co., New York, 1970, p. 89.

68. Kumar, R., and Kuloor, N. R., *The formation of bubbles and drops* in T. B. Drew, *et al.* (eds), 'Advances in Chem. Engg.' Academic Press, New York, 1970, Vol. 8, p. 255.

69. Kumar, R., *Chem. Engg. Sci.*, 1971, **26**, 177.

70. Desai, D. and Kumar, R., *Chem. Engg. Sci.*, 1982, **37**, 1361.

71. Joly, M., *Surface viscosity* in J. F. Danielli, *et al.* (eds), 'Recent Progress in Surface Science', Vol. I, Academic Press, New York, 1964.

72. Joly, M., *Rheological Properties of Monomolecular films*, in E. Matijevic, (ed.), 'Surface and Colloid Science', Vol. 5, Wiley-Interscience, New York, 1972.